RESEARCH ARTICLE

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Vibro-Acoustic Response of a Thermally Stressed Reinforced Conical Shell

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Numerical study on virbo-acoustic response of a reinforced conical shell in a reverberant enclosure is presented in this paper based on hybrid finite element method and statistical energy analysis (Hybrid FEA-SEA). The shell clamped at two ends is assumed to be subjected to a uniform temperature rise. Using the commercial software NASTRAN, critical buckling temperature of the structure is obtained through an eigenvalue buckling analysis. With the critical buckling temperature as a parameter, hybrid FEM-SEA is used to analyze the virbo-acoustic responses of the reinforced conical shell thermally stressed by various temperature loads. It is found that due to the softening effect of thermal stress, the response peaks shift to lower frequencies. The overall radiation pressure and radiation efficiency decrease with the increase of temperature, while the velocity response of a concerned point doesn't have a monotonic increase since the mode shapes of the structure change significantly as the temperature approaches the critical buckling temperature.

Keywords: Hybrid FEM-SEA, Vibro-Acoustic, Thermal Stress, Reinforced Conical Shell.

1. INTRODUCTION

One of the problems encountered by hypersonic aircraft, such as NASA X43A, is high thermal and acoustic environment, to which the aircraft is subjected during a great portion of the flight envelope. The high thermal load generated by aerodynamic heating causes internal stress and may possibly buckle the panels of the structure, thus changing the dynamic characteristics significantly.

Numerical studies have been progressed on the response of thermally stressed structures. A finite element model of a thin isotropic beam, including the effect of thermal and acoustic loads, was formulated by James Lock and Chuh Mei.[1] Jeyaraj Chandramouli Padmanabhanl et al.[2] have studied on the vibration and acoustic response characteristics of an isotropic rectangular plate in a thermal environment with critical buckling temperature as a parameter, using coupled boundary element method and finite element method (coupled BEM-FEM). The same analysis procedure as in Ref. [2] is employed to investigate the response of a composite plate with inherent material damping and functionally graded elliptic disc in Refs. [3] and [4] respectively. Behnke et al.[5] use Abaqus to investigate the response of a representative integrated thermal protection system under combined thermal, aerodynamic and acoustic loading. Thermal effect is taken into account by means of thermal prestress, the same way as in Ref. [2], based on which analysis is carried on.

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Basically speaking, there are three kinds of main methods of analyzing vibro-acoustic response, FEM, BEM and SEA (Statistical energy analysis).

When external coupled vibro-acoustic problems are concerned, coupled BEM-FEM is often employed to investigate the dynamic response of structures coupled with internal or external acoustics field.[2–6] In an analysis using coupled BEM-FEM, the structure is usually modeled with finite elements and the acoustic field modeled with boundary elements. As a deterministic approach, although it needs much smaller degrees of freedom than FEM, the model is still too large when the coupled system is complex or high-frequency characteristics is concerned. Therefore coupled BEM-FEM is limited for a low-frequency problem in which a system is entirely deterministic.

Statistical energy analysis (SEA) represents a way to study the response characteristics for a high-frequency problem. In SEA, statistical descriptions are employed and a system is entirely statistical.[7,8] SEA describes the storage and transfer of vibration energy between subsystems of weakly coupled models. Therefore SEA is often used in a high-frequency analysis.[9]

Shorter and Langley have developed a hybrid method combining FEM and SEA, i.e., hybrid FEM-SEA, providing a flexible way to account for necessary deterministic details in a vibro-acoustic analysis without requiring that the entire system be modeled deterministically. This method provides a potential solution for the mid-frequency problem.[6,10]

It is observed that limited work has been done to reveal the vibro-acoustic response of thermally stressed structure based on hybrid FEM-SEA. In this paper, the vibro-acoustic response of a reinforced conical shell, a typical structure widely used in aerospace industry, is investigated in a wide frequency band up to 1200 Hz. The commercial software NASTRAN is used to evaluate the natural frequencies of the thermally stressed shell, and VA One is used to calculate vibro-acoustic response using hybrid FEM-SEA method.

2. THEORETICAL BASIS

2.1. Eigenvalue of a Prestressed Structure

As in Ref. [2], by assuming that the conical shell is subjected to a uniform temperature increase, the critical buckling temperature is first evaluated. Then it is used as a parameter to investigate the effect of thermal load on the vibro-acoustic response.

