Abstract—This paper investigates the disturbance observer based composite fuzzy control for a class of uncertain nonlinear systems. With fuzzy logic system (FLS) approximating the unknown nonlinearities, composite learning is constructed on the basis of a serial-parallel identifier. By introducing the intermediate signal, the disturbance observer is developed to provide efficient learning of the compounded disturbance which includes the effect of time-varying disturbance, fuzzy approximation error and unknown dead zone. Based on the disturbance estimation and fuzzy approximation, the adaptive fuzzy controller is synthesized with novel updating law. The stability analysis of the closed-loop system is rigorously established via Lyapunov approach. The performance of the proposed controller is verified via simulation that faster convergence and higher precision are obtained.

Index Terms—disturbance observer, serial-parallel identification model, composite fuzzy control, output feedback control

I. INTRODUCTION

Adaptive control [1], [2], [3], [4], [5], [6] is gaining great attention in recent several decades. Different nonlinear systems are analyzed such as strict-feedback systems [7], [8], [9], pure-feedback systems [10], [11], stochastic nonlinear systems [12], [13], switching systems and multi-agent systems [14], [15], [16], [17], [18]. Since the mathematical model of the controlled system is not available exactly in reality, universal function approximation based design is widely studied. Furthermore, to deal with the specific nonlinear systems, by considering the system structure, neural networks (NNs) [7], [19], [20], [21], [22], [23] or FLS [24], [25], [26], [27] are combined with other techniques, such as back-stepping scheme or dynamic surface design.

For the design of intelligent control in continuous-time and discrete-time, with different motivation, it can be classified into two groups roughly. One is referred as indirect control [28]. For indirect control, the idea is to approximate each part of the nonlinear functions. Then if the intelligent system is working efficiently, the perfect feedback is achieved. In [4], the design is proposed with adaptive dead-zone to improve robustness under the approximation errors and disturbances. The other way is the direct design [29]. In the method, the desired control input is presented according to the dynamics. By analyzing the inputs of the functions, the intelligent system is used to approximate the ideal controller. Also much concern has been on dealing with the specific constraint. For example for the problem of actuator saturation [30], [31], [32], [33], backlash [34] or dead-zone input [35], [36], [37], the intelligent control is developed with auxiliary compensation since the presence of such nonlinearity may deteriorate the system performances or even result in the instability. For the output constraint, the Barrier Lyapunov Function based adaptive controller design [38] is widely employed. Also the prescribed tracking performance can be achieved by error transformation [39]. Furthermore, some other interesting topics can be found as global tracking [40], [41], reinforcement learning [10], [42] or adaptive dynamic programming [43], [44], [45], [46], [47], self-organizing neural control [48], [49], etc.

Though there is great development of intelligent control, in the above-mentioned methods, the tuning of intelligent system is directly proposed with tracking error which means the controller is aiming at achieving asymptotic stability and tracking. However, the motivation of using intelligent system to approximate nonlinear functions was ignored, thus the accuracy of the desired identified models is out of attention. As a result, the tracking performance is difficult to evaluate since mostly only the tracking error can be guaranteed to be bounded. With the discussed concern, an interesting work is presented in [50] that the modeling error is included in fuzzy weight updating law. However, the method requires the $n$th derivative to be known thus it cannot be used in real application. To deal with this problem, the design with low-pass filter is proposed in [51], [52], [53] and better tracking performance is achieved with faster adaptation. Furthermore, in [8], the composite neural control is proposed for the strict-feedback systems by introducing new modeling error and more accurate tracking is achieved.

Time-varying unknown disturbance [54], [55] exists in many physical systems. For example, the ground effect and crosswind are time-varying for unmanned flight system while there exist wind, waves, and ocean currents for the surface vehicles. To tackle the disturbance, one way is to construct the robust item to make the system stable. However, it is some kind of energy waste and the tracking precision cannot be expected.
As a result, there came disturbance observer based control, whose idea is like some kind of feedforward design that the effect of disturbance can be estimated and further used for compensation. In [56], the 3-DOF helicopter dynamics with disturbance is controlled with neural design. In [57], the output feedback design with nonlinear disturbance observer and state observer is provided for strict-feedback systems with time-varying disturbance using back-stepping technique. In [58], the composite learning based control is proposed with fuzzy approximation and disturbance observer.

