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Static and dynamic responses analysis for 1–3 piezocomposite laminates with Interdigitated Electrodes

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Abstract. The static and dynamic responses analysis for the 1–3 piezocomposite laminates equipped with Interdigitated Electrodes (IDE) is performed with analytical method. In the static analysis, the solution of the derived governing differential equations is obtained through the power series expansion and Fourier expansion methods. In the dynamic analysis, the governing equations for the transverse vibration are established based on the thin plate theory, and the solutions are obtained through the separation of variables and Fourier expansion methods. The illustrative analysis is carried out to investigate the static and dynamic characteristics of the laminated plates. The numerical results show that, the magnitude of shear curvature gradually increases with the increment of the stiffness anisotropy and the free-strain anisotropy of the piezocomposite and the vibration suppression can be also achieved by adopting a time-dependent control voltage to the IDE of the piezocomposite.

Keywords: 1–3 piezocomposite, static and dynamic responses, Analytical method, The Interdigitated electrode

1. Introduction

In the recent years the study of adaptive structures has attracted significant researchers, due to their potential benefits in a wide range of applications [1,2]. The use of piezoelectric piezoelectric materials, in the form of reinforced fibers embedded in the laminated composite structures, can provide structures that combine the superior mechanical properties of composite materials and the capability to sense and adapt their response. Different from traditional electrodes, an Interdigitated Electrode, i.e. IDE, was proposed by Hagood et al. [3] (see Fig. 1). With interdigitated electrodes, the piezoelectric fibers in piezocomposite layer are poled along the fiber length, causing an anisotropy piezoelectric deformation, and thus the effective constant of piezocomposite, e_{11}, will play an important dominant role on the active control.

Investigators have developed several numerical and analytical models for the static and dynamic control of laminated plates with piezoelectric ceramic sensors and actuators. A pioneering work is due to Alik and Hughes [4], which analyzed the interactions between electricity and elasticity. Ray et al. [5] studied the exact static analysis of piezoelectric plates under cylindrical bending. Comparing with the analysis of laminate plates with piezoelectric layers, the work reported in the area of laminated plate with 1–3 piezocomposite layers is still quite limited, especially for the analytical analysis of the static and dynamic control of laminates with 1–3 piezoelectric composite layer equipped with the IDE. In this paper, the static

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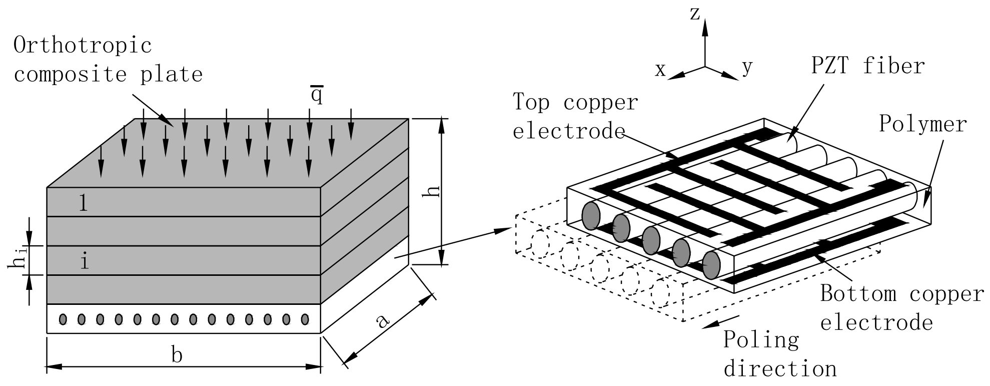

Fig. 1. A N-layer rectangular composite laminate with 1–3 piezoelectric composite layer.

and dynamic responses analysis has been conducted for rectangular composite laminates which consists of 1–3 piezoelectric fiber-reinforced composites layers subjected to the electromechanical loadings by the analytical method. The numerical examples are presented to investigate static and dynamic characteristics of the laminated plates.

2. Static response analysis

The rectangular laminated plate considered in the static case illustrated in Fig. 1. A pressure is applied on the upper surface of the laminated plate and a control voltage is applied to the 1–3 piezocomposite layer with interdigitated electrodes.

