Experimental study of elastic constant softening prior to stress-induced martensitic transformation

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There has been a large body of experimental evidence for the existence of precursor lattice softening prior to temperature-induced martensitic transformation, but little is known about whether such softening effect also exists prior to stress-induced martensitic transformation. Recent molecular dynamics simulations suggested such a possibility, but direct experimental evidence remains unclear. In the present study, we established a simple experimental method that can measure elastic constant C' from zero stress up to the critical stress prior to stress-induced martensitic transformation. By applying this method to a Cu-Al-Ni single crystal, we found C' of $(101)[\bar{1}01]$ shear mode does soften with increasing applied stress along the [001] direction, until a stress-induced martensitic transformation occurs at a critical stress. This is direct evidence for the existence of lattice softening prior to a stress-induced martensitic transformation. Our result completes an important conclusion that the softening of elastic constant C' is a common feature prior to both temperature- and stress-induced martensitic transformations. Furthermore, we also analyzed the dependence of C' on uniaxial tensile stress through a theoretical model and obtained similar result as that of our experiment.

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I. INTRODUCTION

Martensitic transformation (MT) is a diffusionless solid-solid phase transition with a predominantly shear distortion of the lattice. Martensitic transformation in shape memory alloys has attracted much attention in the past decades since it is the origin of the unique shape memory effect and superelasticity.[1–3]

MT can be induced either by mere cooling (temperature-induced MT) or by stress (stress-induced MT). Prior to the temperature-induced MT, numerous studies have shown that the lattice of the parent phase softens with approaching the transformation temperature,[1,2,4–10] as manifested by the softening of the elastic constant C' $[\equiv(C_{11}-C_{12})/2]$ (Refs. 1–9) and TA2 phonon.[1,2,10–12] These precursory lattice softening effects provide important clues on the physical nature of martensitic transformation. On the other hand, stress can also induce martensitic transformation, but little is known about whether the elastic constant softening also exists prior to the stress-induced transition. Previous investigations[1,13–19] on third order elastic constants and Gruneisen parameters indicate that the vibrational anharmonicity of crystal increases near the transformation temperature. Such information seems to indirectly suggest a possible lattice softening with increasing stress and/or pressure, but the anharmonicity effect was measured only at low stress level. Neutron inelastic scattering studies[11,12] seem to indiate that stress can cause the softening of the whole TA2 branch of the phonon dispersion curve, including the long wavelength phonons; this can also be viewed as an indirect indication of the stress-induced elastic softening. Thus, it is unclear how the stiffness (i.e., elastic modulus) of the lattice changes from zero stress up to the critical stress for stress-induced MT.

Very recently, molecular dynamics simulations[20] of stress-induced MT suggested the existence of the softening of elastic constant C' from zero stress up to the critical stress for stress-induced MT. It was found that under [100]-[001] biaxial tension compression, C' softens in $\{110\}[1\bar{1}0]$ and $\{101\}[\bar{1}01]$ shear modes. Such a result was further supported by an analytical theory based on Landau free-energy expansion.[20] Despite the simulation result and theoretical prediction, direct experimental evidence for the C' softening prior to the stress-induced MT is still lacking.

The purpose of the present work is to provide direct experimental evidence for the C' softening from zero stress up to the critical stress for stress-induced martentsitic transformation. For this purpose, a critical step is to determine C' at arbitrary bias stress, in particular, at high stress level close to the critical stress for the MT. In the present study, we establish an experimental method by using a simple tensile test that can measure C' of one of the $\{110\}\langle 1\bar{1}0\rangle$ shear modes at arbitrary finite uniaxial stress. By applying this method to a Cu-Al-Ni single crystal, the dependence of C' of $(101)[\bar{1}01]$ shear mode on uniaxial tensile stress along the [001] direction was measured. Our experimental results clearly show the softening behavior of elastic constant C' from zero stress up to the critical stress. Combining the generally observed C' softening prior to temperature-induced MT, the present finding completes an important conclusion that the softening of elastic constant C' is a common feature prior to both temperature- and stress-induced MTs. Finally, by means of a Landau-type analytical model, we also analyzed the variation of C' under uniaxial tension, and the analytical result is consistent with our experimental results.