Motivated by the above-mentioned discussion, in this paper, a class of uncertain nonlinear systems with time-varying disturbance and unknown dead zone is studied. Different from previous design toward asymptotic stability, the novel serial-parallel identification model is developed to construct the modeling error. Furthermore, by incorporating the fuzzy approximation, the novel disturbance observer is designed to compensate the effect of approximation error, unknown dead zone and time-varying disturbance.

The organization of the paper is as follows. Section II presents the class of nonlinear systems with unknown dead zone. In Section III, the adaptive fuzzy control is designed with disturbance observer while the composite learning is constructed. In Section IV, simulation is presented to show the effectiveness of the proposed approach. The final conclusion is included in Section V.

II. PROBLEM DESCRIPTION

A. System Dynamics

Let’s consider the nonlinear systems with the controllability canonical form

\[
\begin{align*}
\dot{x}_i &= x_{i+1}, \quad i = 1, 2, \ldots, n - 1 \\
x_n &= f(x_n) + g(x_n)u + b(t) + d(t) \\
y(t) &= x_1
\end{align*}
\]

where \(x_n = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n\), are system states, \(y \in \mathbb{R}\) is system output, \(u \in \mathbb{R}\) is the control input, \(f \in \mathbb{R}\) and \(g \in \mathbb{R}\) are unknown functions, and \(d(t) \in \mathbb{R}\) denotes the unknown external disturbance.

The input nonlinearity is described as

\[
\begin{align*}
u(t) &= \left\{
\begin{array}{ll}
\lambda(u_v - b_r) & \text{if } u_v \geq b_r \\
0 & \text{if } -b_l < u_v < b_r \\
\lambda(u_v + b_l) & \text{if } u_v \leq -b_l
\end{array}\right.
\end{align*}
\]

where \(\lambda, b_l, b_r\) are unknown constants.

Signal \(u_v\) is applied as

\[
\begin{align*}
u_v &= \text{sat}(u_c) \\
&= \left\{
\begin{array}{ll}
u_c & \text{if } |u_c| \leq u_{\text{max}} \\
\text{sign}(u_c) u_{\text{max}} & \text{if } |u_c| > u_{\text{max}}
\end{array}\right.
\end{align*}
\]

where \(\text{sign}(\cdot)\) is the sign function, \(u_{\text{max}}\) is the bound of \(u_v\).

Then we have

\[
u(t) = \lambda u_v + \psi(t)
\]

where

\[
\psi(t) = \left\{
\begin{array}{ll}
-\lambda b_r & \text{if } u_v \geq b_r \\
-\lambda u_v & \text{if } -b_l < u_v < b_r \\
\lambda b_l & \text{if } u_v \leq -b_l
\end{array}\right.
\]

System can be further written as

\[
\begin{align*}
\dot{x}_i &= x_{i+1}, \quad i = 1, 2, \ldots, n - 1 \\
x_n &= f(x_n) + g_{\text{new}}(x_n)u_v + b(t) + d(t) \\
y(t) &= x_1
\end{align*}
\]

where \(g_{\text{new}}(x_n) = \lambda g(x_n), b(t) = g(x_n)\psi(t)\).

Remark 1: For the disturbance, it is bounded and its change rate is bounded such that \(|d(t)| \leq \bar{d}, |d'(t)| \leq \bar{d}_v\). The constants \(\bar{d}\) and \(\bar{d}_v\) will be used in the stability analysis and don’t need to be known for the controller design.

B. Fuzzy Logic System

To approximate the nonlinear function \(f(X_m)\), the applied FLS [24] is considered

\[
\hat{f}(X_m) = \hat{w}^T \Theta(X_m)
\]

where \(X_m \in D_X \subset \mathbb{R}^M\) is the input vector of the FLS, \(D_X = \Omega_{x_1} \times \Omega_{x_2} \times \cdots \times \Omega_{x_M}\) is a fuzzy approximation region, \(\hat{w} \in \mathbb{R}^N\) is the FLS output, \(\Theta(\cdot) : \mathbb{R}^M \rightarrow \mathbb{R}^N\) is a nonlinear vector function of the inputs and the elements in \(\Theta(X_m)\) are given by

\[
\theta_i(X_m) = \frac{\prod_{l=1}^{M} \mu_{A^1_l}(x_i)}{\sum_{l=1}^{L} \prod_{l=1}^{M} \mu_{A^1_l}(x_i)}
\]

where \(A^1_l\) is the fuzzy partitions on \(\Omega_x\), \(\mu_{A^1_l}\) is the membership function of \(A^1_l\), \(\mu_{A^1_l}(x_i) = \exp \left[\frac{\left(\frac{x_i - a_{l_1}^1}{\sqrt{2b_l^1}}\right)^2}{\|w^*_l\|^2} \right]\) with \(a_{l_1}^1\) and \(b_l^1\) as the centers and widths of \(\mu_{A^1_l}\) respectively.