In the absence of body forces and electric charge density, the field equation of elastic equilibrium and Gauss' law of electrostatics are

$$\sigma_{ij,j} = 0; \quad D_{i,i} = 0 \tag{1}$$

where σ_{ij} is stress tensor, D_i the electric displacement vector, a comma denotes partial differentiation with respect to the coordinate x_i. The constitutive equations of 1–3 piezocomposite medium are

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k; \quad D_i = e_{ikl}\varepsilon_{kl} + \in_{ik} E_k \tag{2}$$

Substituting Eqs (2) into (1), the governing differential equations of the 1–3 piezocomposite laminated plate are obtained as follows

$$C_{11}u_{,xx} + C_{66}u_{,yy} + C_{55}u_{,zz} + (C_{12}+C_{66})v_{,xy} + (C_{13}+C_{55})w_{,xz} + e_{11}\phi, xx + e_{26}\phi, yy + e_{35}\phi_{,zz} = 0 \tag{3}$$

$$C_{66}v_{,xx} + C_{22}v_{,yy} + C_{44}v_{,zz} + (C_{12}+C_{66})u_{,xy} + (C_{23}+C_{44})w_{,yz} + (e_{12}+e_{26})\phi, xy = 0 \tag{4}$$

$$C_{55}w_{,xx} + C_{44}w_{,yy} + C_{33}w_{,zz} + (C_{13}+C_{55})u_{,xz} + (C_{23}+C_{44})v_{,yz} + (e_{35}+e_{13})\phi, xz = 0 \tag{5}$$

$$e_{11}u_{,xx} + e_{26}u_{,yy} + e_{35}u_{,zz} + (e_{12}+e_{26})v_{,xy} + (e_{13}+e_{,35})w_{,xz} - \in_{11}\phi_{,xx} - \in_{22}\phi_{,yy} - \in_{33}\phi_{,zz} = 0 \tag{6}$$

Here, we adopt the simply supported conditions. The interdigitated electrodes are applied on the top and bottom surfaces of the layer, the electric potential on these two surfaces can be expressed as $\phi = \bar{\phi}(x)$ for $z = 0, b$. Then, the displacement functions satisfying the simply supported edge boundary conditions are given by

$$u = \sum_{k=0}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}(k) z^k \cos(m\pi x/a)\sin(n\pi y/b) \tag{7}$$

$$v = \sum_{k=0}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}(k) z^k \sin(m\pi x/a)\cos(n\pi y/b) \tag{8}$$

$$w = \sum_{k=0}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} D_{mn}(k) z^k \sin(m\pi x/a)\sin(n\pi y/b) \tag{9}$$

The electric voltage induced by the IDE also can be expanded into Fourier series, as follows:

$$\Phi = \sum_{k=0}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \phi_{mn}(k) z^k \cos(m\pi x/a)\sin(n\pi y/b) \tag{10}$$

After substituting Eqs (7)–(10) into the governing Eqs (3)–(6), the recurrence relations of $A_{mn}(k), B_{mn}(k), D_{mn}(k), \phi_{mn}(k)$ can be obtained. Given m and n, there are six unknowns for each layer i.e. $A_{mn}(0)$, $A_{mn}(1)$, $B_{mn}(0)$, $B_{mn}(1)$, $D_{mn}(0)$ and $D_{mn}(1)$. These unknowns can be obtained by satisfying the boundary conditions on the outer surface and the continuity conditions on the interface between two adjacent layers. For details please refer to the reference [6]. Thus, the static displacements of the laminates can be solved.

3. Transverse dynamic response analysis

The laminated plate considered in dynamic case consists of a orthotropic graphite/epoxy composite layer sandwiched between two 1–3 fiber-reinforced piezoelectric composite layers, and the control voltages are applied to the 1–3 piezocomposite layers equipped with the IDE. According to the thin plate theory, the displacements of the laminates are

$$u(x,y,z,t)=u_0(x,y,t) - z\frac{\partial w}{\partial x}; \quad v(x,y,z,t)=v_0(x,y,t) - z\frac{\partial w}{\partial y}; w(x,y,z,t)=w(x,y,z,t) \tag{11}$$

Ignoring the body force, the vibration equation for transverse response of the laminated plate can be expressed as

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\partial y} + \frac{\partial^2 M_y}{\partial y^2} - \bar{\rho}h\frac{\partial^2 w}{\partial t^2} = 0 \tag{12}$$

where M_x, M_y and M_{xy} are stress couples and $\bar{\rho}h = 2\rho^p h_p + \rho^e h_e$. The superscripts "$e$" and "$p$" denote the orthotropic composite material and the 1–3 piezoelectric composites material, respectively. ρ is the density. The simply supported boundary conditions are also adopted here, and the initial conditions are

$$w(x,y,0) = \Phi_w(x,y), \quad \dot{w}(x,y,0) = \Psi_w(x,y) \tag{13}$$

Table 1
Material properties of the elastic composites

C_{11}	C_{22}	C_{33}	C_{12}	C_{13}	C_{23}	C_{44}	C_{55}	C_{66}	(Units)
13.03	7.563	7.815	2.721	3.036	2.766	4.305	5.158	2.542	10^9Pa

