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Thickness-induced resonance-based complex permittivity measurement technique for barium strontium titanate ceramics at microwave frequency

Song Xia, Zhuo Xu, and Xiaoyong Wei
Electronic Materials Research Laboratory, Key Laboratory of the Ministry of Education, Xi'an Jiaotong University, Xi'an 710049, People's Republic of China

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Thickness-induced resonance would be the major factor of uncertainty for complex permittivity measurement by using transmission/reflection method when the thickness of sample is several integer multiples of the half wavelength. A new technique for complex permittivity measurement was presented in this paper using the thickness-induced resonance for barium strontium titanate (BST) ceramics at microwave frequency. Simulated and experimental results show that there are some resonance peaks on the transmittance versus frequency curve and the complex permittivity can be calculated from the resonance. © *2009 American Institute of Physics.* [doi:10.1063/1.3237244]

I. INTRODUCTION

Complex permittivity is one of the main characterizations of microwave dielectric materials. Many techniques, such as cavity resonator method and transmission/reflection (T/R) method,[1–3] were introduced and developed to deal with.

The resonant frequencies and quality factor of cavity resonators will change when the samples are put in the cavity under the microwave frequency. The dielectric constant and loss tangent of the sample can be calculated from the variances of resonance frequency and the quality factor before and after sample loaded, respectively. This method is very useful for low-loss dielectrics with higher precision. But the calculation procedure is complex and the resonant model is difficult to distinguish. And it is not fit for those with higher dielectric loss materials.

The scattering parameters of the transmission-line made by the sample could be measured by the traditional T/R method. The complex permittivity can be retrieved by these scattering parameters accordingly even for continuous frequency measurements. However, the measurement uncertainty of this method is very poor for the extreme mismatch of impedance for high-dielectric-constant materials. Lanagan *et al.*[4] proposed a new method of calculating the complex permittivity by the phase variation in transmittance versus frequency. It is effective for high-dielectric-constant materials with high dielectric loss. Additionally, for overcoming the resonance that occurs when the sample thickness is greater than the half wavelength, Baker-Jarvis *et al.*[5] proposed an iterative procedure to bypass the inaccuracy peaks applicable to dielectric materials.[6]

In this paper, the thickness-induced resonance was used for the calculation of dielectric constant and loss tangent by using traditional T/R setup,[1–3] including a network analyzer and a set of rectangular waveguide and modified calculation procedures. It is a convenient method to obtain the microwave complex permittivity for barium strontium titanate (BST) ceramics.

II. THEORY

The thickness-induced resonance-based method proposed in this paper is derived from the traditional T/R method. For convenience of samples prepared, a rectangular waveguide is used usually. A transverse electric (TE_{10}) wave mode was propagating in the empty and sample loaded rectangular waveguide, respectively. Equation (1) gives the formulas of transmittance of transmission-line filled with sample material for measurement.

$$s_{21} = \frac{\left(Z_m - \frac{1}{Z_m}\right)\sinh \gamma l}{2\cosh \gamma l + \left(Z_m + \frac{1}{Z_m}\right)\sinh \gamma l}, \tag{1}$$

where Z_m is the normalized characteristic impedance of the sample, γ is the propagation coefficient of the sample, l is the thickness of the sample, and λ_0 and λ_c are the free space wavelength and the cutoff wavelength of waveguide, respectively.

For low-loss dielectrics ($\tan\delta < 0.1 \ll 1$, $\mu_r = 1$), Z_m can be considered to be a real number approximately and its imaginary part can be neglected. For low-loss high-dielectric-constant dielectrics $(\lambda_0/\lambda_c)^2 \ll \varepsilon_r$, the propagation coefficient γ can be approximated as

$$\gamma = \alpha + j\beta \approx \frac{\pi}{\lambda_0}\sqrt{\varepsilon_r'}\tan\delta + j\frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r'}, \tag{2}$$

where ε_r' is the real part of the relative permittivity and is equivalent to the dielectric constant, $\tan\delta(=\varepsilon_r''/\varepsilon_r')$ is the dielectric loss tangent, and α and β are the attenuation coefficient and phase shift constant of the sample in the waveguide, respectively.

