A robust band-dependent variable step size NSAF algorithm against impulsive noises

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Abstract
Proposed is a new subband adaptive filter (SAF) algorithm by minimizing Huber’s cost function that is robust to impulsive noises. Generally, this algorithm works in the mode of the normalized SAF (NSAF) algorithm, while it behaves like the sign SAF (SSAF) algorithm only when the impulsive noises appear. To further improve the robustness of this algorithm against impulsive noises, the subband cutoff parameters are updated in a recursive way. Moreover, the proposed algorithm can be interpreted as a variable step size NSAF algorithm, thus it exhibits faster convergence rate and lower steady-state error than the NSAF. Simulation results, using different colored input signals in both impulsive and free-impulsive noise environments, show that the proposed algorithm works better than some existing algorithms.

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1. Introduction

In adaptive filtering algorithms, one of the simple algorithms is the normalized least mean square (NLMS), and it has been widely used in many practical areas such as system identification, channel estimation, and echo cancellation [1]. However, the NLMS algorithm converges slowly when the input signals are colored. To address this problem, an attractive approach is to use the subband adaptive filter (SAF), because it divides the colored input signal into almost mutually exclusive multiple subband signals and each subband signal is approximately white [2]. In [3], Lee and Gan proposed the normalized SAF (NSAF) algorithm, which converges faster than the NLMS for the colored input signals, due to the inherent decorrelating property of SAF [4]. Besides, the NSAF has almost the same computational complexity as the NLMS, especially for applications of long adaptive filter such as echo cancellation. However, like the conventional NLMS algorithm, the NSAF algorithm also exists a tradeoff between the convergence rate and steady-state error for the choice of the step size. Subsequently, to obtain both fast convergence rate and low steady-state error, several variable step size NSAF algorithms were developed [5,6]. Regrettably, these algorithms may diverge when the impulsive noises are present, since they stem from the $L_2$-norm optimization. Recently, to suppress the impulsive interferences, a sign SAF (SSAF) algorithm was presented in [7] by minimizing the $L_1$-norm of the a posteriori error vector of subband filter. Meanwhile, to lower the steady-state error of the SSAF, the authors proposed its variable regularization parameter version, called the VRP-SSAF. Following these works, to overcome a tradeoff problem of the SSAF between the convergence rate and steady-state error, many variable step size SSAF algorithms were proposed based on different principles of the step size update [8–10]. However, their convergence rates are not satisfactory. This paper proposes a new SAF algorithm based on Huber’s cost function by applying the gradient descent method. The proposed algorithm provides an automatic mechanism to switch between the NSAF and SSAF by iteratively updating...
the cutoff parameters. When the impulsive noises happen, this algorithm behaves like the SSAF which has good robustness. Moreover, the proposed algorithm can be considered as a variable step size version of the NSAF, thus it obtains fast convergence rate and low steady-state error in both impulsive and free-impulsive noise scenarios. According to the above properties, we term the proposed algorithm as the robust variable step size NSAF (RVSS-NSAF).

2. The conventional NSAF algorithm

Let us consider the output signal of the unknown system, i.e., the desired signal \( d(n) \)
\[
d(n) = u^T(n)w_o + \eta(n),
\]
where \((\cdot)^T\) indicates transpose, \(w_o\) is the unknown \(M\)-dimensional vector that we want to estimate, \(u(n) = [u(n), u(n - 1), ..., u(n - M + 1)]^T\) is the input signal vector, \(\eta(n)\) is the system noise. In many practical scenarios, \(\eta(n)\) may include the white Gaussian background noise \(\epsilon(n)\) and impulsive noise \(\delta(n)\). Fig. 1 shows the multiband-structure diagram of SAF, where \(w(k) = [w_1(k), w_2(k), ..., w_M(k)]^T\) denotes the tap-weight vector of adaptive filter and is also an estimate of \(w_o\) at time index \(k\), and \(N\) denotes number of subbands. The desired signal \(d(n)\) and input signal \(u(n)\) are partitioned into the subband signals \(d_i(n)\) and \(u_i(n)\) through the analysis filter bank \( \{H_i(z), i \in [0, N-1]\} \), respectively. Then, the subband signals \(y_i,k\) and \(d_i,k\) are generated by critically decimating \(y_i(n)\) and \(d_i(n)\), respectively. Here, \(n\) indicates the time index in the decimated sequences. It is easy to know that the \(i\)th subband error signal can be computed by
\[
e_{i,k}(k) = d_{i,k}(k) - y_{i,k}(k) = d_{i,k}(k) - u_i^T(k)w(k)
\]
where \(u_i,k = [u_i(kN), u_i(kN - 1), ..., u_i(kN - M + 1)]^T\), and \(d_{i,k}(k) = d_i(kN)\).