This paper is organized as follows: In Sec. II, we introduce the principle of measuring elastic constant C' at finite uniaxial stress using a different method. Section III describes the sample and experimental setup. Section IV presents the experiment results. Then, in Sec. V, with a theoretical model,

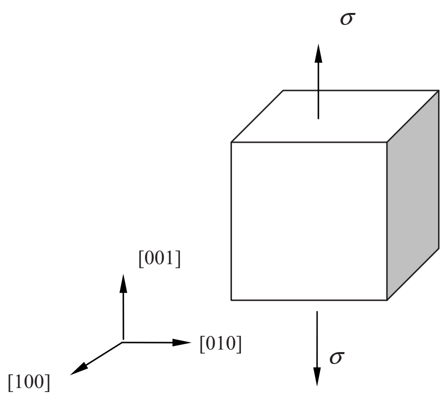

FIG. 1. Geometrical configuration of the uniaxial stress σ relative to orientation of the crystal under test.

we prove the softening of elastic constant C' prior to uniaxial-tension-induced MT and a comparison to the experimental results is made. Finally, the conclusions are made in Sec. VI.

II. ANALYTICAL FORMULA OF ELASTIC CONSTANT C' UNDER UNIAXIAL STRESS

The physical meaning of C' is the elastic modulus of a $\{110\}\langle 1\bar{1}0\rangle$ shear mode, which can be defined as $C' \equiv (C_{11} - C_{12})/2$ for a cubic crystal. Accordingly, elastic constant C' at a finite uniaxial stress σ can be expressed by

$$C'(\sigma) \equiv \frac{C_{11}^{\sigma} - C_{12}^{\sigma}}{2}, \tag{1}$$

where C_{11}^{σ} and C_{12}^{σ} are the C_{11} and C_{12} at given uniaxial stress σ, respectively.

Equation (1) indicates that, elastic constant C' at finite uniaxial stress σ can be obtained by measuring C_{11}^{σ} and C_{12}^{σ}. In the following, we show if using a single crystal sample and performing uniaxial tensile testing along its [001] direction, C_{11}^{σ} and C_{12}^{σ} can be obtained by measuring the variation of uniaxial strain and transverse strain as a function of the uniaxial tensile stress σ.

According to the generalized Hooke's law,[21] a small change of stress $d\sigma_{ij}^{\sigma}$ at a given uniaxial stress σ will produce a small change of strain $d\varepsilon_{kl}^{\sigma}$, and their relationship can be expressed as

$$d\sigma_{ij}^{\sigma} = C_{ijkl}^{\sigma} d\varepsilon_{kl}^{\sigma}, \tag{2}$$

where C_{ijkl}^{σ} are the elastic modulus tensor of the crystal at a given stress σ.

For a cubic single crystal, if the uniaxial tensile stresses σ is applied along the [001] direction, as shown in Fig. 1, Eq. (2) can be simplified as follows:

$$0 = C_{11}^{\sigma} d\varepsilon_{11}^{\sigma} + C_{12}^{\sigma} d\varepsilon_{22}^{\sigma} + C_{12}^{\sigma} d\varepsilon_{33}^{\sigma}, \tag{3a}$$

$$0 = C_{12}^{\sigma} d\varepsilon_{11}^{\sigma} + C_{11}^{\sigma} d\varepsilon_{22}^{\sigma} + C_{12}^{\sigma} d\varepsilon_{33}^{\sigma}, \tag{3b}$$

$$d\sigma^{\sigma} = C_{12}^{\sigma} d\varepsilon_{11}^{\sigma} + C_{12}^{\sigma} d\varepsilon_{22}^{\sigma} + C_{11}^{\sigma} d\varepsilon_{33}^{\sigma}, \tag{3c}$$

where C_{11}^{σ}, C_{12}^{σ}, and C_{44}^{σ} are the three independent elastic stiffness of the cubic crystal at given bias stress σ; $d\sigma^{\sigma}$ and $d\varepsilon_{33}^{\sigma}$ are the infinitesimal increment in uniaxial stress and in uniaxial strain in the [001] direction at the given uniaxial stress σ, respectively; $d\varepsilon_{11}^{\sigma}$ and $d\varepsilon_{22}^{\sigma}$ are the corresponding infinitesimal increment in transverse strain in the [100] and [010] directions, respectively.