Internal stress induced by thermal increase ΔT is used to calculate the differential stiffness matrix $[K]_\sigma$. An eigenvalue buckling analysis taking account of the effect of thermal stress can be formulated as,

$$([K]+\lambda_i[K]_\sigma))\{\phi\}=\{0\} \tag{1}$$

where $[K]$ is the structural stiffness matrix and λ_i is an scalar multiplier for the applied thermal load ΔT. The product of the lowest value λ_1 and the temperature increase ΔT yields the critical buckling temperature T_{cr}.

Once the critical buckling temperature T_{cr} is determinate, the natural frequencies of the structure preloaded to various thermal loads below T_{cr} can be gained by solving the eigenvalue equation

$$(-\omega^2[M]+([K]+[K]_\sigma))\{\phi\}=\{0\} \tag{2}$$

where $[M]$ is the structural mass matrix.

2.2. Hybrid FEM-SEA Theory

In an analysis of hybrid FEM-SEA, the system is partitioned into a set of coupled subsystems, as illustrated in Figure 1.

A set of degrees of freedom q_1 is used to describe the displacement response of the deterministic subsystems and across the 'deterministic' and 'hybrid' junctions, while another set q_2 is defined to describe the displacement field across the statistical subsystem. Thus the combination of q_1 and q_2 represents the displacement field of the system, written as

$$q=\{q_1^T q_2^T\} \tag{3}$$

The dynamic stiffness matrix for the deterministic subsystems K_d is obtained at a given frequency of interest ω using FEM. The equations of motion for the deterministic subsystems can then be written as

$$K_d q=f_d \tag{4}$$

where f_d is a vector of generalized forces associated with external excitations applied to the deterministic degrees of freedom q.

The response of a statistical subsystem with an uncertain boundary can be described in terms of the superposition of a direct field and a reverberant field. The direct field describes the outgoing displacement field associated with a prescribed displacement of the deterministic boundary in the absence of the random boundary. The reverberant field satisfies a blocked boundary condition across the deterministic boundary the prescribed boundary condition across the random boundary, when added to the direct field.

The equation of motion for the mth statistical subsystem can be derived using a direct boundary element method and written as

$$K^m_{\text{dir}} q=f+f^m_{\text{rev}} \tag{5}$$

where K^m_{dir} is the dynamic stiffness matrix for the mth statistical subsystem, f is the vector of generalized forces and f^m_{rev} is the blocked reverberant force on the connection degrees of freedom.

Assemble the total dynamic stiffness matrix for the system

$$K_{\text{tot}}=K_d+\sum_m K^m_{\text{dir}} \tag{6}$$

Thus, the equation of motion for the system can then be written as

$$K_{\text{tot}} q=f+\sum_m f^m_{\text{rev}} \tag{7}$$

The total dynamic stiffness matrix K_{tot} for the system is found by adding the dynamic stiffness of the deterministic subsystems to the direct field dynamic stiffness of the statistical subsystems.

More details of the Hybrid FEM-SEA theory are documented in Ref. [7].

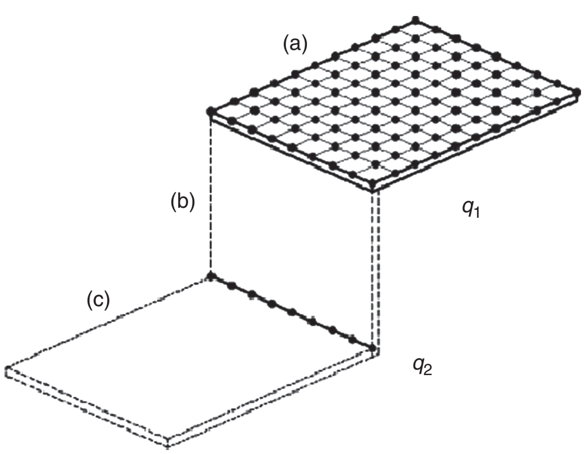

Fig. 1. Degrees of freedom q_1 and q_2 are used to describe response of deterministic subsystem (a) and junction between statistical subsystems (b) and (c).

