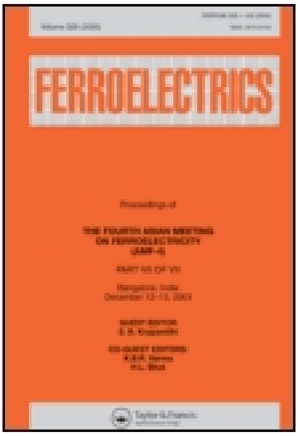

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A Complex Permittivity Measurement Technique for High Dielectric Constant Materials at Microwave Frequency

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The measurement uncertainty of traditional transmission/reflection method is very poor for high dielectric constant (>100) materials or sample whose thickness was several integer multiples of half wavelength. A modified calculating procedure was presented for measuring the high dielectric constant materials. The method is based on the thickness resonance when electromagnetic wave is transmitting through transmission line filled with the sample. It is convenient for complex permittivity measurement of high dielectric constant materials with transmission/reflection method.

Introduction

Transmission/reflection(T/R) method [1–4] is a general complex permittivity measurement technique at microwave frequency. The scattering parameters of the transmission-line made by the sample could be measured, and the complex permittivity can be retrieved by these scattering parameters. However, there were many limits with T/R method. Firstly, the measurement uncertainty is very poor for the extreme mismatch of impedance for high-dielectric constant materials. T. Lanagan [5] proposed a new method for calculating the complex permittivity by the phase variation of transmittance versus frequency. It is effective for high dielectric constant materials with high loss. Another, the measurement uncertainty is very poor for thickness resonance while the sample thickness was several integer multiples of half wavelength. Baker-Jarvis [6, 7] proposed an iterative procedure to bypass the inaccuracy peaks for dielectric materials to overcome the resonance occurred when the sample thickness is greater than the half wavelength.

In this paper, a modified calculating procedure was developed for traditional T/R method to measure the complex permittivity of high dielectric constant materials with low loss at microwave frequency.

Principle of Measurement

As we all know, T/R method could be used with any type transmission line, like coaxial line or free-space with TEM propagation mode, rectangular waveguide with TE10 propagation

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mode, etc. For convenience of samples prepared, rectangular waveguide is usually used, and it is also adopted in this paper.

The traditional calculating procedure for complex permittivity measurement of non-magnetic materials was deduced from the following equations, which were applicable for rectangular waveguide with TE10 propagation mode [1, 2, 7]:

$$s_{21} = \frac{(1-\Gamma^2)T}{1-\Gamma^2 T^2} \tag{1a}$$

$$\Gamma = \frac{Z_m - 1}{Z_m + 1} \tag{1b}$$

$$T = e^{-\gamma l} \tag{1c}$$

$$\gamma = j\frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r - \left(\frac{\lambda_0}{\lambda_c}\right)^2} \tag{1d}$$

$$\gamma_0 = j\frac{2\pi}{\lambda_0}\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2} \tag{1e}$$

$$Z_m = \frac{Z}{Z_0} = \frac{\gamma_0}{\gamma} \tag{1f}$$

where s_{21} is the transmittance of transmission line filled with the sample; Γ and T are the first reflection and the transmission coefficients; γ_0, Z_0 and γ, Z represent the propagation constants and the impedances of the empty and filled waveguide, respectively; Z_m is the relative impedance; λ_0 and λ_c correspond to the free-space and the cutoff wavelength; l is the sample length; and ε_r is the complex permittivity of sample.

For high dielectric constant and low loss dielectrics (ε'_r >100, tan$\delta < 0.1 \ll 1$ and $\mu_r = 1$), the propagation coefficient γ can be approximated as

$$\gamma = \alpha + j\beta \approx \frac{\pi}{\lambda_0}\sqrt{\varepsilon'_r}\tan\delta + j\frac{2\pi}{\lambda_0}\sqrt{\varepsilon'_r} \tag{2a}$$

$$\alpha \approx \frac{1}{2}\beta\tan\delta \tag{2b}$$

where ε'_r is the real part of the relative permittivity and is equivalent to the relative dielectric constant, tanδ ($= \varepsilon''_r/\varepsilon'_r$) is the loss tangent, α and β are the attenuation coefficient and phase shift constant of the sample in the waveguide, respectively.

Z_m can be considered as the real number approximately and its imaginary part can be neglected.

