The question of the stress distribution in a neck formed under tension is complicated and has not been fully solved. Since it is important to know the magnitudes of the stresses at the instant preceding rupture, approximate solutions have been constructed which are based on various assumptions stimulated by experimental data. We consider one of these solutions, put forward by Davidenkov and Spiridonova [105].

When the neck appears the stress distribution ceases to be uniaxial and uniform. The difficulty of the analysis is compounded by the fact that the shape of the neck is unknown, the approximate solution utilizes the experimentally observed fact that in the minimum section of the neck the natural strains in the radial and tangential directions are equal and uniformly distributed. Hence is follows that on the section $z=0$

$$\xi_r = \xi_\varphi = \text{const.}$$

at the given instant of time.

Since the elastic deformations in the neck are negligibly small compared with the plastic deformations , the incompressibility equation gives $\xi_z = -2\xi_r = \text{const.}$, and from the Saint Venant-von Mises relations it follows that

$$\sigma_r = \sigma_\varphi \tag{61.1}$$

In the section $z=0$. Further, we have from the symmetry condition that $\tau_{rz}=0$ when $z=0$. In this section the differential equations of equilibrium (58.1) take the form

$$\frac{d\sigma_r}{dr} + \left(\frac{\partial \tau_{rz}}{\partial z}\right)_{z=0} = 0, \qquad \frac{d\sigma_z}{dr} = 0, \tag{61.2}$$

and the yield criterion is

$$\sigma_z - \sigma_r = \sigma_s. \tag{61.3}$$

We take a meridional plane and consider in it the trajectories of the principal stresses σ_3, σ_1 (fig. 187) close to the plane $z=0$. The angle ω of inclination of the tangent to the trajectory of the stress σ_3 is small, and formulae (58.7), with indices 1, 2 replaced by 1, 3 respectively, take the simple form

$$\sigma_z \approx \sigma_3, \quad \sigma_r \approx \sigma_1, \quad \tau_{rz} \approx (\sigma_3 - \sigma_1)\omega.$$

In consequence we have near the plane $z=0$

$$\sigma_3 - \sigma_1 \approx \sigma_s, \quad \tau_{rz} \approx \sigma_s \omega \tag{61.4}$$

and

$$\left(\frac{\partial \tau_{rz}}{\partial z}\right)_{z=0} = \sigma_s \left(\frac{\partial \omega}{\partial z}\right)_{z=0} = \frac{\sigma_s}{\rho}, \tag{61.5}$$

where ρ is the radius of curvature of the trajectory of the principal stress for $z = 0$. The contour of the neck is one of these trajectories; let $\rho = R$ for the contour. From the differential equation (61.2) we obtain

$$\frac{\sigma_r}{\sigma_s} = \int_r^a \frac{dr}{\rho}, \tag{61.5}$$

since $\sigma_r = 0$ when $r = a$.

When $r = 0,\ \rho = \infty$ and when $r = a,\ \rho = R$; on the basis of observations we assume that

$$\rho = Ra/r.$$

Then

$$\frac{\sigma_r}{\sigma_s} = \frac{a^2 - r^2}{2aR}, \qquad \frac{\sigma_z}{\sigma_s} = 1 + \frac{a^2 - r^2}{2aR}. \tag{61.6}$$

This stress distribution in the neck is shown on the left-hand side of fig. 187. to calculate the stresses it is necessary to have experimental measurements of the qualities *a*, *R*.

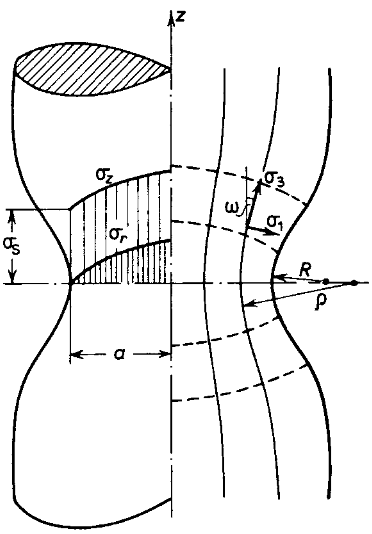

Fig.187

Fig.188

The maximum stresses arise in the central portion of the neck and for this reason rupture begins at the centre. Fig. 188 shows an X-ray photograph (taken from Nadai's book [25]) of the neck of a specimen directly before rupture; it supports the above remark.

(摘自专著 L. M. Kachanov, Fundamentals of the Theory of Plasticity, Dover Publications, 2004, §61. Stress distribution in the neck of a tension specimen, Pages.311-314)