As described in [4], the original NSAF algorithm can be derived by minimizing the following cost function:
\[
J(k) = \frac{1}{2} \sum_{i=0}^{N-1} \left( \frac{e_{i,k}(k)}{||u_i(k)||} \right)^2
\]
where \(||\cdot||\) denotes the \(L_2\)-norm of vector. Applying the gradient descent method, the NSAF algorithm for updating the tap-weight vector is expressed as
\[
w(k + 1) = w(k) + \mu \sum_{i=0}^{N-1} \frac{e_{i,k}(k)u_i(k)}{||u_i(k)||^2}
\]
where \(\mu\) is the step-size.

3. Proposed RVSS-NSAF algorithm

3.1. Derivation of RVSS-NSAF

Applying Huber’s cost function [11] into the SAF, a novel cost function is obtained as
\[
J(k) = \sum_{i=0}^{N-1} \rho\left( \frac{e_{i,k}(k)}{||u_i(k)||} \right)
\]
with a piecewise function given by
\[
\rho(\delta) = \begin{cases} \theta^2/2, & |\delta| \leq \sqrt{\delta}, \\ \sqrt{\delta}|\delta| - \delta/2, & |\delta| > \sqrt{\delta}, \end{cases}
\]
where \(\delta > 0\) is the cutoff parameter.
Taking the gradient of $f(k)$ with respect to $w(k)$, we have

$$\nabla f(k) = \frac{\partial f(k)}{\partial w(k)} = -\sum_{i=0}^{N-1} q_i \left( \frac{e_i(k)}{\|u_i(k)\|} \right) \frac{u_i(k)}{\|u_i(k)\|}$$

(7)

where

$$q_i = \frac{\partial q_i}{\partial \theta} = \begin{cases} \theta, & |b| \leq \sqrt{\delta} \\ \sqrt{\delta} \text{sign}(\theta), & |b| > \sqrt{\delta} \end{cases}$$

(8)

and $\text{sign}(\cdot)$ indicates the sign function. Using the gradient descent method, the resulting update is

$$w(k+1) = w(k) - \nabla f(k)$$

$$= w(k) + \sum_{i=0}^{N-1} \frac{u_i(k) e_i(k)}{\|u_i(k)\|^2} \left( \frac{e_i(k)}{\|u_i(k)\|} \right)$$

$$= w(k) + \left\{ \begin{array}{ll} \frac{\sum_{i=0}^{N-1} u_i(k) e_i(k) e_i(k)}{\sum_{i=0}^{N-1} u_i(k)^2} \leq \sqrt{\delta} & \\
\sqrt{\delta} \frac{\sum_{i=0}^{N-1} u_i(k) \text{sign}(e_i(k)) e_i(k)}{\|u_i(k)\|} > \sqrt{\delta} & \end{array} \right.$$

(9)

Rewrite (9) in a compact way as

$$w(k+1) = w(k) + \sum_{i=0}^{N-1} \min \left\{ \frac{\|e_i(k)\|}{\|u_i(k)\|}, \sqrt{\delta} \right\} u_i(k) \text{sign}(e_i(k)) e_i(k)$$

(10)

Now, a key remaining problem is how to choose the cutoff parameter $\delta$, because it affects the algorithm performances in terms of the convergence rate and noise sensitivity. Namely, if the value of $\delta$ is too small, the algorithm will have strong capability of suppressing outliers (e.g., the impulsive noises), but its convergence rate may be very slow. Conversely, if the value of $\delta$ is too large, a large perturbation sample could damage the stability of the algorithm. Therefore, we wish to dynamically adjust $\delta$ throughout the overall adaptation process without requiring any priori information of noises. To this end, we use a time-varying cutoff parameter $\delta(k)$ for each subband instead of the fixed one in (10), and then (10) becomes

$$w(k+1) = w(k) + \sum_{i=0}^{N-1} \min \left\{ \frac{\|e_i(k)\|}{\|u_i(k)\|}, \sqrt{\delta(k)} \right\} u_i(k) \text{sign}(e_i(k)) e_i(k)$$