Given real continuous function \(f\) on a compact set \(\Omega_{x_n} \subset \mathbb{R}^M\) and an arbitrary \(\varepsilon_M > 0\), there exists FLS in the form of (9) and an optimal parameter vector \(w^*\) such that

\[
f(X_m) = w^*\Theta(X_m) + e
\]

\[
\sup_{X_m \in \Omega_{x_n}} |e| < \varepsilon
\]

Assumption 1: For \(\bar{x}_n \in D_n\) with \(D_n\) as fuzzy approximation region, the following inequalities exist

\[
\hat{f}(\bar{x}_n|w^*_F) \leq f(\bar{x}_n) \leq \hat{f}(\bar{x}_n|w^*_{L})
\]

\[
\hat{g}_{\text{new}}(\bar{x}_n|w^*_G) \leq g_{\text{new}}(\bar{x}_n) \leq \hat{g}_{\text{new}}(\bar{x}_n|w^*_{L})
\]

where \(w^*_F, w^*_G, w^*_{L}, w^*_{U}\) are the bounds of fuzzy weight vector.

Remark 2: In case that \(g(\bar{x}_n) = 1\), \(g_{\text{new}}\) is \(\lambda\), some prior knowledge \(\lambda_0\) is assumed and the other part is considered as disturbance [57]. In [56], the effect of \(\lambda\) is considered as disturbance while to avoid the singularity, the signal is written into the form of \((g_N^2 + \varphi)\) where \(g_N\) is the known part of \(g_{\text{new}}\) and \(\varphi\) is positive constant. In this paper, neither \(\lambda\) nor \(g(\bar{x}_n)\) is ignored and we try to construct the indirect controller by approximating \(g_{\text{new}}\) using the bound \(w^*_G\) and \(w^*_{G}\).

C. Control Goal

Given a bounded reference trajectory \(\bar{y}_d(t) = [y_d(t), \dot{y}_d(t), \cdots, \dot{y}_d^{(n-1)}(t)]^T\), define the tracking error \(e = e(t) \triangleq y_d(t) - y(t) \in \mathbb{R}^n\), \(\bar{e} = \bar{e}(t) \triangleq [\varepsilon, \varepsilon, \cdots, \varepsilon^{(n-1)}]^T \in \mathbb{R}^n\).

Considering the unknown dead zone and time-varying disturbance, this paper is aiming at synthesizing the signal \(u_v\) for system (1) with composite learning and disturbance estimation such that the tracking error is bounded.
III. COMPOSITE LEARNING BASED FUZZY CONTROL

To control system (1), the main challenge is on the unknown nonlinear functions \( f, g \), the dead zone information \( \lambda, b(t) \) and time-varying disturbance \( d(t) \) which make the dynamic inversion algorithm \( u^* = \frac{1}{q} \left[ -f + y_d^n(t) + \bar{k}e \right] \) cannot be implemented where \( \bar{k} = [k_1, \ldots, k_2, k_1]^T \in \mathbb{R}^n \) is chosen such that the polynomial \( s^n + k_1s^{n-1} + \cdots + k_n = 0 \) is Hurwitz.

Now in this paper, for unknown dynamics, the FLS will be employed for approximation while for time-varying disturbance, the disturbance observer will be constructed. We will construct the adaptive fuzzy controller step by step.

Define \( F(\bar{x}_n) = L_FB(\bar{x}_n) \) and \( G(\bar{x}_n) = L_Gg_{new}(\bar{x}_n) \) where \( L_F \) and \( L_G \) are positive design parameters. The dynamics of \( \bar{x}_n \) can be written as

\[
\dot{x}_n = L_F^{-1}F(\bar{x}_n) + L_G^{-1}G(\bar{x}_n)u + b(t) + d(t)
\]

where \( w_F, w_G \) are the optimal FLS weights vectors approximating \( F, G \) separately, and \( |\hat{e}_F(\bar{x}_n)| \leq \varepsilon_F, |\hat{e}_G(\bar{x}_n)| \leq \varepsilon_G \).