Here we assume that the electric field has the opposite sign in the piezocomposite layers symmetric about the middle plane. for the transverse vibration, the corresponding motion equation is expressed as

$$\frac{\partial^2 w}{\partial t^2} + d\frac{\partial w^4}{\partial x^4} + e\frac{\partial^4 w}{\partial x^2 \partial y^2} + f\frac{\partial^4 w}{\partial y^4} = -p\frac{\partial^2 \tilde{V}}{\partial x^2} - q\frac{\partial^2 \tilde{V}}{\partial y^2} \tag{14}$$

In order to use the separation of variables method to solve Eq. (14), the transverse displacement $w(x, y, t)$ is sought in the form

$$w(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \eta_{mn}(t; \tilde{V}) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{15}$$

The control electric field induced by the IDE can be expanded into Fourier series as follows

$$\tilde{V}(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \hat{V}_{mn}(t) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{16}$$

Substituting Eqs (15) and (16) into (14), we have

$$\begin{aligned} &\ddot{\eta}_{mn} + [d(\frac{m\pi}{a})^4 + e(\frac{m\pi}{a})^2(\frac{n\pi}{b})^2 + f(\frac{n\pi}{b})^4]\eta_{mn} \\ &\qquad = \tilde{V}(t)(\frac{4mp}{na^2} + \frac{4nq}{mb^2})(\cos m\pi - 1)(\cos n\pi - 1) \end{aligned} \tag{17}$$

According to Eq. (13), the initial conditions are also expanded into Fourier series. The coupled differential Eq. (7) together with their initial conditions can be solved numerically. For more details please refer to reference [7].

4. Numerical examples

4.1. Static response analysis

The numerical example in this case is for two layers laminated plate made of orthotropic elastic composite with the thickness $h_2 = 3$ mm as top layer and 1–3 piezocomposite with the thickness $h_1 = 2$ mm as bottom layer. The side lengths of two layers are $a = b = 5$ cm. The material properties for elastic composite and 1–3 piezoelectric composite are listed in Tables 1 and 2, respectively, where $Rc = C_{22}/C_{11}$, $Rd = d_{11}/d_{12}$. The uniformly distributed load is applied on the top surface of the plate, i.e. $q_0 = -0.01$MPa. The maximum electric potential induced by IDE is $\phi_0 = -$ 100V with the number of the electrodes $R = 50$.

Since the numerical analysis indicates that the maximum value of shear curvature at each layer of the plate appears at the four angular points, the analyses for the effects of different factors on the shear curvature of the laminate at point $x = a, y = 0$ along z-axis are carried out. Figure 2. examined

Table 2
Material properties of the 1–3 piezoelectric composites

C_{11}	C_{22}	C_{33}	C_{12}	C_{13}	C_{23}	C_{44}	C_{55}	C_{66}	(Units)
48.35	Rc·C_{11}	Rc·C_{11}	4.995	4.995	4.477	2.962	3.274	3.274	10^9Pa
d_{11}	d_{12}	d_{13}	d_{26}	d_{35}	(Units)	ε_{11}	ε_{22}	ε_{33}	(Units)
Rd·d_{12}	−0.0467	−0.0467	0.0088	0.0088	10^{-9}C·(mN)$^{-1}$	1300ε_0	1.475ε_0	1.475ε_0	CVm^{-1}

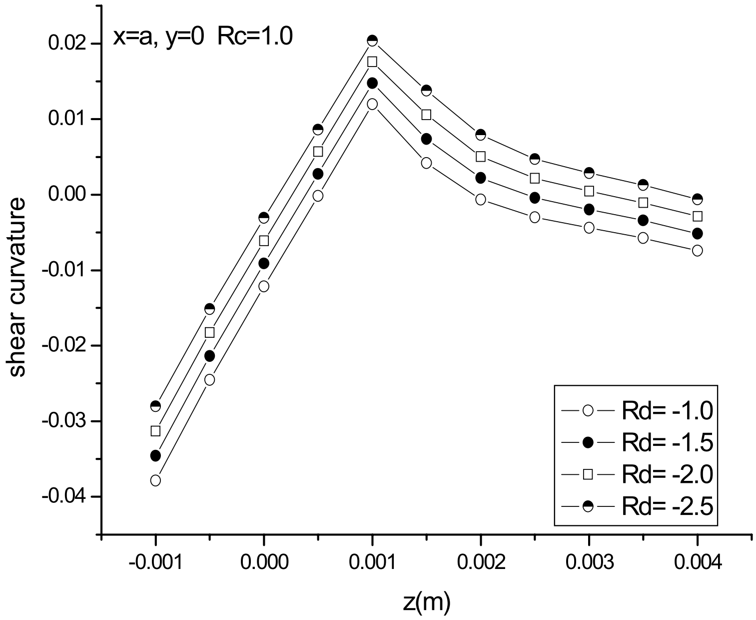

Fig. 2. Distribution of maximum value of shear curvature.