The solution of Eqs. (3a)–(3c) gives the implicit expressions of C_{11}^{σ} and C_{12}^{σ} as

$$\frac{d\sigma^{\sigma}}{d\varepsilon_{33}^{\sigma}} = \frac{(C_{11}^{\sigma})^2 + C_{11}^{\sigma}C_{12}^{\sigma} - 2(C_{12}^{\sigma})^2}{C_{11}^{\sigma} + C_{12}^{\sigma}}, \tag{4}$$

$$\frac{d\varepsilon_{11}^{\sigma}}{d\varepsilon_{33}^{\sigma}} = -\frac{C_{12}^{\sigma}}{C_{11}^{\sigma} + C_{12}^{\sigma}}. \tag{5}$$

By combination of Eq. (1), (4), and (5), we obtain the expression of C' at given uniaxial stress σ as

$$C'(\sigma) = \frac{A + 2AB}{2(1 + B - 2B^2)}, \tag{6}$$

where

$$A = \frac{d\sigma^{\sigma}}{d\varepsilon_{33}^{\sigma}}, \quad B = \frac{d\varepsilon_{11}^{\sigma}}{d\varepsilon_{33}^{\sigma}}.$$

Equation (6) indicates that if the value of $d\sigma^{\sigma}/d\varepsilon_{33}^{\sigma}$ and $d\varepsilon_{11}^{\sigma}/d\varepsilon_{33}^{\sigma}$ at bias stress σ can be measured, we can obtain C' at this bias stress. It should be noted that for the uniaxial tension configuration shown in Fig. 1, the measured $C'(\sigma)$ is the modulus of $(101)[\bar{1}01]$ shear mode, that is, $C'_{101}(\sigma)$.

To calculate the value of $C'_{101}(\sigma)$ at a given uniaxial stress σ, we first apply the bias uniaxial stress σ along the [001] direction, followed by superimposing an infinitesimal increment of uniaxial stress $d\sigma^{\sigma}$ in the same direction and then measure the resultant small change in uniaxial strain $d\varepsilon_{33}^{\sigma}$ and transverse strain $d\varepsilon_{11}^{\sigma}$. This can be readily done by measuring σ vs ε_{33} and ε_{33} vs ε_{11} curves, and then calculating their differential curves $d\sigma^{\sigma}/d\varepsilon_{33}^{\sigma}$ vs σ and $d\varepsilon_{11}^{\sigma}/d\varepsilon_{33}^{\sigma}$ vs $\varepsilon_{33}^{\sigma}$ curves (the latter can be converted into $d\varepsilon_{11}^{\sigma}/d\varepsilon_{33}^{\sigma}$ vs σ curve). Substituting the $d\sigma^{\sigma}/d\varepsilon_{33}^{\sigma}$ and $d\varepsilon_{11}^{\sigma}/d\varepsilon_{33}^{\sigma}$ values into Eq. (6) gives the $C'_{101}(\sigma)$. This is the principle of our experimental method for measuring C' [$C'_{101}(\sigma)$ here] as a function of uniaxial stress σ.

III. EXPERIMENT

In the present study, we used the above-mentioned uniaxial tension test to measure the stress-induced elastic constant C' softening prior to a stress-induced MT for a single crystal of Cu-13.8Al-4Ni (wt. %) alloy. The single crystal was spark cut to a plate tensile specimen (Fig. 2), whose surface normals are in three the $\langle 100\rangle$ directions with an uncertainty less than 2°. Differential scanning calorimetry (DSC) was used to determine the transformation temperatures of the tensile specimen, and the DSC sample was spark cut from the same single crystal.