Furthermore, using FLS to approximate \( F, G \), we have

\[
\hat{F} = w_F^T\theta_F(\bar{x}_n)
\]

\[
\hat{G} = w_G^T\theta_G(\bar{x}_n)
\]

Define \( \hat{w}_F = w_F - \hat{w}_F, \hat{w}_G = w_G - \hat{w}_G, \Delta u = u_{\text{new}} - u_c \). For sake of simplicity, denote \( \theta_F = \theta_F(\bar{x}_n), \theta_G = \theta_G(\bar{x}_n) \). Then the derivative of \( x_n \) can be written as

\[
\dot{x}_n = L_F^{-1}\hat{F} + L_G^{-1}\hat{G}u_c + L_F^{-1}\left[ w_F^T\theta_F - \hat{F} \right]
\]

\[
+ \left[ w_G^T\theta_G - \hat{G} \right] u_c
\]

\[
+ L_F\varepsilon_F + L_G\varepsilon_Gu_c + b(t) + d(t)
\]

\[
= L_F^{-1}\hat{F} + L_G^{-1}\hat{G}u_c + L_F^{-1}\hat{w}_F^T\theta_F
\]

\[
+ L_G^{-1}\hat{w}_G^T\theta_Gu_c + D(t)
\]

where \( D(t) = L_F\varepsilon_F + L_G\varepsilon_Gu_c + b(t) + d(t) + L_G^{-1}\hat{G}\Delta u \).

Remark 3: The compounded disturbance \( D(t) \) is composed of the effect of fuzzy approximation error, saturation effect, dead zone and time-varying disturbance. There exists unknown positive constant \( D_{\text{max}} \) such that \( |D(t)| \leq D_{\text{max}} \).

Remark 4: [59] Following the boundedness of

\[
|\hat{e}_F| \leq \varepsilon_F, |\hat{e}_G| \leq \varepsilon_G, |u_c| \leq v_{\text{max}}, |u_c| \leq v_{\text{max}}
\]

and from Assumption 1, we know there exists an unknown positive constant \( v_D \) such that \( |\hat{D}| \leq v_D \).

Assumption 2: [60] The parameter vectors \( \hat{w}_F \) and \( \hat{w}_G \) belong to compact \( \Omega_F \) and \( \Omega_G \), respectively, which are defined as \( \Omega_F = \{ \hat{w}_F : ||\hat{w}_F|| \leq M_F \} \) and \( \Omega_G = \{ \hat{w}_G : ||\hat{w}_G|| \leq M_G \} \), where \( M_F, M_G \in \mathbb{R}^+ \) are user-defined finite constants.

Define

\[
\Lambda = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -k_n & -k_{n-1} & \cdots & -k_1 \end{bmatrix}
\]

and \( B = [0, \ldots, 0, 1]^T \).

The indirect composite fuzzy controller is proposed as

\[
u_c = u_n + u_h\]

where \( u_n \) is the adaptive item given as

\[
u_n = \frac{1}{L_G^{-1}G}[-L_F^{-1}\dot{F} + y_d^n(t) + \bar{k}e - \dot{D}(t)]
\]

with \( \dot{D}(t) \) as the disturbance estimation and \( u_h \) is the \( H^\infty \) term given as

\[
u_h = -\frac{1}{L_G^{-1}G}u_{h0}
\]

with \( u_{h0} = -\frac{1}{r}B^TP\bar{c}, r \) is positive constant and matrix \( P = P^T \geq 0 \) is the solution of the following Riccati-like equation

\[
\Lambda^TP + PA + Q - \frac{2}{r}PPBB^T + \frac{1}{r^2}PB^TPB^TP = 0
\]

where \( Q \) is an arbitrary \( n \times n \) positive-definite symmetric matrix to be given.

Remark 5: For the controller design in (18), the compensation is composed of two parts where FLS is used to approximate the unknown system functions while disturbance observer is designed to estimate the compounded part of approximation error and time-varying disturbance.

From (17), it is known that

\[
y_d^n(t) = -\bar{k}e + \hat{D} + L_G^{-1}\hat{G}u_c + L_F^{-1}\hat{F} + u_{h0}
\]

Subtracting (14) from (21), the error dynamics is obtained as

\[
\dot{\hat{e}} = \Lambda\bar{e} + B\left[ \hat{D} + L_G^{-1}\hat{G}u_c + L_F^{-1}\hat{F} - f - gu - b - d \right]
\]

\[
= \Lambda\bar{e} - B\left[ L_F^{-1}\hat{w}_F^T\theta_F + L_G^{-1}\hat{w}_G^T\theta_Gu_c + \hat{D} - u_{h0} \right]
\]

(22)

where \( \hat{D} = D - \hat{D} \).