Using these approximations, the magnitude of transmittance is

$$|s_{21}| = \frac{1-\Gamma^2}{\sqrt{e^{\beta l\tan\delta} + \Gamma^4 e^{-\beta l\tan\delta} - 2\Gamma^2\cos 2\beta l}} \tag{3}$$

The extremum conditions of $|s_{21}|$ can be obtained as follow.

$$\frac{\partial\left(1/|s_{21}|^2\right)}{\partial(\beta l)} = \frac{\left(e^{\beta l\tan\delta} - \Gamma^4 e^{-\beta l\tan\delta}\right)\tan\delta + 4\Gamma^2\sin 2\beta l}{\left(1-\Gamma^2\right)^2} = 0 \tag{4}$$

Using the approximation, $(e^{\beta l \tan\delta} - \Gamma^4 e^{-\beta l \tan\delta})\tan\delta \approx 0$, $1/|s_{21}|^2$ and $|s_{21}|$ would be a extremum when $\sin 2\beta l = 0$, namely, $\beta l = \frac{k}{2}\pi$ $(k = 1, 2, \ldots)$.

$$\frac{\partial^2\left(1/|s_{21}|^2\right)}{\partial(\beta l)^2} \approx \left(\frac{2\Gamma}{1-\Gamma^2}\right)^2 2\cos 2\beta l \tag{5}$$

When $\beta l = \frac{2k-1}{2}\pi$ and $\frac{\partial^2(1/|s_{21}|^2)}{\partial(\beta l)^2} < 0$, $1/|s_{21}|^2$ is a maximum extremum and $|s_{21}|$ is a minimum extremum.

When $\beta l = k\pi$ and $\frac{\partial^2(1/|s_{21}|^2)}{\partial(\beta l)^2} > 0$, $1/|s_{21}|^2$ is a minimum extremum and $|s_{21}|$ is a maximum extremum.

The magnitude of transmittance would be a maximum and the thickness resonance would occur when the sample thickness is several integer multiples of half-wavelength

1) Calculate Relative Dielectric Constant ε_r'

For two adjacent resonance peaks at f_a and f_b $(f_b > f_a)$, we have

$$\beta l|_{f_a} = k\pi,\ \ \beta l|_{f_b} = (k+1)\pi \tag{6a}$$

Namely,

$$\lambda_{ga} = 2l/k, \lambda_{gb} = 2l/(k+1) \tag{6b}$$

where λ_{ga}, λ_{gb} are the waveguide wavelength in the filled waveguide at frequency f_a and f_b, respectively. In the waveguide filled with sample material, we have

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2} \tag{7}$$

Where λ is the wavelength in sample material given by

$$\lambda = \frac{c}{\sqrt{\varepsilon_r} f} \tag{8}$$

From (6)–(8), we get the equations

$$\varepsilon_r = \left[\left(\frac{k}{2l}\right)^2 + \frac{1}{\lambda_c^2}\right]\frac{c^2}{f_a^2} \tag{9a}$$

$$\varepsilon_r = \left[\left(\frac{k+1}{2l}\right)^2 + \frac{1}{\lambda_c^2}\right]\frac{c^2}{f_b^2} \tag{9b}$$

By solving the equation (9), it can be obtained that

$$k = \frac{f_a^2 \pm \sqrt{f_a^4 - \left(f_b^2 - f_a^2\right)\left[4\frac{l^2}{\lambda_c^2}\left(f_b^2 - f_a^2\right) - f_a^2\right]}}{f_b^2 - f_a^2} \tag{10}$$

where k must be a positive integer. Using the root, the dielectric constant at f_a and f_b can be calculated.

2) Calculate Loss Tangent tanδ

From equation (3), the maximum transmission at resonance is

$$|s_{21}|^2_{f_a} = \frac{\left(\frac{1}{\Gamma} - \Gamma\right)^2}{2\cosh\left(k\pi \tan\delta - 2\ln|\Gamma|\right) - 2} \tag{11a}$$

At the frequencies slightly deviated from the resonance frequency, the transmission is

$$|s_{21}|^2_{f_a+\Delta f} \approx \frac{\left(\frac{1}{\Gamma} - \Gamma\right)^2}{2\cosh\left[(k\pi + \Delta\theta)\tan\delta - 2\ln|\Gamma|\right] - 2\cos 2\Delta\theta} \tag{11b}$$

Where $\Delta\theta$ is the differential of phase shift between $f_a+\Delta f$ and f_a.

