Neural Diffusion Distance for Image Segmentation

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1. Introduction

What is diffusion distance?

Diffusion Distance: pairwise distance of graph nodes considering global structure.

Graph: $G = (V, E)$; $V = (v_1, \cdots, v_N)$

Similarity matrix:

$$w_{ij} = \exp(-\mu ||\mathbf{f}_i - \mathbf{f}_j||_2^2), \text{ for } j \in S_N(i),$$

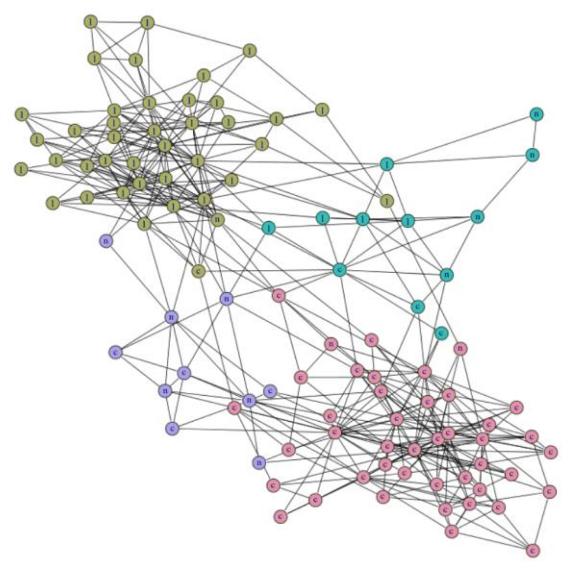
Transition matrix:

$$P = D^{-1}W, \text{ where } D = \mathrm{diag}(W\vec{1}).$$

Diffusion distance:

$$D_t(i,j) = \sum (p(k,t|i) - p(k,t|j))^2 \tilde{w}(k),$$
$$= \sum_{m=0}^{N-1} \lambda_m^{2t} (\Phi_m(i) - \Phi_m(j))^2,$$

$\{\lambda_m, \Phi_m\}_{m=0}^{N-1}$ are eigenvalues and eigenvectors of P.



A graph $G = (V, E)$

1. Introduction

Properties and challenges for diffusion distance

Properties of Diffusion Distance:

- Small if there are a large number of short paths connecting two points
- Decrease when t increases
- Varying t induces multi-scale distance

Applications:

- ☐ Spectral clustering
- ☐ Dimension reduction
- ☐ …..

Challenges:

- **How to design feature of each node?**
- **How to set an optimal t?**
- **Effectiveness in real applications?**

Coifman, R.R.; S. Lafon. Diffusion maps. *Applied and Computational Harmonic Analysis*. **21**: 5–30, 2016
Zeev Farbman, Raanan Fattal, and Dani Lischinski. Diffusion maps for edge-aware image editing. ACM Transactions on Graphics (TOG), 29(6):145, 2010.

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Our contributions:

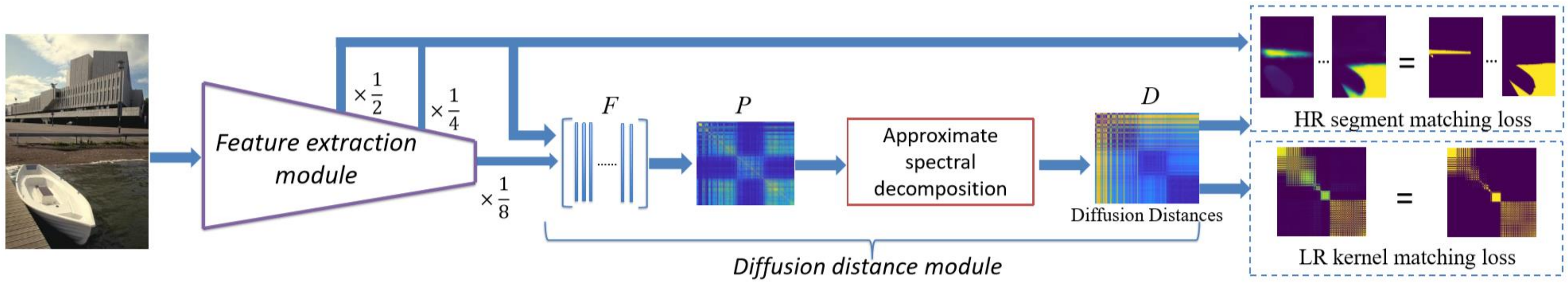
Neural diffusion distance + applications in image segmentation

