(i) *Method of solution*. The plane surface of a semi-infinite block of plastic-rigid material is penetrated normally by a smooth rigid wedge of total angle 2θ. In Fig. 54 (right hand half) *ABDEC* is the region of plastically deforming material; *AC* is the displaced surface (whose shape is to be determined); *AB* is the line of contact with the wedge, and *BDEC* is a slip-line. The most convenient starting slip-line is *BD*. When its position has been assumed, the condition that slip-lines meet the wedge at 45° defines the field *ABD* uniquely (third boundary-value problem). Since the free surface will not necessarily meet the wedge orthogonally, the point *A* must be a stress singularity. This, with the slip-line *AD*, defines the field *ADE*, which may be continued round *A* through any desired angle (first boundary-value problem, special case). The slip-line *AE*, together with the requirement that *AC* must be a free surface, defines the field *AEC* and, incidentally, the shape of *AC* (converse of second boundary-value problem). Now the point *C* must lie on the original plane surface; this determines the angular span ψ of the field *ADE*. We have next to examine whether, with our initial choice of *BD*, the velocity boundary conditions are satisfied. Along *AB* the component of velocity normal to the wedge is equal to the normal component of the speed of penetration; along *BDEC* the normal component of velocity is zero since the material underneath is rigid. The velocity solution may therefore be begun in *ABD* (third boundary-value problem), and extended successively to *ADE* and *AEC* (first boundary-value problem). The calculated velocities of elements on the free surface must be such that the surface is continually displaced in such a way that geometric singularity is preserved. This is the condition which controls the shape of the starting slip-line *BD*. In the unit diagram the curve corresponding to the free surface must be the trajectory for surface elements. Hence, according to the interpretation of (1), the tangent at any point on this curve must pass through the associated focus with position vector **v**. if the tentative solution has this property, similarity is maintained.

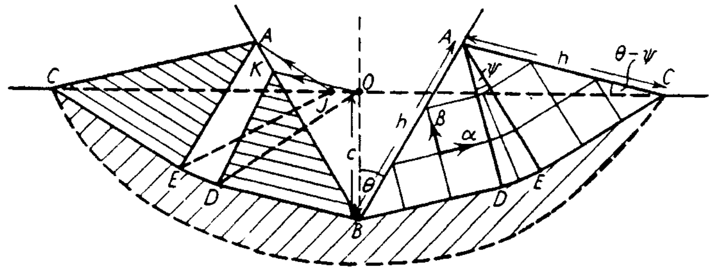

FIG. 54. Indentation of a plane surface by a smooth wedge, showing the slip-line field on the right and the main features of the distortion on the left.

(ii) *Position of the displaced surface*. We now verify that there is a possible solution when *BD* is straight and has a certain specific length. The displaced surface *AC* and the slip-line in *ABD* and *AEC* are then also straight, while *ADE* is a field of radii and circular arcs. For a given choice of the length of *BD*, the magnitude ψ of the angle *DAE* is determined by the condition that *C* should fall on the original surface. This is so if the height of *C* above *B* is equal to *c*; that is, if

$$AB\cos\theta - AC\sin(\theta-\psi) = OB,$$

or $$h[\cos\theta - \sin(\theta-\psi)] = c. \tag{2}$$

Since v is zero on the plastic-rigid boundary *BDEC*, it is zero everywhere by Geiringer's equation for the variation of v along the straight β-lines. It follows that u is constant on each α-line, and hence, by the boundary condition on *AB*, it is universally equal to $\sqrt{2}\sin\theta$ (the downward speed of the wedge is unity on the scale c). thus, at any moment, all elements are moving with the same speed along the α-lines. The surface *AC* is therefore displaced to a parallel position, and the new configuration can be made geometrically similar by a suitable choice of the length of *BD* or, equivalently, the position of *A*.

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The mean compressive stress has the value k on the free surface in compression, and hence, by Hencky's theorem, its value on the wedge face *AB* is $k(1+2\psi)$. The pressure P on the wedge is therefore distributed uniformly, and is of amount

$$P = 2k(1+\psi) \tag{5}$$

The load per unit width is $2Ph\sin\theta$, and the work expended per unit volume of the impression below *OC* is $Ph\sin\theta/c$. The relation between P and θ is shown in Fig. 56; P rises steadily from $2k$ to $2k(1+\frac{1}{2}\pi)$ as the angle increases. This should be contrasted with the experimental observation by Bishop, Hill, and Mott [*Proc. Phys. Soc*. 57 (1945), 147] that, when cold-worked copper is indented by a lubricated *cone*, the mean resistive pressure decrease as the cone becomes less pointed; for $\theta > 30^\circ$ the decrease is slight and the pressure has an approximately constant value of $2\cdot 3Y$.

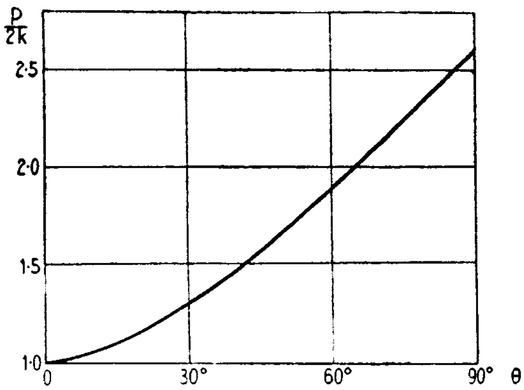

Fig. 56. Relation between the pressure and the semi-angle in wedge-indentation.

The distribution of stresses in rigid material is not known, but there is no reason to suppose that the material is incapable of supporting the calculated stresses along

BDEC. It is observed in the indentation of hard materials by a smooth wedge that the plastic region extends a little way below the tip (more if the wedge is rough or the material is annealed), but that the strains are small; this corresponds to the rigid part of the plastic region (sketched diagrammatically in Fig. 54) for our hypothetical plastic-rigid body. The present solution would continue to hold even for a block of finite dimensions, provided it could be associated with a non-plastic state of stress in the rigid material. In other words, to the approximation achieved by the hypothetical material, the state of stress in the plastically deforming region can remain similar even if the block is finite, though the non-plastic stress distribution, of course, can not. As the penetration increased, however, a stage would be reached where a possible state of stress in the rigid material could not be found; this would imply that plastic *deformation* had begun elsewhere.

(摘自专著 R. Hill, The Mathematical Theory of Plasticity, Oxford University Press, 1998, Section VIII.2 Wedge-indentation, Pages.215-219)