the effect of piezoelectric free-strain anisotropy Rd on the maximum value of shear curvature across thickness. As we can see, Fig. 2. displayed that the maximum value of shear curvature changes slowly at the top layer but increases rapidly at the bottom piezoelectric composite layer. The shear curvature of plate increases continuously with the magnitude of piezoelectric free-stain anisotropy Rd. Figure 3 illustrates the effects of Rc on the distribution of the maximum value of shear curvature along direction z when only the compression load is applied. The negative shear curvature of plate is induced, and it changes slowly in the bottom piezocomposite layer but changes rapidly in the upper elastic composite layer. Furthermore, as the value of Rc increases, the stiffness along direction y and direction z is higher. Thus the shear curvature of plate is smaller.

4.2. Forced response of transverse vibration with active feedback control

In this section, the numerical example is for three layers laminated plate, which consists of a orthotropic elastic composite middle layer with the thickness $h_e = 2$ mm sandwiched by two 1–3 piezo-composite layers with the thickness $h_p = 1$ mm. In this case, the electric field with opposite sign is applied to the top and bottom piezocomposite layers, and induces bending moments as control forces. The active

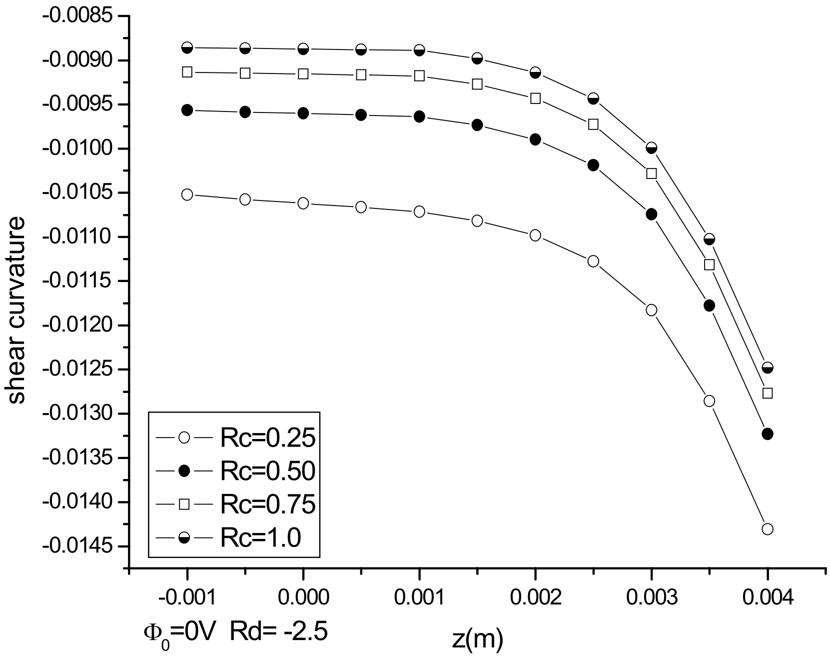

Fig. 3. Distribution of maximum value of shear curvature with different Rc under uniform pressure.

velocity feedback electric field is designed as

$$\tilde{V}(x,y,t) = -r\frac{\partial^3 w}{\partial x \partial y \partial t}(a/20, b/20, t) = -r\frac{mn\pi^2}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\dot{\eta}_{mn}(t)\cos\frac{m\pi}{20}\cos\frac{n\pi}{20}$$

where $\frac{\partial^3 w}{\partial x \partial y \partial t}(a/20, b/20, t)$is the velocity of the shear curvature at x=a/20,y=b/20 and $r$is the control gain given by $r = 10$. The initial conditions are taken as

$$w(x,y,0) = \Phi_w(x,y) = 0 \quad \dot{w}(x,y,0) = \Psi_w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} 0.02\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

In this analysis, the Newmark method is employed to obtain the transient displacements of the plate. The numbers of modes are set to $m = n = 11$ which provide a 121-term approximation for the solutions of transverse vibration. In Figs 4 and 5, the transverse displacement at $x = a/2, y = a/2$ is plotted with time for the two cases of the free vibration and forced vibration. It is obviously that, under the active control electric field, the transverse displacement decreased with time.

