International Journal of Modern Physics B
Vol. 17, Nos. 1 & 2 (2003) 183–187

World Scientific
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SIMULATION OF FLUID FLOW AND HEAT TRANSFER IN A PLANE CHANNEL USING THE LATTICE BOLTZMANN METHOD

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Received 9 August 2002

Forced convective flow and heat transfer between two parallel plates are studied using the lattice Boltzmann method (LBM) in this paper. Three kinds of thermal boundary conditions at the top and bottom plates are studied. The velocity field is simulated using density distribution function while a separate internal energy distribution function is introduced to simulate the temperature field. The results agree well with data from traditional finite volume method (FVM) and analytical solutions. The present work indicates that LBM may be developed as a promising method for predicting convective heat transfer because of its many inherent advantages.

1. Introduction

The recent development of the lattice Boltzmann method has provided a new simulation tool for computational fluid dynamics. Based on the kinetic theory, the lattice Boltzmann method simulates fluid flows by tracking the evolution of fluid particles taking on a few discrete speeds in discrete space at discrete time steps. LBM is fully parallel and can easily model fluid flows with complicated boundary conditions.

Heat transfer problems were simulated through adopting multi-speed thermal fluid lattice Boltzmann models firstly in Ref. 1. In general, the available thermal lattice Boltzmann models fall into three categories. In the firstly proposed multi-speed approach, only the density distribution function is needed. These multi-speed models suffer severe numerical instability and the temperature variation is limited to a narrow range.[1-3] The second approach is to solve the energy equation combined with momentum equations.[4,5] Its main advantage is the enhancement of the numerical stability. However, the viscous heat dissipation and compression work done by the pressure cannot be taken into account in this approach. In the third approach,[6] an independent internal energy distribution function was introduced to obtain the temperature field. This model has a better numerical stability and the viscous heat dissipation and compression work done by the pressure can be solved fundamentally. In this paper we adopted the third scheme to simulate fluid flow and heat transfer in a 2-D plane channel. The inlet fluid has a uniform velocity and temperature. The top and bottom plates are stationary. Three kinds of thermal boundary conditions are discussed, i.e. the top and bottom plates are at the same

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temperature, the top and bottom plates have different temperatures, and the bottom is adiabatic while the top has a fixed temperature.

2. Thermal Lattice Boltzmann Model

Within the lattice Boltzmann scheme, the evolution of the density distribution function in fluid systems obeys the Boltzmann equation. By using the well-known single time relaxation BGK model for the collision operator,[7] the velocity Boltzmann equation can be written as:

$$\frac{\partial f}{\partial t}+(\mathbf{c}\cdot\nabla)f=-\frac{1}{\tau_v}\left(f-f^{eq}\right) \tag{1}$$

Here f is the single particle density distribution function. The relaxation time τ_v controls the rate of approaching equilibrium. The macroscopic variables, such as the density ρ, the velocity $\mathbf{u}$, and the internal energy per node are defined in terms of the particle distribution function as follows:

$$\rho=\sum_i f_i\text{, }\ \rho\mathbf{u}=\sum_i f_i\mathbf{c}\text{, }\ \rho e=\sum_i\frac{1}{2}f_i(\mathbf{u}-\mathbf{c}_i)\cdot(\mathbf{u}-\mathbf{c}_i) \tag{2}$$

A new variable, the internal energy density distribution function, is introduced in Ref. 6:

$$g=\frac{1}{2}f_i(\mathbf{u}-\mathbf{c}_i)\cdot(\mathbf{u}-\mathbf{c}_i) \tag{3}$$

By the definition of g, the evolution equation of the internal energy distribution function can be obtained. Similar to Eq. (1), we use an internal energy relaxation time τ_e to replace collision term, then, the energy equation can be written as:

$$\frac{\partial g}{\partial t}+(\mathbf{c}\cdot\nabla)g=-\frac{1}{\tau_e}\left(g-g^{eq}\right)-fq \tag{4}$$

Here, q is the heat flux per unit mass flow along a given $\mathbf{u}$ direction at each lattice, and can be determined by.[6]

$$q=(\mathbf{c}-\mathbf{u})\cdot[\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{c}\cdot\nabla)\mathbf{u}] \tag{5}$$

τ_v and τ_e are related with kinetic viscosity ν and thermal diffusivity a of the fluid.