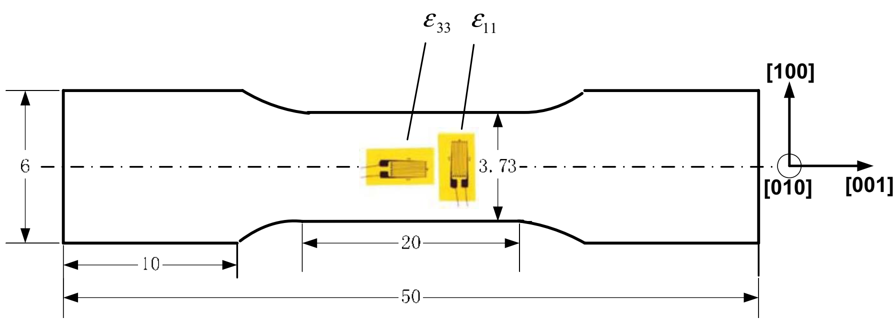

FIG. 2. (Color online) Dimensions (unit: mm) and orientation of tensile specimen and the strain gauge configuration.

All the specimens for uniaxial tension and DSC measurement were sealed into quartz tubes filled with low-pressure argon gas. They were solution treated at 1273 K for 1 h and quenched into brine. Then, all of the specimens were chemically etched to remove the surface layer that might be slightly oxidized during heat treatment.

Uniaxial tensile tests were done with a SHIMAZU AG-20KNIT tensile machine. The tensile strain ε_{33} and transverse strain ε_{11} were measured by two strain gauges placed along the [001] and [100] direction, respectively, as shown in Fig. 2. The loading rate was 3 N/s. A water bath heating unit was used to control the temperature of the sample, which has an accuracy better than ± 0.3 K. A differential scanning calorimeter (Rigaku ThermoPlus DSC 8230) was used to measure the transformation temperature; the heating and cooling rates were 10 K/min.

The DSC measurement shows that the transformation temperatures of the tensile sample at zero stress are $M_s = 314.5$ K and $A_f = 328.5$ K, respectively. The testing temperatures for tensile measurement were 316 K ($M_s + 1.5$ K) and 320 K ($M_s + 5.5$ K). To guarantee, the tensile testing was done in a full parent phase state, the tensile specimen was first heated to above A_f, and then cooled down to the testing temperature before the test. A complete uniaxial tension starts from zero stress, in the parent β phase, and stops at the critical stress for the stress-induced β to γ' (Refs. 22–24) martensitic transformation, at which the tensile strain begins to change abruptly. Although it is desirable to perform the measurement over a wider temperature range above M_s, the particular transformation nature of our Cu-Ni-Al alloy restricts our measurement to a narrow temperature range (from M_s to $M_s + 14$ K). This is because beyond this temperature range, the stress-induced martensite (β') is no longer the same as the temperature-induced one (γ'),[22–24] thus making it difficult to compare the physical meaning of the stress effect on lattice softening at different temperatures. The estimated maximum uncertainty in determining the elastic constants by the present method ranges from −5% to −3%.

IV. EXPERIMENTAL RESULTS

Figure 3(a) shows the measured uniaxial stress σ vs uniaxial strain ε_{33} relation (open circle) at 320 K. Figure 3(b) shows the corresponding transverse strain ε_{11} vs uniaxial strain ε_{33} relation (open circle) at the same temperature (the uncertainty in strain measurement in Figs. 3(a) and 3(b) ranges from −1.5% to 0). The solid lines in the two figures are the least-squares best fitting curves of the data by a polynomial function up to the eight power. The correlation coefficient between the fitted polynomial and the experimental data is better than 0.995.

$d\sigma^{\sigma}/d\varepsilon_{33}^{\sigma}$ vs σ and $d\varepsilon_{11}^{\sigma}/d\varepsilon_{33}^{\sigma}$ vs σ relations are then calculated from the above best-fitting polynomial functions representing σ vs ε_{33} and ε_{11} vs ε_{33} relations. They are shown in Figs. 4(a) and 4(b), respectively. By substituting the $d\sigma^{\sigma}/d\varepsilon_{33}^{\sigma}$ and $d\varepsilon_{11}^{\sigma}/d\varepsilon_{33}^{\sigma}$ values into Eq. (6), we can obtain the value of $C'_{101}(\sigma)$. Figure 5 shows the calculated $C'_{101}(\sigma)$ as a function of uniaxial tensile stress σ at 316 K ($M_s + 1.5$ K) and 320 K ($M_s + 5.5$ K), respectively. It is noted that our fitting and differentiation procedure is analogous to a procedure used in obtaining high order elastic constants.[13,25–28]