Remark 6: Through selection of \( \bar{k} \) to make the polynomial \( s^n + k_1s^{n-1} + \cdots + k_n = 0 \) Hurwitz, it is known that \( \Lambda \) is a stable matrix and there exists a unique positive-definite symmetric \( n \times n \) matrix \( P \) so that the Lyapunov equation \( \Lambda^TP + PA = -Q \) holds.

In previous design, the fuzzy adaptive law is constructed with tracking error while the motivation of using FLS to approximate the nonlinear function is not seriously considered. To include the factor of modeling error, the following signal is defined

\[
\kappa_F = x^{(n-1)} - x_n
\]

by introducing the following serial-parallel identification model with a low-pass filter

\[
\begin{align*}
\dot{x}_{i-1} &= \hat{x}_i, i = 2, \cdots, n \\
\dot{x}_n &= -\alpha_F\kappa_F + L_G^{-1}\hat{G}u_c + L_F^{-1}\hat{F} + \hat{D}
\end{align*}
\]
where $\hat{x}_i$ is the estimation of $x_i$, $\alpha_F \in \mathbb{R}$ is positive design constant.

Remark 7: Since the nonlinear functions $f$ and $g$ are unknown, the information $x_n$ is not available. Thus we try to construct new information to include the performance how the FLS is working as approximator. The design in (24) is different from the previous design in [52] since the disturbance observer is designed as (28) with updating algorithm (29). Then the semiglobally uniformly ultimate boundedness of all the closed-loop system signals included in (32) can be guaranteed.

Proof: Consider the following Lyapunov function candidate

$$V = V_1 + V_2 + V_3 + V_4 + V_5$$

where $V_1 = \frac{1}{2} \bar{e}^T \bar{P} \bar{e}$, $V_2 = \frac{1}{2} \tilde{D}^2$, $V_3 = \frac{1}{2} \gamma_F \tilde{w}^T \tilde{w}_F$, $V_4 = \frac{1}{2} \gamma_G \tilde{w}^T \tilde{w}_G$, $V_5 = \frac{\gamma_x}{2} \tilde{P}^T \tilde{P} \hat{w}_F$.

For the first item, $V_1$ is calculated as

$$V_1 = \frac{1}{2} (\hat{e}^T \bar{P} \hat{e} + \hat{e}^T \hat{e}^T P \hat{e})$$

For the second item, $V_2$ is calculated as

$$V_2 = \hat{D} (\hat{D}^T \hat{D} + \tilde{D} \tilde{D}^T + \tilde{D} \tilde{D} + \bar{D} \tilde{D})$$

For the third item, $V_3$ is calculated as

$$V_3 = \frac{1}{\gamma_F} \tilde{w}^T \tilde{w}^T \tilde{w}_F$$

For the fourth item, $V_4$ is calculated as

$$V_4 = \frac{1}{\gamma_G} \tilde{w}^T \tilde{w}_G$$

For the fifth item, $V_5$ is calculated as

$$V_5 = \gamma_x \tilde{w}^T \tilde{w}_F$$

Finally the derivative of $V$ is obtained as

$$\dot{V} = -\frac{1}{2} \varepsilon^T \bar{P} \bar{e} - \frac{1}{2} \tilde{D} \tilde{D} - \frac{1}{2} \tilde{D} \tilde{D}^T - \frac{1}{2} \varepsilon^T \bar{P} \bar{e} - \frac{1}{2} \varepsilon^T \bar{P} \bar{e}$$