- **Learn features and diffusion distance by end-to-end trainable network**
- **Produce high quality diffusion distance map on images**
- **A new image segmentation method using neural diffusion distance**
- **A new weakly supervised semantic segmentation method**

2. Neural Diffusion Distance

Definition of Neural Diffusion Distance

Neural Diffusion Distance (NDD): deep architecture simultaneously learns node features, optimal hyper-parameters for diffusion distance computation.



Approximate spectral decomposition (*differentiable*):

$$Z_{n+1} = PU_n,\ \{U_{n+1}, R_{n+1}\} = \mathrm{QR}(Z_{n+1}), n = 0, \cdots, T,$$

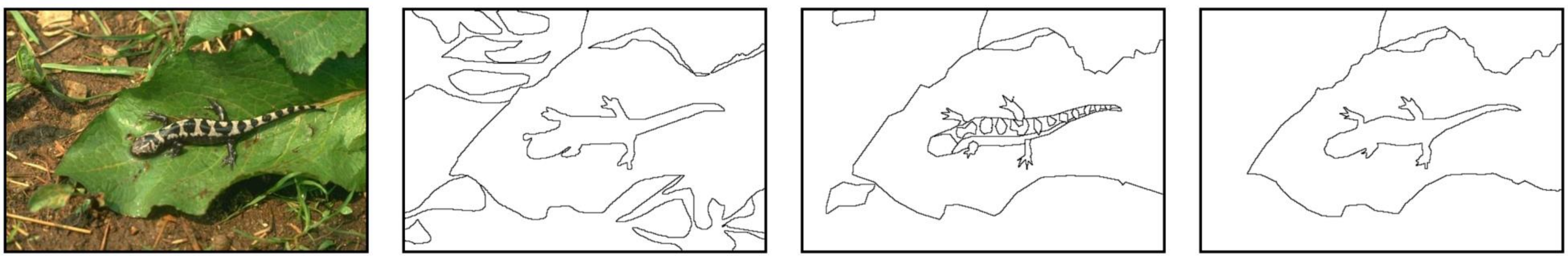
→ **Taken as network block**

2. Neural Diffusion Distance

Learning Neural Diffusion Distance

Proposition 1. Assume eigenvalues of P satisfy $\lambda_0 > \lambda_1 > \cdots > \lambda_{N_e-1} > \lambda_{N_e}$, and all leading principal sub-matrices of $\Gamma^T U_0$ (Γ is a matrix with columns $\Phi_1, \cdots, \Phi_{N_e}$) are non-singular, then columns of U_n converge to top N_e eigenvectors in linear rate of $(\max_{k\in[1,N_e]}\{|\lambda_k|/|\lambda_{k-1}|\})^{2t}$, and diagonal values of R_n converge to corresponding top N_e eigenvalues $\lambda_0^{2t}, \cdots, \lambda_{N_e-1}^{2t}$ in same rate.