3. VALIDATION STUDIES

An isotropic plate with the dimensions $0.5\times0.4\times0.1$ m^3 investigated in Ref. [2] is now considered for validation studies. The plate is of the density $\rho_s=7850$ kg/m,^{3} Young's modulus $E=2.1\times10^{11}$ N/m^2 and Poisson's ratio $\nu=0.3$. The critical buckling temperature (T_{cr}) and thermally stressed natural frequencies of the plate with all edges clamped are obtained. The results are compared with Ref. [2] in Tables I and II.

It is clear that the results are in excellent agreement with Ref. [2].

Table I. Validation of the critical buckling temperature.

Critical buckling temperature (°C)		
Ref. [2]	Present	Error (%)
139	139.7	0.504

Table II. **Validation of natural frequencies.**

Natural frequency (Hz)		Uniform temperature applied on the plate (°C) 0 T_{cr}	0.5 T_{cr}	0.99 T_{cr}
1st mode	Ref. [2]	465	333	47
	Present	464.3	331.0	46.6
	Error (%)	0.151	0.601	0.851
2nd mode	Ref. [2]	818	670	481
	Present	809.9	663.6	475.6
	Error (%)	0.990	0.955	1.12
3rd mode	Ref. [2]	1066	922	756
	Present	1058.2	916.61	750.7
	Error (%)	0.731	0.584	0.701

4. ANALYSIS AND RESULTS

The reinforced conical shell clamped at two ends in a reverberant enclosure is assumed to be subjected to a uniform temperature rise to the critical buckling temperature obtained in the first step, shown in Figure 2. The structure has the dimensions of diameter $d_{\text{bottom}} = 1.5$ m, $d_{\text{top}} = 0.9$ m, height $h = 1.2$ m and thickness $t = 0.012$ m. The shell is assumed to be subjected to a uniform temperature rise to the critical buckling temperature obtained in the first step. The acoustic pressure applied on the structure in the vibro-acoustic analysis step is 160 dB. This is a structure much more complex than the rectangular plate analyzed in Ref. [2], which indicates the characteristics of vibro-acoustic response may be different from that of a rectangular plate.

The critical buckling temperature and thermally stressed natural frequencies of the shell are firstly evaluated using NASTRAN by FEM. Thereafter, the characteristics of vibro-acoustic response are obtained by hybrid FEM-SEA using VA One. The acoustic field is modeled with SEA method, while the structure is modeled with finite elements. The finite-element size which is mostly 0.045×0.045 m is chosen after a convergence study of the critical buckling temperature, satisfying an acoustics analysis up-limited to 1200 Hz. This limitation is determinate by the requirement of six elements per wavelength at least.

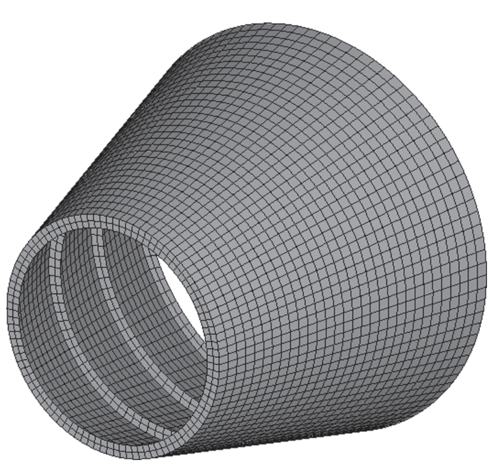

Fig. 2. Reinforced conical shell.

Table III. **Natural frequencies (Hz) of the reinforced conical shell prestressed by various temperature loads.**

	0 T_{cr}	0.5 T_{cr}	0.99 T_{cr}
1st mode	378	346	124
2nd mode	399	366	190
3rd mode	494	444	218
4th mode	497	472	231
5th mode	600	496	234

4.1. Evaluation of the Critical Buckling Temperature and Eigenvalues

The critical buckling temperature of the reinforced conical shell clamped at two ends is 440.66 °C with the assumption that there is no initial stress at 0 °C. With T_{cr} as a parameter, the natural frequencies of the structure are gained by FEM when the shell is subjected to uniform thermal loads of 0 °C, 0.25 T_{cr}, 0.5 T_{cr}, 0.75 T_{cr}, 0.99 T_{cr} respectively, shown in Table III.

Table III demonstrates that natural frequencies decrease with the rise of temperature, which indicates that the peaks of the vibro-acoustic response may shift to lower frequency. Especially for the 0.99 T_{cr} condition, the fifth natural frequency of the structure are lower than the first under others.