The following inequalities exist

$$\tilde{D} \tilde{D} \leq \frac{1}{2} \tilde{D}^2 + \frac{1}{2} \tilde{D}^2$$

$$-\tilde{D} \tilde{D} \tilde{D} \tilde{D} \leq \frac{1}{2} \varepsilon^T \bar{P} \bar{e} + \frac{1}{2} \varepsilon^T \bar{P} \bar{e}$$

$$\tilde{D} \tilde{D} \tilde{D} \tilde{D} \tilde{D} \leq \frac{1}{2} \varepsilon^T \bar{P} \bar{e} + \frac{1}{2} \varepsilon^T \bar{P} \bar{e}$$

where $v_1$ and $v_2$ are scalars.
Furthermore, for \( i = F, G \), we know
\[
\ddot{w}_i = \dot{w}_i (w_i - \hat{w}_i) \leq - \frac{1}{2} e^T \dot{Q} e + \frac{1}{2} \ddot{D}^2 + \frac{1}{2} L_0 L_F^{-1} v_1 \chi_F^2 \ddot{D}^2.
\]
Then the derivative of \( V \) is calculated as
\[
\dot{V} \leq - \frac{1}{2} e^T \dot{Q} e + \frac{1}{2} \ddot{D}^2 + \frac{1}{2} L_0 L_F^{-1} v_1 \chi_F^2 \ddot{D}^2
+ \frac{1}{2} L_0 L_F^{-1} \| \ddot{w}_F \|_2^2 + \frac{1}{2} L_0 L_G^{-1} v_2 \chi_G^2 u_\text{max}^2 \ddot{D}^2
- \frac{1}{2} \delta_F \| \ddot{w}_F \|_2^2 + \frac{1}{2} \delta_G \| \ddot{w}_G \|_2^2 - \gamma_x \alpha_F \kappa_F^2
- \frac{1}{2} e^T \dot{Q} e - \left( \frac{1}{2} \delta_G - \frac{1}{2} L_0 L_G^{-1} \right) \| \ddot{w}_G \|_2^2
- \left( \frac{1}{2} \delta_F - \frac{1}{2} L_0 L_F^{-1} \right) \| \ddot{w}_G \|_2^2 - \gamma_x \alpha_F \kappa_F^2 + C_0
\]
where \( C_0 = \frac{1}{2} \delta_D + \frac{1}{2} \delta_F \| \ddot{w}_F \|_2^2 + \frac{1}{2} \delta_G \| \ddot{w}_G \|_2^2 \).

By selecting parameters \( L_0, L_F, L_G, \delta_F, \delta_G \) to satisfy
\[
K_D = L_0 - \frac{1}{2} L_0 L_F^{-1} v_1 \chi_F^2 - \frac{1}{2} L_0 L_G^{-1} v_2 \chi_G^2 u_\text{max}^2 > 0
\]
\[
K_F = \frac{1}{2} \delta_F - \frac{1}{2} L_0 L_F^{-1} > 0
\]
\[
K_G = \frac{1}{2} \delta_G - \frac{1}{2} L_0 L_G^{-1} > 0
\]
It can be obtained as
\[
\dot{V} \leq - \varrho_0 V + C_0
\]
where \( \varrho_0 \) is given by
\[
\varrho_0 := \min \left[ \frac{\lambda_{\text{min}}(Q)}{\lambda_{\text{max}}(P)}, 2K_D, 2\gamma_F K_F, 2\gamma_G K_G, 2\alpha_f \right]
\]
Then from (40), we know
\[
0 \leq V \leq C_0 \frac{V}{\varrho_0} + \left[ V(0) - C_0 \frac{V}{\varrho_0} \right] e^{-\varrho_0 t}
\]
From (42), it is concluded that as \( t \to \infty \), \( V \to \frac{C}{\varrho_0} \). It may directly show that the signals included in (32) are ultimately uniformly bounded. This concludes the proof.  

IV. SIMULATION

The nonlinear system [28, [58] is used to verify the effectiveness of the proposed approach.
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(\bar{x}_1) + g(\bar{x}_2)u + d(t)
\end{align*}
\]
where
\[
f = \frac{g_c \sin x_1 - m_l x_1 x_2 \cos x_1 \sin x_1}{m_c + m}, \quad g = \frac{m_l x_1}{m_c + m}
\]
\[
l = \frac{4}{3} m \cos^2 x_1
\]
\( b_c = b_l = 0.2, \lambda = 0.8 \).