How to train the network of *Neural Diffusion Distance (NDD)?*



Original Image | Subject 1 | Subject 2 | Subject 3

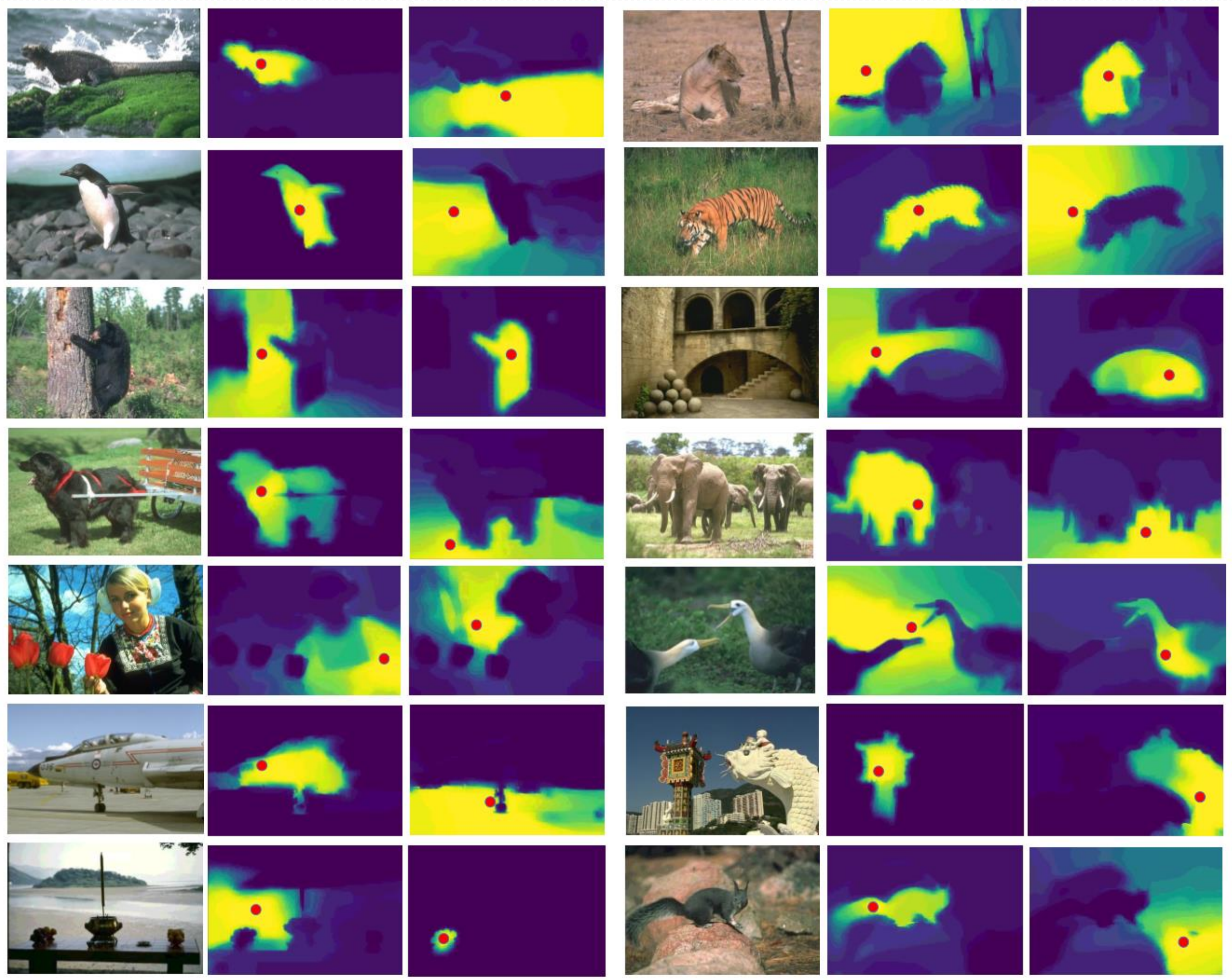
$$L_{hr}(K_D, \hat{K}_{gt}) = \sum_{i\in S} -\left\langle \hat{K}_D^i/||\hat{K}_D^i||, \hat{K}_{gt}^i/||\hat{K}_{gt}^i||\right\rangle,$$

$$L_{lr}(K_D, K_{gt}) = -\left\langle K_D/||K_D||_F, K_{gt}/||K_{gt}||_F\right\rangle.$$