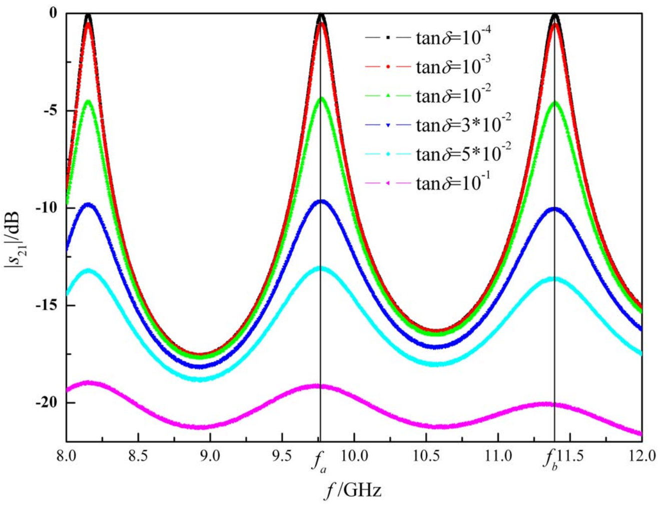

FIG. 1. (Color online) Magnitudes of transmittance vs frequency curve for samples with different loss tangent.

Using these approximations, the transmittance is

$$s_{21} = \frac{4Z_m}{e^{\beta l \tan \delta/2}(Z_m+1)^2 e^{j\beta l} - e^{-\beta l \tan \delta/2}(Z_m-1)^2 e^{-j\beta l}}, \tag{3}$$

and the magnitude of transmittance is

$$|s_{21}| \approx \frac{4Z_m/(1-Z_m^2)}{\sqrt{e^{\beta l \tan \delta}\frac{1}{\Gamma^2} + e^{-\beta l \tan \delta}\Gamma^2 - 2\cos 2\beta l}}, \tag{4}$$

where Γ is the sample interface reflection coefficient.

For low-loss dielectrics, $e^{\beta l \tan \delta/2} \approx e^{-\beta l \tan \delta/2} \approx 1$, then

$$|s_{21}| \approx \frac{4Z_m/(1-Z_m^2)}{\sqrt{\Gamma^2 + \frac{1}{\Gamma^2} - 2\cos 2\beta l}}. \tag{5}$$

A. Extremum of $|s_{21}|$

Because $1/x^2$ is a monotonic decreasing function when x is a positive real number, $1/|s_{21}|^2$ will be a minimum extremum when $|s_{21}|$ is a maximum extremum and vice versa. The extremum conditions of Eq. (5) can be obtained as follows:

$$\frac{\partial(1/|s_{21}|^2)}{\partial(\beta l)} = \left(\frac{1-Z_m^2}{4Z_m}\right)^2 2\sin 2\beta l = 0. \tag{6}$$

At the extremum, $\sin 2\beta l = 0$, namely, $\beta l = k/2\pi (k=1, 2, \ldots)$, when $1/|s_{21}|^2$ is one of the extremum.

$$\frac{\partial^2(1/|s_{21}|^2)}{\partial(\beta l)^2} = \left(\frac{1-Z_m^2}{4Z_m}\right)^2 4\cos 2\beta l. \tag{7}$$

When $\beta l = 2k-1/2\pi$ and $\partial^2(1/|s_{21}|^2)/\partial(\beta l)^2 < 0$, $1/|s_{21}|^2$ is a maximum extremum and $|s_{21}|$ is a minimum extremum.

When $\beta l = k\pi$ and $\partial^2(1/|s_{21}|^2)/\partial(\beta l)^2 > 0$, $1/|s_{21}|^2$ is a minimum extremum and $|s_{21}|$ is a maximum extremum.

The thickness-induced resonance will occur and the magnitude of s_{21} will be at maximum when the sample thickness is several integer multiples of half wavelength.