$$\tau_v=\frac{1}{2}+\frac{3\Delta t}{\left(\Delta x^2\right)}\nu \qquad \tau_e=\frac{1}{2}+\frac{3\Delta t}{2\left(\Delta x^2\right)}a \tag{6}$$

The collision and propagation steps are given by the Boltzmann equation and its discrete expression for the density distribution is:

$$f_i\left(\mathbf{r}+\mathbf{c}_i\Delta t,t+\Delta t\right)=f_i\left(\mathbf{r},t\right)-\frac{\Delta t}{\tau_v}\left[f_i\left(\mathbf{r},t\right)-f_i^{eq}\left(\mathbf{r},t\right)\right] \tag{7}$$

Similarly, the evolution of the internal energy on discrete lattices can be written as:

$$g_i(\mathbf{r}+\mathbf{c}_i\Delta t,t+\Delta t)=g_i(\mathbf{r},t)-\frac{\Delta t}{\tau_e}[g_i(\mathbf{r},t)-g_i^{eq}(\mathbf{r},t)]-f_iq_i\Delta t \tag{8}$$

In this paper, the 9-directional square lattice is adopted, and the equilibrium density distributions are chosen in the following form,[8]

$$f_i^{eq} = \rho\omega_i\left[1+3(\mathbf{c}_i\cdot\mathbf{u})+9(\mathbf{c}_i\cdot\mathbf{u})^2/2-3(\mathbf{u}\cdot\mathbf{u})/2\right],$$

ω_0 =4/9, ω_i =1/9 for i =1,2,3,4, and ω_i =1/36 for i =5,6,7,8.

The parameter c_i is the particle's discrete speed, c_0=0 corresponds to the distribution with zero velocity,

$$c_i = (\cos[(i-1)\pi/2], \sin[(i-1)\pi/2]) \text{ for } i=1,2,3,4,$$

and

$$\mathbf{c}_i = (\cos[(i-5)\pi/2+\pi/4], \sin[(i-5)\pi/2+\pi/4]) \text{ for } i=5,6,7,8.$$

The discrete internal energy density equilibrium distribution takes the form,[6]

$$g_i^{eq} = \begin{cases} -\dfrac{2\rho e}{3}\cdot(\mathbf{u}\cdot\mathbf{u}), & i=0 \\ \dfrac{\rho e}{9}[\dfrac{3}{2}+\dfrac{3(\mathbf{c}_i\cdot\mathbf{u})}{2}+\dfrac{9(\mathbf{c}_i\cdot\mathbf{u})^2}{2}-\dfrac{3(\mathbf{u}\cdot\mathbf{u})}{2}], & i=1,2,3,4 \\ \dfrac{\rho e}{36}[3+6(\mathbf{c}_i\cdot\mathbf{u})+\dfrac{9(\mathbf{c}_i\cdot\mathbf{u})^2}{2}-\dfrac{3(\mathbf{u}\cdot\mathbf{u})}{2}], & i=5,6,7,8 \end{cases} \tag{9}$$

3. Simulation Results

In this paper, incompressible fluid flow and heat transfer between two infinite parallel plates are simulated. The inlet fluid has a uniform temperature T_{in} and a uniform velocity U_{in}. The density of the outflow is specified. The fluid densities at the inlet nodes are set equal to those of the corresponding neighboring interior nodes, while the densities on the walls are calculated with the bounce-back rule. Local equilibrium distribution functions are enforced at the walls and the inlet based on the given densities and velocities. The outflow is supposed to be fully developed and obeys Neumann rule. Three kinds of thermal boundary conditions at the top and bottom plates are discussed in our simulation. For the boundaries with a known temperature, the local thermal equilibrium distribution functions are applied on the wall nodes. As for the adiabatic wall, we simply set energy distributions on the wall to be equal to those of the nearest interior nodes. The following variables are fixed and known as: Re=10, Pr=0.7, T_{in}=20, ρ_{out} =1.2, U_{in}=0.01. The hydrodynamic diameter is twice of the height of the channel.

The predicted profiles of normalized velocity, U/U_{in}, at three cross sections are shown in Fig.1. X and Y stand for dimensionless horizontal and vertical coordinates respectively. The velocity distributions agree well with the results from FVM.