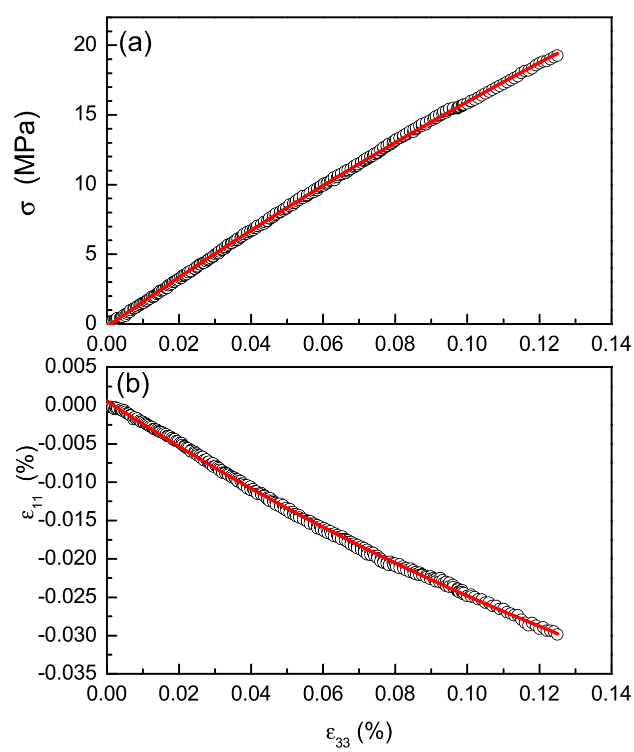

FIG. 3. (Color online) (a) Relationship between tensile stress σ and uniaxial strain ε_{33} for a Cu-13.8Al-4Ni (wt %) single crystal prior to its β to γ' martensitic transformation. (b) Relationship between transverse strain ε_{11} and uniaxial strain ε_{33} under [001] uniaxial tension for the same sample. The solid line is the polynomial best-fitting curves. The uncertainty in the strain measurement in (a) and (b) ranges from −1.5% to 0.

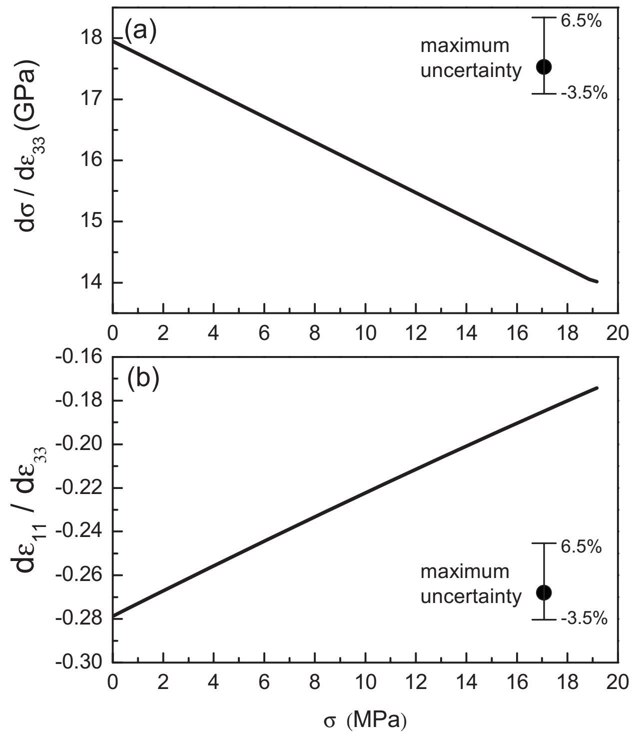

FIG. 4. (a) Relationship between $d\sigma^{\sigma}/d\varepsilon_{33}^{\sigma}$ and tensile stress σ for a Cu-13.8Al-4Ni (wt %) single crystal prior to its β to γ' martensitic transformation. (b) Relationship between $d\varepsilon_{11}^{\sigma}/d\varepsilon_{33}^{\sigma}$ and tensile stress σ under [001] uniaxial tension for the same sample.