The composite learning controller in this paper is marked as CL-AFC while the design in [28] is denoted as \( H_\infty \) FC. Let \( k_1 = 2, k_2 = 1, \alpha_F = 10, \gamma_F = 30, \gamma_G = 10 \). Select
\[
Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \quad \rho = 0.1, \quad r = 2\rho^2, \quad P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}
\]
For the FLS, the initial value of \( \ddot{w}_F(0) \) is set as zero, \( a_i, i = 1, 2 \) is evenly distributed in \([ -\pi/6, \pi/6 ] \) with \( L_N = 25 \).
similar trend of total control input, which means \(u_h(t)\) is the main contributor of \(u_a(t)\). Different from \(H^{\infty}\) FC, the adaptive fuzzy controller \(u_a(t)\) of CL-AFC plays the leading role which confirms the idea that composite learning can greatly enhance the adaptation of the controller. Furthermore, in Fig.5, the curve of disturbance observer exhibits sinusoidal functions while the disturbance is \(d(t) = 5.5 \sin t\) which indicates the disturbance observer can efficiently estimate the external information.

**A. Period Signal Tracking**

The reference signals are given as \(y_d = \pi/4 \sin t\), \(x_{2d} = \dot{y}_d = \pi/4 \cos t\) and there is no limit on the magnitude of the control signal which is supposed to be bounded. The disturbance is set as \(d(t) = 5.5 \sin t\). The parameters \(L_0, L_F, L_G\) are set 20, the bound of FLS update is \(w_G^L = 6\) and \(w_G^U = 40\) while the bound is with \(w_F^L = -20\) and \(w_F^U = 20\), the value of \(\hat{w}_G(0)\) is set as 10 and \(\gamma_G = 30\).

The simulation results are depicted in Figs.1-5. The system tracking performance of \(y_d\) and \(\dot{y}_d\) are shown in Fig.1 and Fig.2. It is obvious that CL-AFC is more closely following the reference signal. It can be observed from Fig.3 that CL-AFC is with better tracking precision since \(e_1\) and \(\dot{e}_1\) are smaller than \(H^{\infty}\) FC. In Fig.4, the signal \(u_a(t)\) is with similar trend for the two methods. However, it is interesting to find that \(u_a\) of \(H^{\infty}\) FC is with small value while the robust item \(u_h(t)\) is with

**B. Steady Signal Tracking**

The reference signal is the step command with magnitude \(\pi/6\) and the limit is set as \(u_{max} = 6.5\). The parameters are selected as \(L_0 = 50, L_F = 80, L_G = 100\), the bound of FLS update is \(w_G^L = 30\) and \(w_G^U = 200\) while the bound is with \(w_F^L = -80\) and \(w_F^U = 80\), the value of \(\hat{w}_G(0)\) is set as 50 and \(\gamma_G = 150\).
The time-varying disturbance is set as
\[
d(t) = \begin{cases} 
0.1 \sin(t) & \text{if } t \leq 20 \\
-5 & \text{if } t > 20
\end{cases}
\] (44)

To obtain the reference signals \( y_d, \dot{y}_d, \ddot{y}_d \), the following filter is used [58]:
\[
\frac{\ddot{y}_d}{\ddot{y}_c} = \frac{w_n^2}{s^2 + 2 \varepsilon_c w_n s + w_n^2}
\] (45)

where \( w_n = 0.9, \varepsilon_c = 0.9 \).

The simulation results are shown in Figs.6-11. From the system tracking depicted in Figs.6-9, CL-AFC achieves faster convergence with higher precision in the beginning of reference tracking and sudden disturbance. The signal \( u_v(t) \) is shown in Fig.10 and there exists saturation at the beginning of reference tracking. It is noticed that \( \hat{D}(t) \) in Fig.11 is the estimation of compound disturbance \( D(t) \) and thus not equal to \( d(t) \). However, it still follows the change trend of \( d(t) \) as...
long as $d(t)$ is the dominant factor of $D(t)$. Through the simulation, it is obvious that the proposed CL-AFC is with better performance.

V. CONCLUSION AND FUTURE WORK

A novel composite learning based adaptive fuzzy control is studied for a class of nonlinear systems with unknown dead zone. To deal with the unknown dynamics, the FLS is employed to approximate the nonlinear functions $F$ and $G$. With the fuzzy approximation, the disturbance observer is constructed by introducing new signal $z$. By constructing the serial-parallel identification model, the signal $\hat{x}^{(n-1)}$ is generated to obtain the filtered modeling error. With the composite fuzzy updating law and the disturbance observer, the closed-loop system stability is analyzed via Lyapunov approach. Two kinds of reference signals are used for simulation and the result demonstrates that the proposed CL-AFC can adapt to disturbance and exhibits faster convergence and higher tracking precision. For future work, it is interesting to see how the method performs on flight dynamics [61], marine vehicles [62], [63] or real robotic platforms [64].

REFERENCES