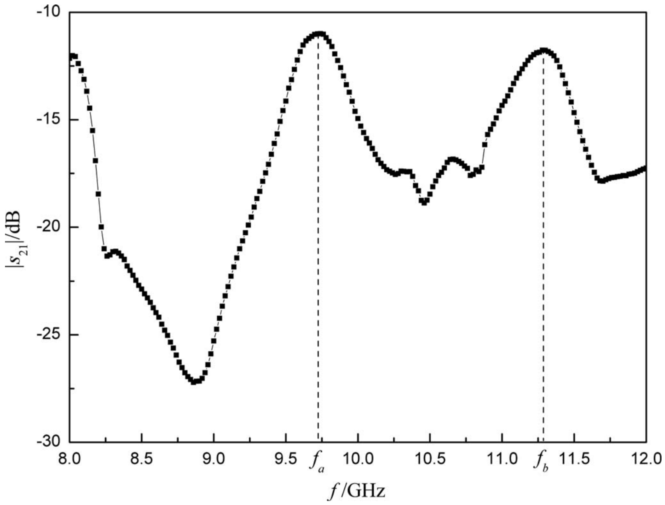

FIG. 2. Magnitudes of transmittance vs frequency curves for BST ceramic.

B. Dielectric constant ε_r'

For two adjacent resonance peaks at f_a and $f_b (f_b > f_a)$, we have

$$\beta l|_{f_a} = k\pi, \quad \beta l|_{f_b} = (k+1)\pi, \tag{8}$$

namely,

$$\begin{cases} \lambda_{ga} = 2l/k \\ \lambda_{gb} = 2l/(k+1) \end{cases}, \tag{9}$$

where l_{ga} and l_{gb} are the waveguide wavelength in sample material at frequency f_a and f_b, respectively. In the waveguide filled with sample material, we have

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}, \tag{10}$$

where λ is the wavelength in sample material given by

$$\lambda = \frac{c}{\sqrt{\varepsilon_r} f}. \tag{11}$$

From Eqs. (9)–(11), we get the equations

$$\varepsilon_r = \left[\left(\frac{k}{2l}\right)^2 + \frac{1}{\lambda_c^2}\right]\frac{c^2}{f_a^2}, \tag{12a}$$

$$\varepsilon_r = \left[\left(\frac{k+1}{2l}\right)^2 + \frac{1}{\lambda_c^2}\right]\frac{c^2}{f_b^2}. \tag{12b}$$

By solving Eq. (12), it can be obtained that

$$k = \frac{f_a^2 \pm \sqrt{f_a^4 - (f_b^2 - f_a^2)\left[4\frac{l^2}{\lambda_c^2}(f_b^2 - f_a^2) - f_a^2\right]}}{f_b^2 - f_a^2}, \tag{13}$$

where k must be a positive integer. Using the root, the dielectric constant at f_a and f_b can be calculated.

TABLE I. The calculation results of the simulation.

	Resonance at f_a			Resonance at f_b		
Trial value of tan δ	ε_r'	tan δ^a	tan δ^b	ε_r'	tan δ^a	tan δ^b
1.0×10^{-4}	105.11	0.28×10^{-4}	2.24×10^{-4}	105.11	0.38×10^{-4}	1.35×10^{-4}
1.0×10^{-3}	105.16	0.95×10^{-3}	0.75×10^{-3}	105.20	0.96×10^{-3}	1.02×10^{-3}
1.0×10^{-2}	105.16	0.99×10^{-2}	1.02×10^{-2}	105.11	0.99×10^{-2}	1.01×10^{-2}
3.0×10^{-2}	104.95	3.00×10^{-2}	3.21×10^{-2}	105.20	2.99×10^{-2}	3.25×10^{-2}
5.0×10^{-2}	105.54	4.98×10^{-2}	5.66×10^{-2}	105.34	4.98×10^{-2}	5.96×10^{-2}
1.0×10^{-1}	106.19	0.99×10^{-1}	⋯	106.46	0.99×10^{-1}	⋯

[a] The result is calculated by method 1.
[b] The result is calculated by method 2.