Training set: BSD 500

2. Neural Diffusion Distance

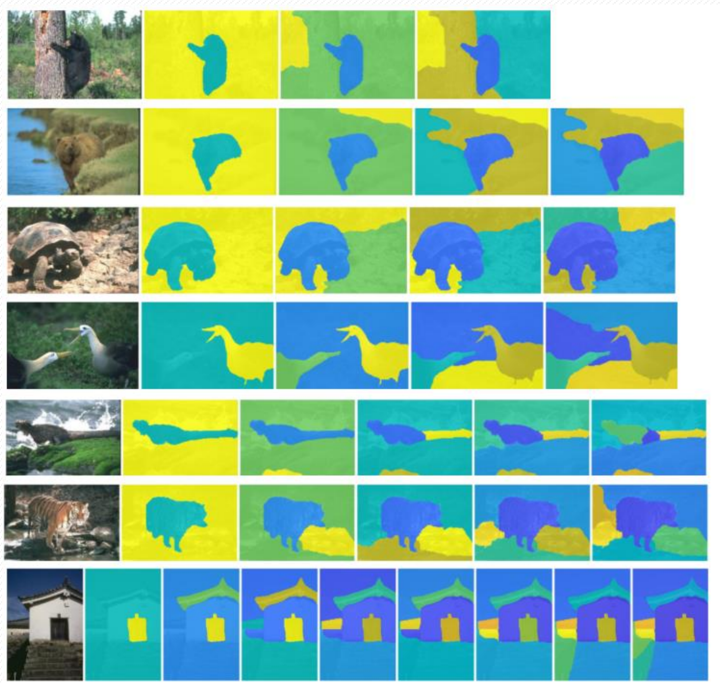
Examples of learned neural diffusion distance (shown by similarity)

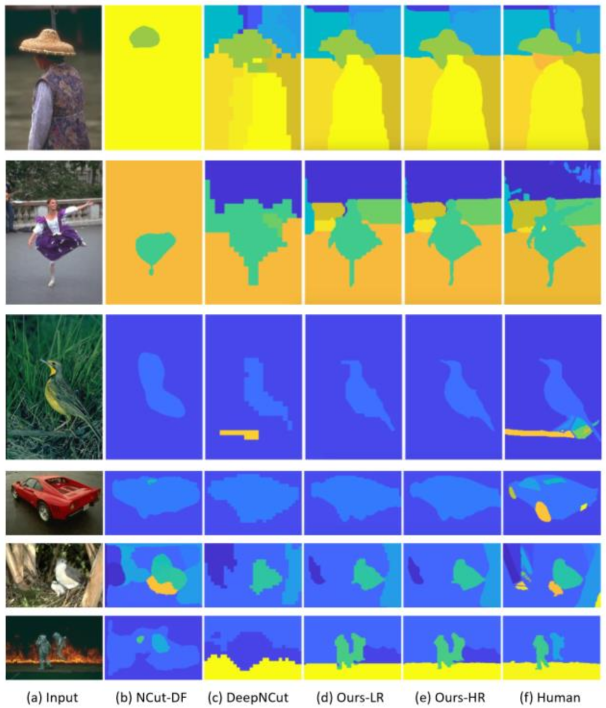


3. Applications and Experiments

Application1: Hierarchical image segmentation

Hierarchical image segmentation (kernel k-means with diffusion distance as kernel)





Methods	NCut [26]	NCut-DF	DeepNCut [13]	Ours-LR	Ours-HR
MAX	0.53	0.56	0.70	0.78	**0.80**
AVR	0.44	0.48	0.60	0.68	**0.69**