5. Conclusion

This study presented an analytical analysis of the static and dynamic responses of laminated plates with 1–3 piezoelectric composite layers equipped with the IDE. The solutions are obtained through the separation of variables and Fourier expansion methods for the static and transverse vibration governing equations. The solution can be obtained to any desired degree of accuracy by truncating the power series

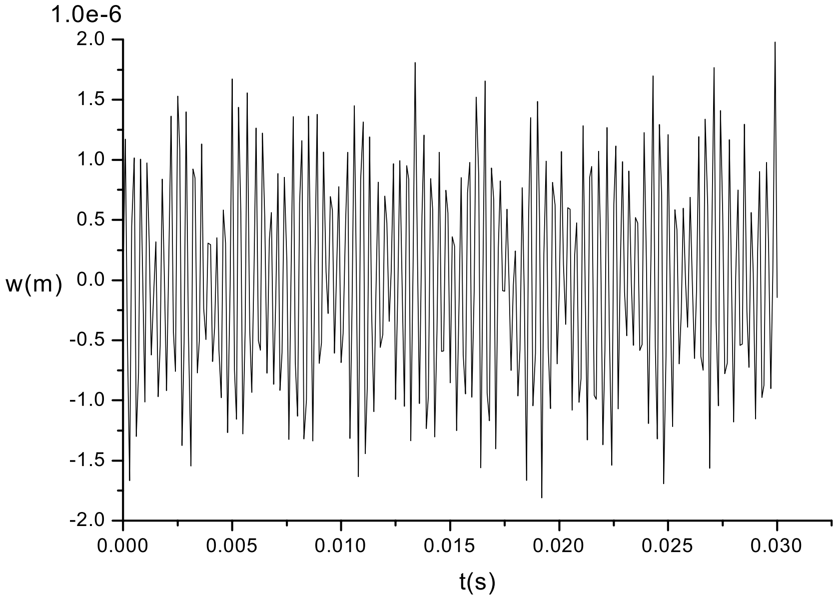

Fig. 4. The transient of the transverse displacement w for free vibration.

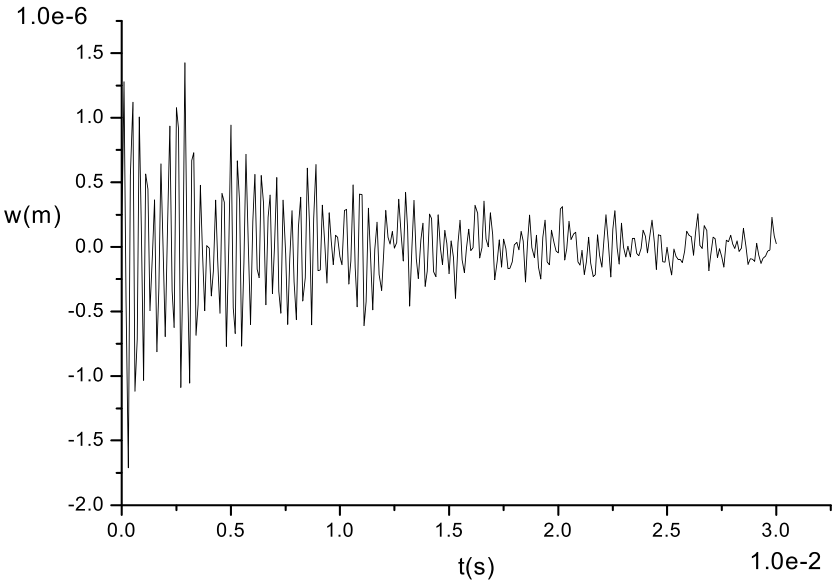

Fig. 5. The transient of the transverse displacement w for the forced vibration.

to appropriate number of terms, which also has some other merits, such as simplicity, convenience and so on. Due to the fact that, with the IDE, the piezoelectric effect along the fiber direction is more significant than that perpendicular to the fiber direction, the anisotropic properties would be created in the laminated plate. Therefore, the effects of stiffness anisotropy, piezoelectric free-strain anisotropy on the shear curvature of a two-ply piezoelectric composite laminate are investigated in the static analysis. In the dynamic analysis, the active control capability and dynamic characteristics of the laminated plates with

1–3 piezocomposite layers is studied through a numerical example. The numerical results show that the maximum magnitude of shear curvature of the laminate increased with the magnitude of Rd, and it also increased as the Rc decreased. Active damping control of vibration response can be implemented by adopting a negative velocity feedback. Results presented here can be used to enhance the understanding of the static and dynamic response behavior of the 1–3 piezoelectric composite structures.

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