From Fig. 5, we can see that under the [001] uniaxial tension, $C'_{101}(\sigma)$ decreases with the increase of uniaxial stress σ until σ reaches the critical stress σ_{cr} (as shown in Fig. 5). This is the direct evidence for the existence of C' softening

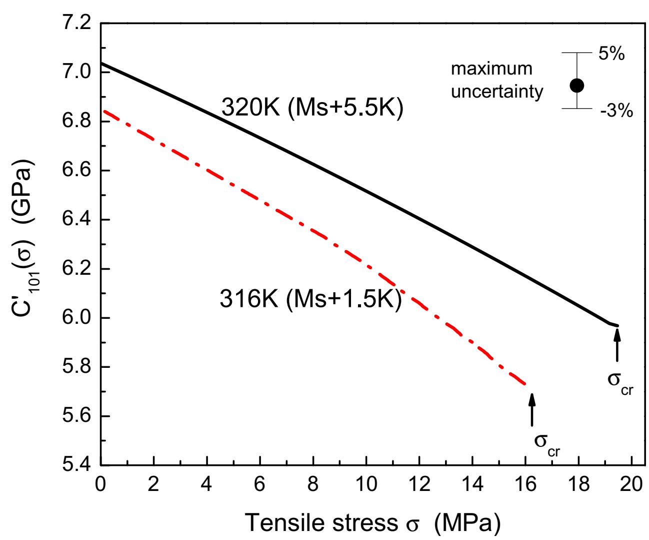

FIG. 5. (Color online) Dependence of the elastic constant $C'_{101}(\sigma)$ on [001] tensile stress σ at two different temperatures for a Cu-13.8Al-4Ni (wt %) single crystal prior to its β to γ' martensitic transformation. C'_{101} is the elastic constant corresponding to (101) $[\bar{1}01]$ shear mode. σ_{cr} is the critical stress for stress-induced martensitic transformation.

from zero stress up to the critical stress at which stress-induced MT occurs. The softening behavior of C' under stress is quite similar to the softening of C' prior to temperature-induced MT in Cu-Al-Ni alloy.[1,5,29,30] This result also confirms the result of recent molecular dynamics simulations.[20] Based on all the above, we can conclude that the softening of C' exists not only in temperature-induced MT but also in stress-induced MT, and it seems a general effect for transforming systems.

Besides, it is noticed that $C'_{101}(\sigma)$ at 320 K (M_s+5.5 K) shows almost a linear relation with the uniaxial stress σ. However, when the temperature is very close to M_s [i.e., 316 K (M_s+1.5 K)], nonlinearity in the $C'_{101}(\sigma)$ vs σ curve becomes apparent but only at high stress; $C'_{101}(\sigma)$ keep a linear relation with σ at small stress. It indicates that the nonlinearity in the $C'_{101}(\sigma)$ vs σ curve enhances in the vicinity of M_s, which is also quite similar to the enhancement of the anharmonicity prior to temperature-induced MT.[1,13] We will discuss this point in Sec. V.

By extrapolating the C'_{101} vs σ (uniaxial tensile stress) curve to zero stress, we obtain the value of C' in the absence of stress, which is C'_{101} (σ=0)=6.84 GPa at 316 K (M_s +1.5 K). This value is close to the reported value of 7.02–7.23 GPa (Refs. 5, 29, and 30) for Cu-Al-Ni alloys of similar compositions by a pulse-echo method, which is a standard method for measuring elastic constant with no bias stress.

V. ANALYTICAL MODEL FOR ELASTIC CONSTANT SOFTENING UNDER UNIAXIAL TENSION

In our recent paper,[20] we have established an analytical theory to investigate the dependence of C' on *biaxial tensile-compressive stress*, and the results are in good agreement with that of our molecular dynamics simulations.[20] In this section, based on the same idea, we establish a theoretical model to investigate the dependence of C' on *uniaxial tensile stress*. This will enable us to make a direct comparison between theory and our current experiment. We shall show that the theoretical result also indicates the existence of C' softening under uniaxial tension, being consistent with our experimental results.