When the transmission at $f_a+\Delta f$ is half that of the resonance, there is

$$|s_{21}|^2_{f_a+\Delta f} = \frac{1}{2}|s_{21}|^2_{f_a} \tag{12}$$

Namely,

$$\cosh\left[(k\pi + \Delta\theta)\tan\delta - 2\ln|\Gamma|\right] - \cos 2\Delta\theta = 2\cosh\left(k\pi \tan\delta - 2\ln|\Gamma|\right) - 2 \tag{13}$$

Using mathematical approximations $\cosh(x + \Delta x) \approx \cosh(x) + \sinh(x)\Delta x$ and $\cos 2\Delta\theta \approx 1 - 2\Delta\theta^2$, equation (13) can be simplified.

$$\cosh\left[(k\pi - \Delta\theta)\tan\delta - 2\ln|\Gamma|\right] = 2\Delta\theta^2 + 1 \tag{20}$$

Namely,

$$\tan\delta = \frac{\operatorname{arccosh}\left(2\Delta\theta^2 + 1\right) + 2\ln|\Gamma|}{k\pi - \Delta\theta} \tag{21}$$

The loss tangent can be calculated with $\Delta\theta = \frac{\pi l}{\lambda_{ga}}\left(\frac{\lambda_{ga}}{\lambda}\right)^2\frac{1}{Q}$ [8], where Q is the quality factor of resonance.

To validate the calculating procedure, we simulated it at X-band rectangular waveguide with different relative dielectric constant or loss tangent.

1) The relative dielectric constant ε_r' is from 95 to 105, loss tangent tanδ is 10^{-2}, and thickness l is 10.0mm. The magnitude of transmittance is calculated and the calculated results are shown in figure 1(added by 0.5% random error).
   It can be found that the resonant frequency is decreased for the increase of relative dielectric constant. The magnitude of s_{21} at the resonant frequency and the Q value of resonance is near for those with same loss tangent.
2) The relative dielectric constant ε_r' is 100, loss tangent tanδ is from 10^{-4} to 10^{-1}, and the thickness l is 10.0 mm. The magnitude of transmittance is calculated and the results are shown in Fig. 2 (added by 0.5% random error).

It can be found that the thickness resonance occurs at near frequency for the samples with same relative dielectric constant and different loss tangent. The magnitude of s_{21} at the resonant frequency and the Q value of resonance are smaller for those with higher loss tangent. Figure 3 shows the error of the complex permittivity calculated by the new procedure.

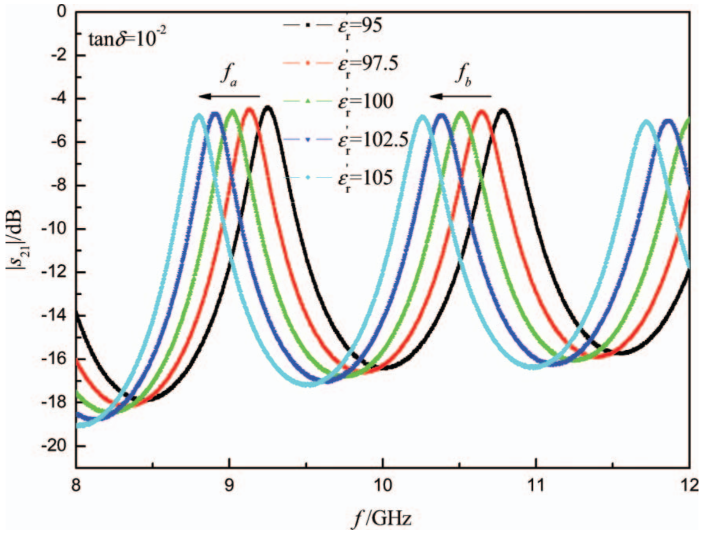

Figure 1. Magnitudes of transmittance versus frequency curve for samples with different relative dielectric constant. (See Color Plate XIX)

From the simulated, it can be proved that this procedure can be used for measuring complex permittivity of high dielectric constant, low loss dielectric materials.

Experimental

A 9 mm-thick BST ceramic was used as sample and its magnitudes of transmittance versus frequency curves was measured at microwave frequency. The result was plotted in Fig. 2.