(11)

As in [12,13], a recursive way for computing $\delta(k)$ is

$$\delta(k+1) = \lambda \delta(k) + (1-\lambda) \min \left\{ \frac{\|e_i(k)\|}{\|u_i(k)\|}, \sqrt{\delta(k)} \right\}$$

(12)

where $\lambda$ is the forgetting factor which is determined by $\lambda = 1 - N/\kappa M$ with a typical range of $1 \leq \kappa \leq 6$ [5]. The initial values of $\delta(k)$ for $i \in [0,N-1]$ can be set by $\delta(0) = \sigma_0^2/\kappa M$, where $\sigma_0^2$ represents the power of the input signal and the desired signal, respectively.

3.2. Discussions

**Remark 1.** Clearly, (11) reveals how the proposed algorithm works. In general case, i.e., in the absence of impulsive noise, the algorithm operates in the mode of the NSAF with unit step size. However, when the impulsive noises appear, the magnitude of the normalized subband error $|e_i(k)/\|u_i(k)\||$, will be larger than $\sqrt{\delta(k)}$. At this moment, the algorithm behaves like the SSAF with a step size $\sqrt{\delta(k)}$ which has good robustness against impulsive noises. Hence, the proposed algorithm can prevent the influence of impulsive noises on the tap-weight vector update by automatically switching between the NSAF and SSAF algorithms. From another perspective, the proposed algorithm...
Table 2
Computational complexity of various SAF algorithms for each fullband input sample. The integer \( L \) denotes the length of the prototype filter of the filter bank.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Multiplications</th>
<th>Additions</th>
<th>Square-roots</th>
<th>Divisions</th>
<th>Comparisons</th>
<th>Absolutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSAF</td>
<td>( 3M + 3NL + 1 )</td>
<td>( 3M + 3N(L-1) )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>VSSM-NSAF [5]</td>
<td>( 6M + 3NL + 8 )</td>
<td>( 5M + 3N(L-1) + 3 )</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SSAF</td>
<td>( M + 2M/N + 3NL )</td>
<td>( 2M + M/N + 3NL/2N-1 )</td>
<td>( 1/N )</td>
<td>( 1/N )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>VSS-SSAF [9]</td>
<td>( M + (2M+3)/N + 3NL )</td>
<td>( 2M + (M+2)/N + 3NL/2N )</td>
<td>( 1/N )</td>
<td>( 2/N )</td>
<td>( 2/N )</td>
<td>1</td>
</tr>
<tr>
<td>BDVSS-SSAF [10]</td>
<td>( 3M + 3NL + 5 + 9/M )</td>
<td>( 3M + 3N(L-1) + 2/M + 4 )</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Proposed RVSS-NSAF</td>
<td>( 3M + 3NL + 5 )</td>
<td>( 3M + 3N(L-1) + 1 )</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 2. The NMSD curves of various SAF algorithms in the absence of impulsive noise (i.e., \( \Pr = 0 \)): (a) SNR = 30 dB and (b) SNR = 20 dB. VSSM-NSAF: \( \kappa = 6 \); VSS-SSAF: \( K = 1 \), \( \kappa = 5 \); BDVSS-SSAF: \( \kappa = 2 \); RVSS-NSAF: \( \kappa = 1 \), \( V_T = 3M \). can also be considered as a NSAF algorithm with band-dependent variable step size (VSS) \( \mu_i(k) \), i.e.,

\[
\mu_i(k) = \min \left\{ \frac{\sqrt{\delta_i(k)}}{\| \mathbf{u}_i(k) \|} \right\} \quad \text{for} \quad i \in [0, N-1]. \tag{13}
\]

This property indicates that the proposed RVSS-NSAF algorithm provides faster convergence rate and lower steady-state error as compared to the original NSAF. As a result, the RVSS-NSAF algorithm is summarized in Table 1.