In a stressed crystal, the thermodynamic potential Φ is related to the Helmholz free energy F, the stress σ_{ik}, and strains ε_{ik} as follows:[31]

$$\Phi = F - \sum_{i,k} \sigma_{ik}\varepsilon_{ik}. \tag{7}$$

The Helmholz free energy (F) of a cubic crystal in terms of the symmetry adapted strain components is given in Ref. 32. By considering only symmetry-breaking terms, the Helmholz free energy (F) in our case[1,13,14] is

$$F = F_0 + \frac{1}{2}C'(\eta_1^2 + \eta_2^2) - \frac{1}{3}C_3\eta_1(\eta_1^2 - 3\eta_2^2) + \frac{1}{4}C_4(\eta_1^2 + \eta_2^2)^2, \tag{8}$$

where $C_3=(1/8\sqrt{3})(3C_{112}-2C_{123}-C_{111})$ is a combination of third-order elastic constants, C_4 is a combination of fourth-

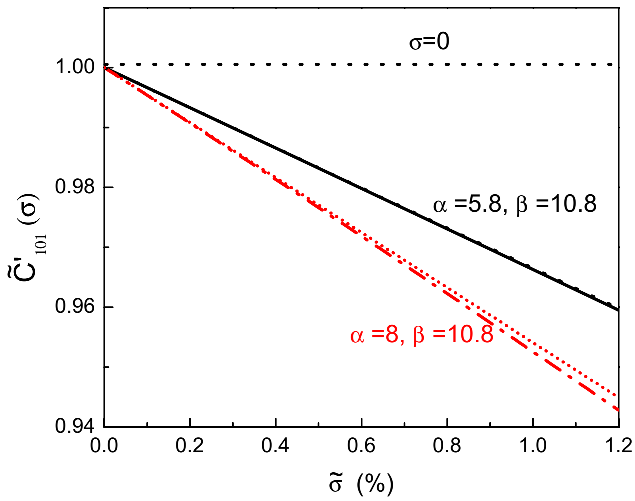

FIG. 6. (Color online) Calculated normalized elastic constant $\tilde{C}'_{101}(\sigma)$ as a function of normalized uniaxial stress $\tilde{\sigma}$ at temperature well above M_s (black line, $\alpha=5.8$ and $\beta=10.8$) and close to M_s (red line, $\alpha=8$ and $\beta=10.8$). The deviation from the straight line (dotted line) reflects the nonlinearity in the $\tilde{C}'_{101}(\sigma)$ vs $\tilde{\sigma}$ relation. The uniaxial stress is applied along the [001] orientation, as shown in Fig. 1.

order elastic constants, $\eta_1=(2\varepsilon_{33}-\varepsilon_{11}-\varepsilon_{22})/\sqrt{3}$ and $\eta_2=(\varepsilon_{11}-\varepsilon_{22})$ are the deviatoric shear strains expressed in terms of Lagrangian strain tensor components (ε_{ij}), and C' is $\{110\}\langle 1\bar{1}0\rangle$ shear modulus in the absence of external stress.

Considering the volume invariance condition $\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}=0$, $\Sigma_{i,k}\sigma_{ik}\varepsilon_{ik}$ for the present uniaxial tension (shown in Fig. 1) can be expressed as

$$\sum_{i,k}\sigma_{ik}\varepsilon_{ik}=\sigma\varepsilon_{33}=\frac{1}{\sqrt{3}}\sigma\eta_1, \tag{9}$$

where σ is the tensile stress along the [001] direction (shown in Fig. 1).

Following a similar derivation as our previous work,[20] we can obtain elastic constant $C'(\sigma)$ of the $(101)[\bar{1}01]$ shear mode under [001] uniaxial tensile stress σ.

$$\tilde{C}'_{101}(\sigma)=C'_{101}(\sigma)/C'=1-\frac{1}{\sqrt{3}}\alpha\tilde{\sigma}+\left(-\frac{1}{3}\alpha^2+\frac{5}{6}\beta\right)\tilde{\sigma}^2, \tag{10}$$

where $\tilde{C}'_{101}(\sigma)$ represent the normalized elastic constant $C'(\sigma)/C'$ of $(101)[\bar{1}01]$ shear mode at given uniaxial tensile stress σ; $\alpha=C_3/C'$, and $\beta=C_4/C'$; $\tilde{\sigma}$ is the normalized uniaxial tensile stress $\tilde{\sigma}=\sigma/C'$.