C. Loss tangent tan δ

1. Method 1

From Eq. (4), the maximum transmittance is

$$|s_{21}|_{\max} \approx \frac{4Z_m/(1-Z_m^2)}{\sqrt{2\cosh(k\pi \tan\delta - 2\ln|\Gamma|) - 2}}, \tag{14}$$

then

$$\tan\delta \approx \frac{\operatorname{arccosh}\dfrac{\left[\dfrac{4Z_m}{|s_{21}|(1-Z_m^2)}\right]^2 + 2}{2} + 2\ln|\Gamma|}{k\pi}. \tag{15}$$

2. Method 2

From Eq. (4), the maximum transmission at resonance is

$$|s_{21}|_{f_a}^2 \approx \frac{8Z_m^2/(1-Z_m^2)^2}{\cosh(k\pi\tan\delta - 2\ln|\Gamma|) - 1}. \tag{16}$$

At frequencies slightly deviated from the resonance frequency, the transmission is

$$|s_{21}|_{f_a+\Delta f}^2 \approx \frac{8Z_m^2/(1-Z_m^2)^2}{\cosh[(k\pi+\Delta\theta)\tan\delta - 2\ln|\Gamma|] - \cos 2\Delta\theta}, \tag{17}$$

where $\Delta\theta$ is the differential of the phase shift between $f_a+\Delta f$ and f_a.

When the transmission at $f_a+\Delta f$ is half that of the resonance, there is

$$|s_{21}|_{f_a+\Delta f}^2 = \tfrac{1}{2}|s_{21}|_{f_a}^2, \tag{18}$$

namely,

$$\cosh[(k\pi+\Delta\theta)\tan\delta - 2\ln] - \cos 2\Delta\theta = 2\cosh(k\pi\tan\delta - 2\ln|\Gamma|) - 2. \tag{19}$$

Using the mathematical approximations $\cosh(x+\Delta x) \approx \cosh(x) + \sinh(x)\Delta x$ and $\cos 2\Delta\theta \approx 1 - 2\Delta\theta^2$, Eq. (19) can be simplified as

$$\cosh[(k\pi - \Delta\theta)\tan\delta - 2\ln|\Gamma|] = 2\Delta\theta^2 + 1, \tag{20}$$

namely,

$$\tan\delta = \frac{\operatorname{arccosh}(2\Delta\theta^2 + 1) + 2\ln|\Gamma|}{k\pi - \Delta\theta}. \tag{21}$$

The loss tangent can be calculated with $\Delta\theta = \pi l/\lambda_{ga}(\lambda_{ga}/\lambda)^2 1/Q$,[7] where Q is the quality factor of resonance.

III. SIMULATION AND EXPERIMENTAL

A. Simulated

In the X-band rectangular waveguide with 9.0 mm as the sample thickness l, 105 as the dielectric constant ε_r', 10^{-4} to 10^{-1} as the loss tangent tan δ, the magnitude of transmittance is calculated, and the calculated results are shown in Fig. 1 (by added 0.5% random error).

It can be found that the thickness-induced resonance occurs at near frequency for the samples with same dielectric constant and different loss tangent. The magnitude of s_{21} at the resonant frequency and the Q value of resonance are smaller for those with higher loss tangent.

From the calculation results of the simulation, it can be proved that this method can be used for measuring complex permittivity of high-dielectric-constant low-loss microwave dielectric materials (see Table I).

B. Experimental

A 9-mm-thick BST ceramic was used as sample and the magnitudes of transmittance versus frequency curves of it was measured at microwave frequency. The result was plotted in Fig. 2.

From the Fig. 2, it can be seen that there are two thickness-induced resonances at 9.74 and 11.28GHz. From the two resonant peaks, the complex permittivity of BST can be calculated. The dielectric constant ε_r' is 105.86 and 107.31 at the resonances. The loss tangent tan δ calculated by method 1 is 0.037 and 0.039. The loss tangent tan δ calculated by method 2 is 0.031 and 0.032.

IV. RESULTS

In this paper, the simulation and experiment are based on X-band rectangular waveguide with propagation of TE_{10} mode. The results show that the thickness-induced resonant method can be applied to complex permittivity measurement for BST ceramics. This method is also applicable to other types of transmission-line systems, such as coaxial, circular waveguide, and microstrip lines. This method is advantageous because it only needs to measure the magnitude of transmittance versus frequency curve to calculate the dielectric constant and loss tangent for dielectric materials.

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