According to Table III and Eq. (2), it is concluded that the increase of temperature has a softening effect on the structure. Based on an eigenvalue buckling analysis, it makes sense that the first natural frequency approaches zero as temperature reaches T_{cr}.

Figure 3 illustrates the mode shapes of the clamped conical shell prestressed by different thermal loads. For this reinforced conical shell, although the internal stress distribution due to thermal load is the same for various temperatures since both the boundary condition and the model are symmetrical, the mode shapes change significantly except several low-order modes when the temperature is relatively lower. The mode shapes under high temperature load are comprised of many local modes. This is different from the rectangular plate discussed in Ref. [2], of which the changes of mode shapes induced by temperature rise can be neglected.

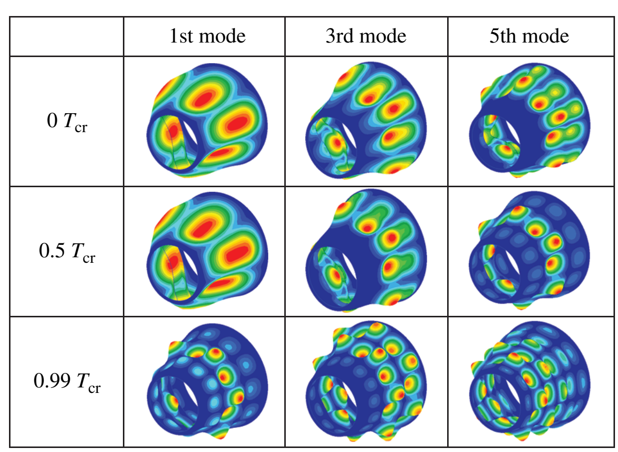

Fig. 3. Mode shapes of the reinforced conical shell subjected to different uniform temperature loads.

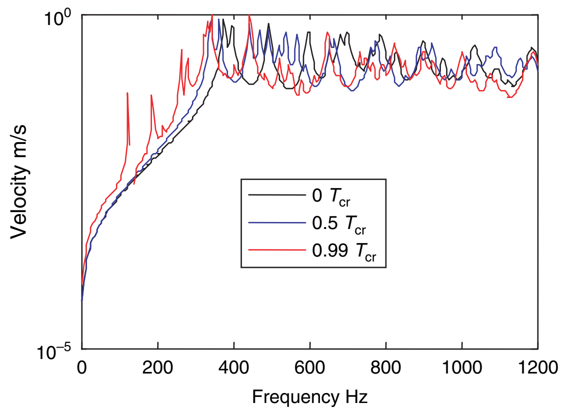

Fig. 4. Velocity of a concerned point with the reinforced conical shell subjected to different uniform temperature loads.

4.2. Characteristics of Vibro-Acoustic Response

Based on the eigenvalue analysis accomplished by NASTRAN, the commercial software VA One is used to investigate the vibro-acoustic response by hybrid FEM-SEA.

Radiation efficiency and velocity of a concerned point of the shell are obtained with the structure subjected to various uniform temperature loads. Radiation pressure from structure to air is calculated at a reference point.

It can be seen that the peaks of all these response curves shift to lower frequencies as the temperature increases, which is due to the softening effect of the thermal stress.

Figure 4 shows the velocity response of the concerned point, and Figure 5 shows the RMS of velocity in the whole analysis frequency band. It can be found that as a relatively complex system, RMS doesn't have a monotonic change as that of the plate discussed in Ref. [2], since the influence of thermal load on the structure becomes lager and the mode shapes of the reinforced conical shell change significantly with the increase of the uniform load temperature.

In Figure 6, the radiation efficiency decreases significantly with the increase of temperature. As the load temperature approves T_{cr}, the peaks shift to lower frequencies and the similarity between the curves becomes poorer.