According to Landau theory of ferroelastic transformation,[1,13,14] C', C_3, and C_4 are positive, thus $\alpha=C_3/C'>0$. With the information above, from Eq. (10) it is clear that $C'(\sigma)$ of $(101)[\bar{1}01]$ shear mode softens with increasing the uniaxial tensile stress $\tilde{\sigma}$ along [001].

Figure 6 shows the variation of $\tilde{C}'_{101}(\sigma)$ with the uniaxial tensile stress $\tilde{\sigma}$ at a temperature not close to M_s (solid line in Fig. 6), where the values of α and β ($\alpha=5.8$ and $\beta=10.8$) are derived from the experimentally determined C' (7.42 GPa), C_3 (43.26 GPa), and C_4 (80 GPa) values for a $Cu_{2.72}Al_{1.122}Ni_{0.152}$ shape memory alloy at room temperature (M_s+75 K).[14] It can be seen that $\tilde{C}'_{101}(\sigma)$ softens with the increase of [001] uniaxial tensile stress $\tilde{\sigma}$, and $\tilde{C}'_{101}(\sigma)$ vs $\tilde{\sigma}$ relation is almost linear, which has a similar tendency as our experimental result at 320 K (solid line in Fig. 5).

When the temperature is close to M_s, it is well known that C' further decreases; at the same time, it has been also experimentally found that C_3, which reflects the vibrational anharmonicity of the crystal, increases significantly.[13] Combining these two effects, the value of α ($\alpha=C_3/C'$) should have a significant increase on approaching M_s. Figure 6 also shows $\tilde{C}'_{101}(\sigma)$ as a function of $\tilde{\sigma}$ with a larger α ($\alpha=8$, and $\beta=10.8$) (dash-dotted line in Fig. 6). Clearly, we can see $\tilde{C}'_{101}(\sigma)$ vs $\tilde{\sigma}$ curve exhibits apparent nonlinearity for a larger α. Thus, our analytical result also indicates that the $\tilde{C}'_{101}(\sigma)$ vs $\tilde{\sigma}$ curve become more nonlinear with the temperature approaching M_s. This is quite similar to our experimental results shown in Fig. 5 (dash-dotted line). Therefore, our analytical model well reproduces all the experimentally observed effects.

From Eq. (10), we can conclude that in general $C'(\sigma)$ is not linearly dependent on stress σ. However, when stress is low or when anharmonicity is weak (C_3 and C_4 are small), $C'(\sigma)$ can be approximated by a linear function of stress σ, $C'(\sigma)=C'+S\sigma$ (S is a coefficient). The linear relation between $C'(\sigma)$ and σ, which is known as the Thurston–Brugger relation,[33] has long been used to measure third-order elastic constants, which reflects the anharmonicity of the lattice. Nevertheless, it should be noted that such a linear relation does not fully reflect the effect of stress on elastic constant C', in particular, in the vicinity of martensitic transformation where the lattice is highly anharmonic or at high stress level. A more complete description of the anharmonic effect needs Eq. (10), from which even fourth order elastic constant can also be derived.

Finally, it should be noted that, recent molecular dynamic simulation[20] suggests that C' may harden in some of the crystallographically equivalent $\{110\}\langle 1\bar{1}0\rangle$ shear modes prior to stress-induced MT, i.e., not all the $\{110\}\langle 1\bar{1}0\rangle$ shear modes soften under stress. This can also be experimentally verified by means of the present experimental method.

VI. CONCLUSION

In the present study, the elastic constant softening under uniaxial tension was investigated by means of a simple tensile test, and the following results are obtained:

(1) An experimental method is established for measuring C' of one of the $\{110\}\langle 1\bar{1}0\rangle$ shear modes for a cubic crystal under finite uniaxial stress along [001].

(2) By means of this method, we show direct evidence for elastic constant C' softening prior to stress-induced martensitic transformation. The stress vs C' relation is nonlinear in

the vicinity of M_s but becomes almost linear at higher temperature. The stress-induced C' softening effect, in combination with the well-observed C' softening prior to temperature-induced martensitic transformation, suggests that the softening of C' is a common feature for both thermally and stress-induced martensitic transformations.

(3) A theoretical model for the dependence of C' on uniaxial tensile stress is formulated, and the calculated result is consistent with our experimental result.

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