**Remark 2.** Based on the commonly used diagonal assumption in the previous SAFs, i.e., \( \mathbf{u}_i^T(k) \mathbf{u}_i(k) \approx 0 \), \( i \neq l \) [3], from (11) we can get

\[
\| \mathbf{w}(k+1) - \mathbf{w}(k) \|^2 = \sum_{i=0}^{N-1} \min \left\{ \left\| \frac{\delta_i(k)}{\| \mathbf{u}_i(k) \|} \right\| \sqrt{\delta_i(k)} \right\}^2 = \sum_{i=0}^{N-1} \delta_i(k) \tag{14}
\]

This relation illustrates that the square norm of the update of the filter tap-weight vector is always less than or equal to the amount \( \sum_{i=0}^{N-1} \delta_i(k) \) at each iteration, regardless of any noise perturbations (it may be the impulsive noises or not). Moreover, it has been found in [12] that the sequences of \( \delta_i(k) \) computed by (12) is strictly decreasing as the iteration goes on, and having \( \lim_{k \to \infty} \delta_i(k) = 0 \) regardless of \( \delta_i(0) \) and \( \lambda \). Therefore, we can obtain \( \lim_{k \to \infty} \| \mathbf{w}(k+1) - \mathbf{w}(k) \|^2 = 0 \) which guarantees the stability of the proposed algorithm, even if there are the impulsive noises.

**Remark 3.** The monotonically decreasing behavior of the sequences \( \delta_i(k) \) towards zero ensures the robustness of the RVSS-NSAF algorithm against the impulsive noises, but also loses its tracking capability when the unknown system suddenly changes. To remedy this shortcoming, a reset method similar to Yoo et al. [10] and Rey Vega et al. [12] is applied into the RVSS-NSAF, as shown in Table 1. Its role is to reset the values of \( \delta_i(k), i \in [0, N-1] \) by \( \delta_i(k+1) = \delta_i(0) \) when the change of the unknown system is detected.

**Remark 4.** Table 2 shows the computational complexities of the proposed RVSS-NSAF and some existing SAF algorithms in terms of the total number of additions, multiplications, divisions, square-roots, comparisons and absolutions for each fullband input sample. In this table, the computational cost of the reset algorithms applied into the BDVSS-SSAF and RVSS-NSAF algorithms are not been included, because they are performed only every \( V_T \) input samples. As compared to the NSA, the additional computation of the RVSS-NSAF stems from (12) and (13). Since the calculation of \( \| \mathbf{u}_i(k) \| \) is already available from the NSA in (4), the RVSS-NSAF requires additional 4 multiplications, 1 division, 2 comparisons, 1 square-root, 1 addition, and 1 absolutions for each fullband input sample. Fortunately, this moderate increase is worth, since the RVSS-NSAF enhances significantly performances.
In our simulations, the unknown \( \mathbf{w}_0 \) is the impulse response of an acoustic echo path with \( M = 512 \) taps; and a cosine modulated filter bank with number of subbands \( N = 4 \) is used in all the SAF algorithms [2]. The background noise \( \nu(n) \) is white Gaussian with a signal-to-noise ratio (SNR) of 30 dB, unless otherwise specified. The normalized mean square deviation (NMSD), 
\[
10\log_{10}\left( \frac{\| \mathbf{w}_0 - \mathbf{C}_0 \mathbf{w}(k) \|_2^2}{\| \mathbf{w}_0 \|_2^2} \right) \text{ (dB)},
\]
is used as a measure of the algorithm performance. All results are obtained by averaging over 50 independent runs, except for Section 4.2 (single realization).

### 4. Simulation results

In our simulations, the unknown \( \mathbf{w}_0 \) is the impulse response of an acoustic echo path with \( M = 512 \) taps; and a cosine modulated filter bank with number of subbands \( N = 4 \) is used in all the SAF algorithms [2]. The background noise \( \nu(n) \) is white Gaussian with a signal-to-noise ratio (SNR) of 30 dB, unless otherwise specified. The normalized mean square deviation (NMSD), 
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\]
is used as a measure of the algorithm performance. All results are obtained by averaging over 50 independent runs, except for Section 4.2 (single realization).