Figure 2 shows the normalized temperature profiles when the wall temperatures at the top and bottom plates are set equal, T_w =10. Figure 3 shows the results when the wall temperature at the bottom plate is set to T_{w1} =0 while the top plate with T_{w2} =10. Figure 4 shows the results when the bottom plate keeps adiabatic while the top plates with T_{w2} =10. T_b is the bulk temperature of the section. The temperature profiles agree well with the results from FVM except that there is a little deviation near the top in Fig.3. The detailed contours of friction factor and local Nusselt numbers along the flow direction are not plotted here because of the article's space limitation. The product of Darcy friction factor and Reynolds number keeps at 95.8 in the velocity fully developed region, deviating from the analytical result, 96, with an error of 0.2% only. The local Nusselt numbers at the top plate are 7.2, 3.9 and 4.5 respectively in the thermal fully developed region. Compared with the corresponding analytical solutions, 7.54, 4.0 and 4.86, the errors are 4.5%, 2.5% and 7.4% respectively. We think the key measure to enhance the accuracy is to develop high-order thermal boundary treatments, and we're actively doing this work.

4. Conclusions

The fluid flow and heat transfer between two parallel plates are studied using a thermal lattice Boltzmann model. The key point in the scheme is the application of two sets of distributions: the density distribution to simulate hydrodynamics and the internal energy distribution to simulate the thermodynamics. Another important problem is how to handle the velocity and thermal boundary conditions. Compared with the results of finite volume method and analytical solutions, the agreement of hydrodynamic results is excellent while that of thermal results is reasonably good. Further improvement for treating the thermal boundary condition is needed to enhance the accuracy of thermal results.

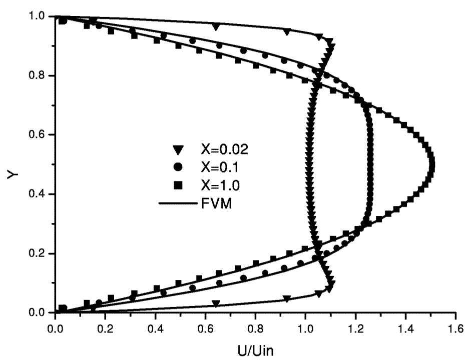

Fig. 1. Velocity profiles at cross sections

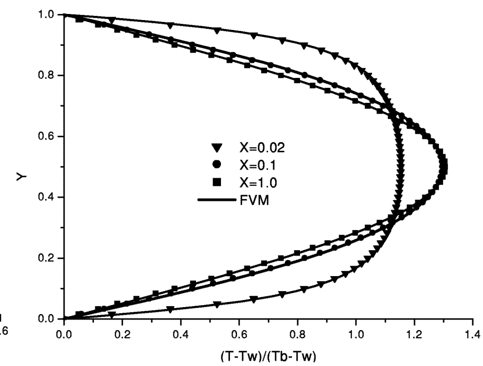

Fig. 2. Temperature profiles at cross sections

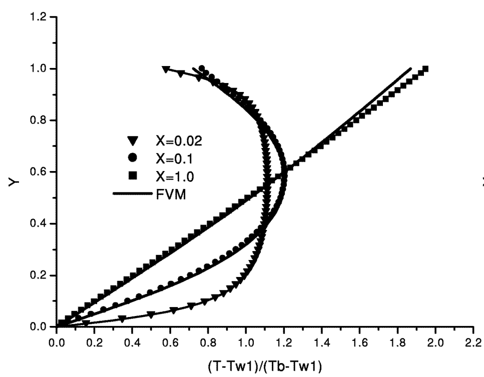

Fig. 3. Temperature profiles at cross sections

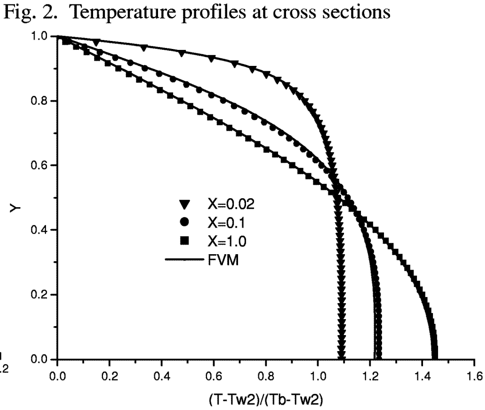

Fig. 4. Temperature profiles at cross sections

Acknowledgements

The project is financially supported by the National Key of Fundamental R&D of China (No.G2000026303) and National Natural Science Foundation of China (No.50076034).

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