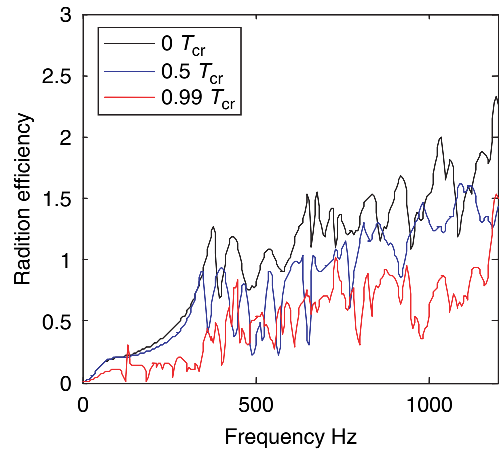

Fig. 6. Radiation efficiency for the reinforced conical shell subjected to different uniform temperature loads.

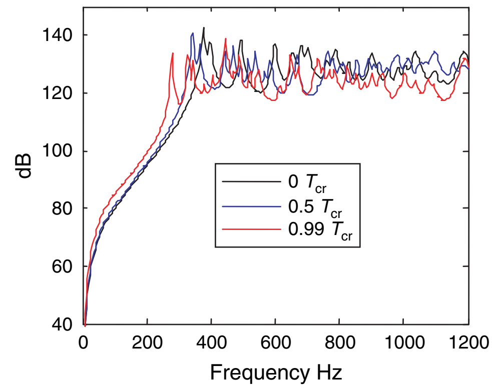

Fig. 7. Radiation pressure from the reinforced conical shell to reference point with the shell subjected to different uniform temperature loads.

Figure 7 illustrates the radiation pressure from the conical shell to reference point in this reverberant acoustic field. The peaks of the radiation pressure shift to lower frequencies and its overall value shown in Figure 8 decreases with the increase of temperature.

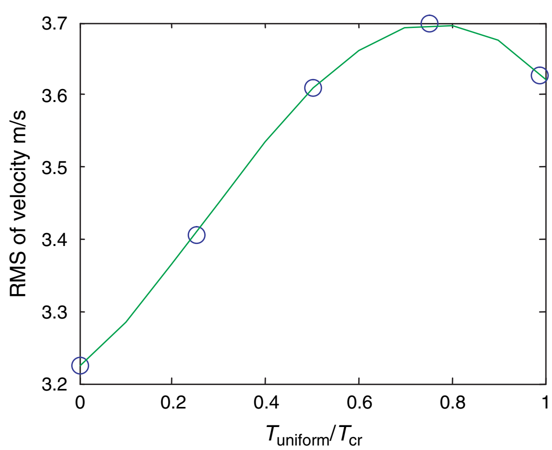

Fig. 5. RMS of velocity of concerned point in the entire frequency band with the reinforced conical shell subjected to different uniform temperature loads.

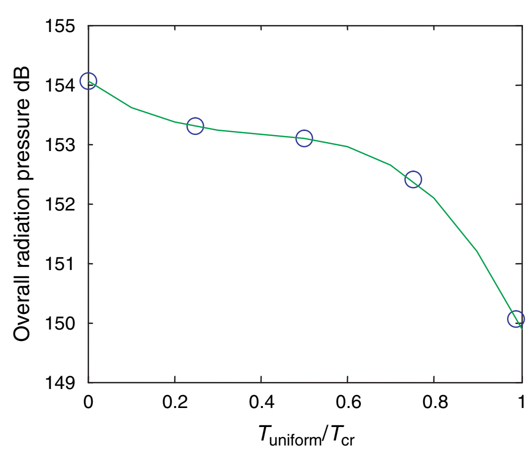

Fig. 8. Overall radiation pressure from the reinforced conical shell to reference point with the shell subjected to different uniform temperature loads.

5. CONCLUSION

Based on hybrid FEM-SEA, the vibro-acoustic response of a reinforced conical shell clamped at two ends and subjected to a uniform temperature rise is investigated in a wide frequency band up to 1200 Hz. The critical buckling temperature is firstly evaluated though an eigenvalue buckling analysis. Thereafter, it is employed as a parameter to investigate the thermally stressed natural frequencies. Then, the commercial software VA One is used to analyze the vibro-acoustic response in a reverberant enclosure by modeling structure with FEM and acoustic field with SEA. It is observed that the thermal stress induced by temperature rise has a significant effect on the vibro-acoustic response. Due to the softening effect of the thermal stress, the response peaks shift to lower frequencies. The overall radiation pressure and the radiation efficiency decrease with the increase of temperature, while velocity response of a concerned point doesn't have a monotonic increase, since the mode shapes of the reinforced conical shell change significantly as the temperature approaches the critical buckling temperature.

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