#### 4.1. System identification under impulsive noises

In this section, the colored input signal \( u(n) \) is a first-order autoregression (AR(1)) process with a pole at 0.95. The impulsive noise \( \vartheta(n) \) is usually modeled as a Bernoulli–Gaussian (BG) process, i.e., \( \vartheta(n) = c(n)A(n) \) [8–14], where \( c(n) \) is a Bernoulli process with the probability density function defined by \( p\{c(n) = 1\} = Pr \) and \( p\{c(n) = 0\} = 1 - Pr \) (with \( Pr \) denoting the probability of the occurrence of the impulsive noises), and \( A(n) \) is a white Gaussian process with zero mean and variance \( \sigma_A^2 = 1000E[\mathbf{u}^T(n)\mathbf{w}_0]^2 \) [12,13]. Also, to assess the tracking capability of the algorithms, the unknown vector is changed from \( \mathbf{w}_0 \) to \( -\mathbf{w}_0 \) at the \( 4.5 \times 10^5 \)th input samples.

Fig. 3. The NMSD curves of various SAF algorithms in the presence of impulsive noises. SNR = 30 dB: (a) \( Pr = 0.001 \) and (b) \( Pr = 0.01 \). The choices of the parameters for these VSS algorithms are the same as Fig. 2.

First, we compare the performances of the proposed RVSS-NSAF algorithm with that of the NSAF, SSAF, VSSM-NSAF [5], VSS-SSAF [9] and BDVSS-SSAF [10] algorithms in the absence of impulsive noise (i.e., \( Pr = 0 \)), as shown in Fig. 2. To obtain a fair comparison, the parameters of these VSS algorithms are set to their recommended values in the literatures [5,9,10]. As expected, the steady-state NMSDs of all the algorithms increase as the SNR decreases. As can also be seen, the SSAF has slower convergence rate than the NSAF when there is no impulsive noise, since it only uses the sign information of the subband error signals to update the tap-weight vector. In contrast to the fixed step size algorithms (i.e., the NSAF and SSAF), their VSS versions obtain a good compromise between fast convergence rate and low steady-state error. Importantly, among these VSS algorithms, the RVSS-NSAF provides the fastest convergence rate, best tracking capability and lowest steady-state error.

Then, Fig. 4 examines the performances of the RVSS-NSAF in the situation that the impulsive noises are present (e.g., \( Pr = 0.001 \) and 0.01) by comparing it with the SSAF, VSSM-NSAF [5], VSS-SSAF [9] and BDVSS-SSAF [10] algorithms. Here, the result of the NSAF is omitted, due to the fact that it is divergent for the impulsive noises. Similar to the NSAF, the VSSM-NSAF is also divergent, while the remaining algorithms can prevent the impulsive noises from damaging the adaptive process since they are based on the \( L_1 \)-norm minimization. Compared with the SSAF, these VSS algorithms (i.e., the VSS-SSAF, BDVSS-SSAF and RVSS-NSAF) improve the convergence rate and reduce the steady-state error. Moreover, both the RVSS-NSAF and BDVSS-SSAF algorithms maintain good tracking capability for abrupt change of the unknown system, since they use the reset methods for the cutoff parameters \( \delta_i(k) \) and step sizes, respectively. Interestingly, the proposed RVSS-NSAF works better than the BDVSS-SSAF.
4.2. Acoustic echo cancellation with double-talk

Here, the performances of the RVSS-NSAF are evaluated in an acoustic echo cancellation application with double-talk. The main goal of echo cancellation is to identify the echo path $w_o$, where the far-end input $u(n)$ is a speech signal. And, the near-end signal that can be considered as the impulsive interference $\theta(n)$ is also a speech signal. In Fig. 5, all algorithms except the VSSM-NSAF are robust against double-talk happened in the period with sample index of $[0.9, 1.3] \times 10^4$. Also, the proposed RVSS-NSAF performs better than other SSAF-type algorithms.

5. Conclusions

In this study, we derived the RVSS-NSAF algorithm by introducing Huber’s cost function into the SAF. This cost function makes the RVSS-NSAF have two operating modes, i.e., the NSA and SSAF. However, the latter is operated only when the impulsive noise appears to achieve good robustness. Moreover, owing to the VSS property of the RVSS-NSAF, it obtains fast convergence rate and low steady-state error. Simulation results have demonstrated that the proposed algorithm is superior to other existing algorithms in both impulsive and free-impulsive noise cases.

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