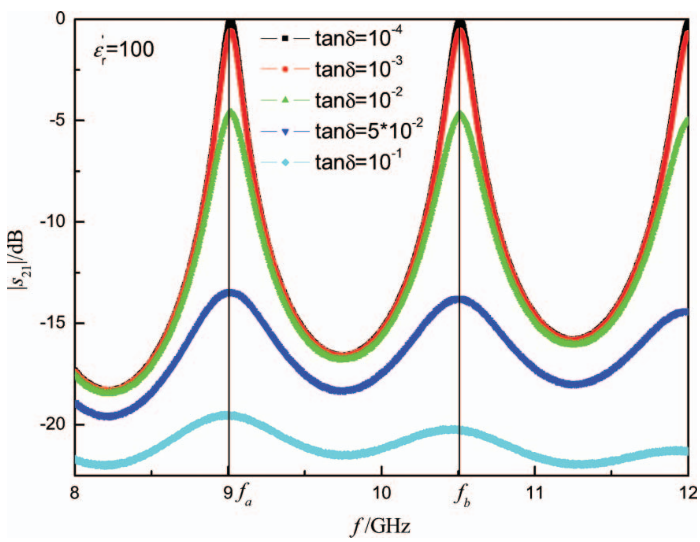

Figure 2. Magnitudes of transmittance versus frequency curve for samples with different loss tangent. (See Color Plate XX)

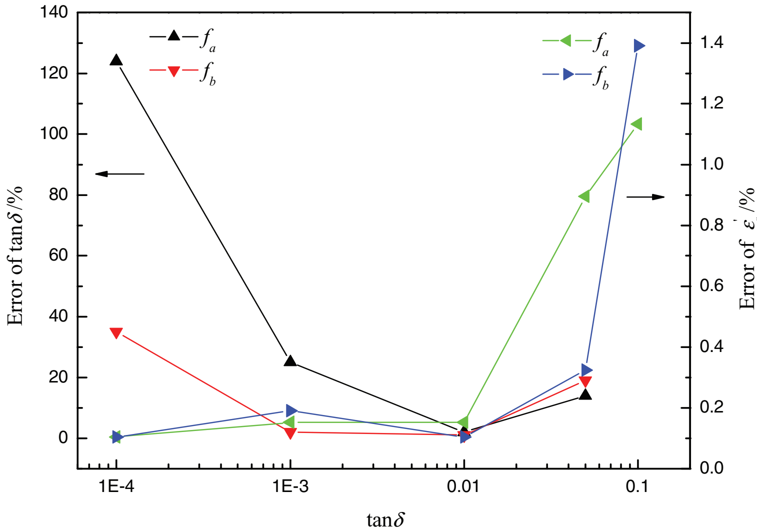

Figure 3. The error of the complex permittivity calculated by the new procedure with different loss tangent. (See Color Plate XXI)

From the Fig. 4, it can be seen that there are two thickness-induced resonances at 9.74 GHz and 11.28 GHz. From the two resonant peaks, the complex permittivity of BST can be calculated. The dielectric constant ε_r' is 105.86 and 107.31 at the resonances. The loss tangent tanδ calculated by method 1 is 0.037 and 0.039. The loss tangent tanδ calculated by method 2 is 0.031 and 0.032.

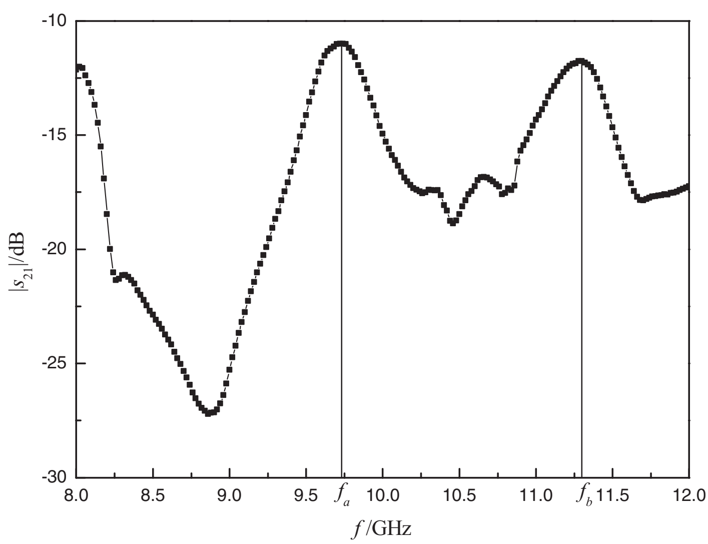

Figure 4. Magnitudes of transmittance versus frequency curves for BST ceramic.

Results

In this paper, the simulation and experiment are based on X-band rectangular waveguide with propagation of TE_{10} mode. The results show that the thickness resonant method can be applied to complex permittivity measurement for high dielectric constant, low loss dielectric materials with traditional transmission/reflection method. This method is also applicable to other types of transmission line systems, such as coaxial, circular waveguide and microstrip lines. This method is advantageous over others because it only needs to measure the magnitude of transmittance versus frequency curve to calculate relative dielectric constant and loss tangent for dielectric materials.

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