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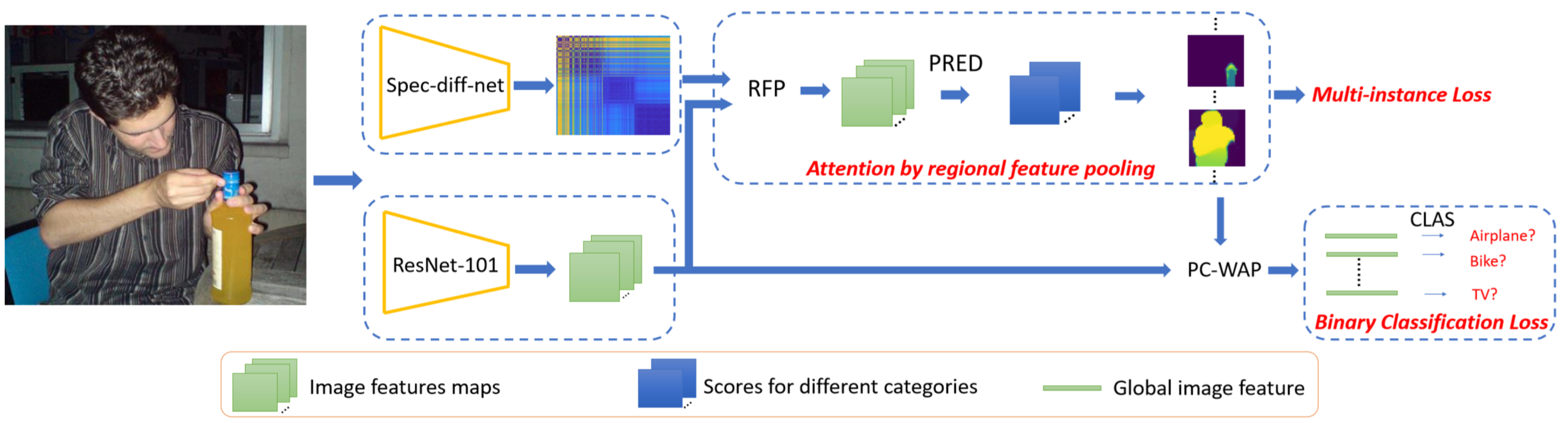
Application 2: Weakly supervised semantic segmentation



Pipeline for weakly supervised semantic segmentation

Basic idea:

- Joint segmentation and classification, and segmentation probability maps are taken as attention for feature aggregation for classification. Only classification label is provided.
- Diffusion distance is utilized for regional feature aggregation (RFP) in segmentation branch.

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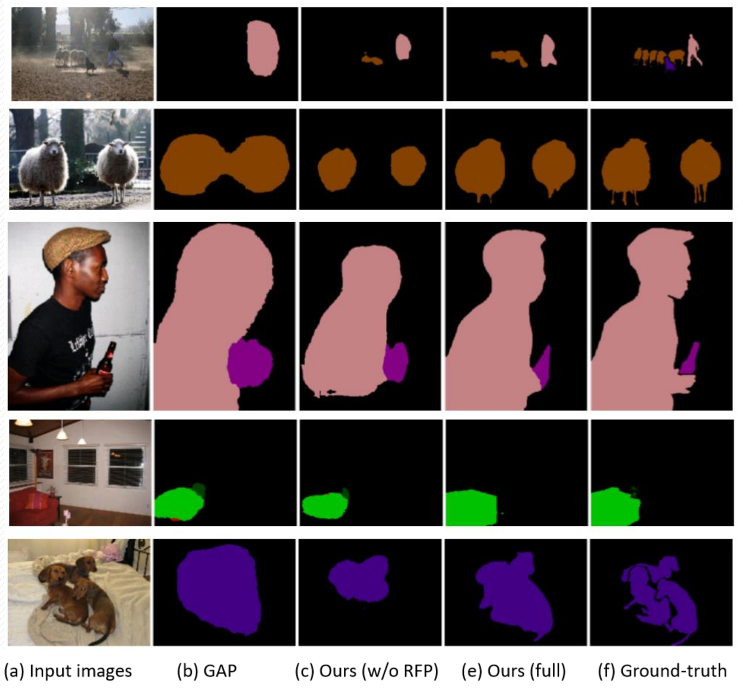
Application 2: Weakly supervised semantic segmentation



Methods	MIL [24]	Saliency [22]	RegGrow [12]	RandWalk [29]	AISI [7]	Ours
Val	42.0	55.7	59.0	59.5	63.6	**65.8**
Test	-	56.7	-	-	64.5	**66.3**

4. Summary and Conclusions

Summary & Conclusions

- Neural diffusion distance bridging gap between diffusion distance and deep learning approach
- Neural diffusion distance can help produce promising segmentation results
- The proposed methodology can be potentially applied to spectral clustering, 3D shape analysis, graph-based diffusion, semi-supervised learning, etc.

Thanks for your attention