Neural Diffusion Distance for Image Segmentation

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Diffusion Distance: pairwise distance of graph nodes considering global structure.

Graph: \( G = (V, E); V = (v_1, \ldots, v_N) \)

Similarity matrix:
\[
    w_{ij} = \exp(-\mu \| f_i - f_j \|_2^2), \text{ for } j \in S_N(i),
\]

Transition matrix:
\[
    P = D^{-1}W, \text{ where } D = \text{diag}(W1).
\]

Diffusion distance:
\[
    D_t(i, j) = \sum (p(k, t|i) - p(k, t|j))^2 \tilde{w}(k),
\]
\[
    = \sum_{m=0}^{N-1} \lambda_m^{2t} (\Phi_m(i) - \Phi_m(j))^2,
\]

\( \{\lambda_m, \Phi_m\}_{m=0}^{N-1} \) are eigenvalues and eigenvectors of \( P \).
1. Introduction

Properties and challenges for diffusion distance

Properties of Diffusion Distance:

- Small if there are a large number of short paths connecting two points
- Decrease when $t$ increases
- Varying $t$ induces multi-scale distance

Applications:

- Spectral clustering
- Dimension reduction
- ….

Challenges:

- How to design feature of each node?
- How to set an optimal $t$?
- Effectiveness in real applications?

1. Introduction

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Our contributions:

*Neural diffusion distance + applications in image segmentation*

- Learn features and diffusion distance by end-to-end trainable network
- Produce high quality diffusion distance map on images
- A new image segmentation method using neural diffusion distance
- A new weakly supervised semantic segmentation method
2. Neural Diffusion Distance

**Definition of Neural Diffusion Distance**

**Neural Diffusion Distance (NDD):** deep architecture simultaneously learns node features, optimal hyper-parameters for diffusion distance computation.

**Approximate spectral decomposition (differentiable):**

\[
Z_{n+1} = PU_n, \quad \{U_{n+1}, R_{n+1}\} = QR(Z_{n+1}), \quad n = 0, \ldots, T,
\]

Taken as network block
2. Neural Diffusion Distance

**Proposition 1.** Assume eigenvalues of $P$ satisfy $\lambda_0 > \lambda_1 > \cdots > \lambda_{N_e - 1} > \lambda_{N_e}$, and all leading principal sub-matrices of $\Gamma^T U_0$ ($\Gamma$ is a matrix with columns $\Phi_1, \cdots, \Phi_{N_e}$) are non-singular, then columns of $U_n$ converge to top $N_e$ eigenvectors in linear rate of $(\max_{k \in [1, N_e]} \{|\lambda_k|/|\lambda_{k-1}|\})^{2t}$, and diagonal values of $R_n$ converge to corresponding top $N_e$ eigenvalues $\lambda_0^{2t}, \cdots, \lambda_{N_e-1}^{2t}$ in same rate.

How to train the network of **Neural Diffusion Distance (NDD)**?

$$L_{hr}(K_D, \hat{K}_{gt}) = \sum_{i \in S} - \left\langle \hat{K}_D^i/\|\hat{K}_D^i\|, \hat{K}_{gt}^i/\|\hat{K}_{gt}^i\| \right\rangle,$$

$$L_{lr}(K_D, K_{gt}) = - \left\langle K_D/\|K_D\|_F, K_{gt}/\|K_{gt}\|_F \right\rangle.$$
2. Neural Diffusion Distance

Examples of learned neural diffusion distance (shown by similarity)
3. Applications and Experiments

Application 1: Hierarchical image segmentation

Hierarchical image segmentation (kernel k-means with diffusion distance as kernel)

![Image of segmentation results]

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3. Applications and Experiments

Application 2: Weakly supervised semantic segmentation

**Basic idea:**
- Joint segmentation and classification, and segmentation probability maps are taken as attention for feature aggregation for classification. Only classification label is provided.
- Diffusion distance is utilized for regional feature aggregation (RFP) in segmentation branch.

**Pipeline for weakly supervised semantic segmentation**
3. Applications and Experiments

Application 2: Weakly supervised semantic segmentation

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4. Summary and Conclusions

- Neural diffusion distance bridging gap between diffusion distance and deep learning approach
- Neural diffusion distance can help produce promising segmentation results
- The proposed methodology can be potentially applied to spectral clustering, 3D shape analysis, graph-based diffusion, semi-supervised learning, etc.
Thanks for your attention