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Cosmological twinlike models with multi scalar fields

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In this study, we consider cosmological models driven by several canonical or noncanonical scalar fields, and we show the manner in which twinlike models for a canonical model can be constructed from noncanonical ones using the superpotential method. We conclude that it is possible to construct twinlike models for multifield cosmological models, even with a nonzero spatial curvature. This work extends the discussions of [D. Bazeia, and J. D. Dantas, Phys. Rev. D **85**, 067303 (2012)] to cases with multi scalar fields and with non-vanished spatial curvature, by using a different superpotential method.

cosmological twinlike models, superpotential method, noncanonical scalar field

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1 Introduction

Scalar field plays an important role in cosmology. It can model the inflation that happened in the early universe [1], or act as a dark energy candidate such as quintessence, to explain the recently observed accelerated expansion of the universe (see for example, refs. [2-4]). In quintessential inflation models, scalar field can account for both early inflation and dark energy [5], and canonical scalar field is economical for constructing inflation and dark energy models. However, with the development of effective field theory, increasing attention is on nonrenormalizable field models. The K-field is a typical class of noncanonical scalar field models, whose Lagrangian $\mathcal{L}(\phi, X)$ is an arbitrary function of the scalar ϕ and its kinetic term $X = -\frac{1}{2}g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi$. The K-field was initially introduced to describe early inflation [6-8], and was later been applied in the study of topological defects [9-11], and brane world [12-16], and so on.

In 2010, a new interesting application of K-field was reported in ref. [17]. The authors found that under certain conditions a K-field model can have the same background topological defect solutions than a canonical model. Such a K-field model is dubbed as a "doppelgänger", or a twinlike model of the canonical one. Despite the uniformity at the background level, twinlike models usually have different linear perturbation structures, and are distinguishable in principle (see also refs. [18, 19]). However, it was later found that when some further conditions are satisfied, the twinlike models can even have the same linear structure [20]. Such pairs of twinlike models are referred to as special twinlike models [21, 22]. To distinguish the special twinlike models, the perturbation equations to the nonlinear orders should be considered.

It is interesting to construct twinlike models in various types of scalar field theories. They have been constructed for the self-dual Abelian-Higgs model [23], for compacton solutions [24] and for thick brane world solutions [19, 22, 24]. In addition to models with single scalar fields, there are also works on multifield twinlike models in the case with flat

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space-time [25].

Since scalar field plays an important role in cosmology, it is also interesting to consider cosmological twinlike models. The first cosmological twinlike model was constructed in ref. [26]. Using the first-order formalism [1)], the authors successfully constructed cosmological twinlike models in the case with a single scalar field and with vanished spatial curvature. However, they failed to find twinlike models with nonzero spatial curvatures because of the form of their first-order formalism. They concluded that the appearance of a spatial curvature prevents the construction of twinlike models in a cosmological scenario.

The aim of this study is to show the possibility of constructing twinlike models irrespective of a vanished spatial curvature. To achieve this, first, we apply a more convenient first-order formalism for constructing twinlike models than that used in ref. [26]. Second, we apply our first-order formalism to multiscalar fields cosmology, which proves to be successful. This study corrects the erroneous conclusion of ref. [26] and furthers the work of ref. [25], where multifield twinlike models were constructed only in two-dimensional flat space-time.

This paper is organized as follows. In sect. 2 we first introduce the model and our conventions. In sect. 3, we establish a first-order formalism for cosmological models with n canonical scalar fields and with an arbitrary spatial curvature. Three types of twinlike models are constructed for these canonical models in sect. 4 by using the first-order formalism given in sect. 3. Some explicit examples of twinlike models are given in sect. 5. Our results are summarized in sect. 6.

2 The model

In cosmological models, the space-time geometry is described by the Friedmann-Robertson-Walker (FRW) metric

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu \\ &= -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \end{aligned} \tag{1}$$

where $a(t)$ is the scale factor, and the constant $k = 1$, 0, or -1 corresponds to spherical, flat, or hyperbolic geometry, respectively. Space-time indices are denoted by Greek letters $\mu, \nu = 0, 1, 2, 3$.

The action of our model comprises the following two parts: gravitational and matter parts:

$$S = S_{\rm HE} + S_{\rm M}. \tag{2}$$

As usual, the gravitational part is given by the Hilbert-Einstein action:

$$S_{\rm HE} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R, \tag{3}$$

where G is the gravitational coupling and $g = \det g_{\mu\nu}$ is the determinant of the metric. For convenience, and to compare with ref. [26], we take $4\pi G = 1$. The matter part is described by the following action:

$$S_{\rm M} = \int d^4x\sqrt{-g}\mathcal{L}(\mathcal{G}_{IJ}, X^{IJ}, \phi^I), \tag{4}$$

where $\mathcal{G}_{IJ} = \mathcal{G}_{IJ}(\phi^K)$ is the metric of the field space such that $\phi_I = \mathcal{G}_{IJ}\phi^J$, and $X^{IJ} = -g_{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J/2$ is the kinetic term for n scalar fields $\phi^I = \phi^I(t)$ with $I, J, K = 1, 2, \cdots, n$.

From the Hamiltonian variation principle $\delta S/\delta g_{\mu\nu} = 0$, one can easily obtain the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2T_{\mu\nu}, \tag{5}$$

where the energy-momentum tensor is defined as:

$$T_{\mu\nu} = -2\frac{1}{\sqrt{-g}}\frac{\delta S_{\rm M}}{\delta g_{\mu\nu}}. \tag{6}$$

In particular, as eq. (4) is considered, we have

$$T_{\mu\nu} = \mathcal{L}_{X^{IJ}}\partial_\mu\phi^I\partial_\nu\phi^J + g_{\mu\nu}\mathcal{L}. \tag{7}$$

Here, we have defined $\mathcal{L}_{X^{IJ}} = \partial\mathcal{L}/\partial X^{IJ}$.

After a simplification, the Einstein equations give

$$H^2 = \frac{2}{3}\left(\mathcal{L}_{X^{IJ}}\dot\phi^I\dot\phi^J - \mathcal{L}\right) - \frac{k}{a^2}, \tag{8}$$

$$\dot H = \frac{k}{a^2} - \mathcal{L}_{X^{IJ}}\dot\phi^I\dot\phi^J. \tag{9}$$

An over dot is used to represent the derivative with respect to t, and $H = \dot a/a$ is the Hubble parameter. The Einstein equations can also be rewritten in terms of the energy density ρ and the pressure p of the matter fields:

$$H^2 = \frac{2}{3}\rho - \frac{k}{a^2}, \tag{10}$$

$$\dot H = \frac{k}{a^2} - (\rho + p), \tag{11}$$

where ρ and p are defined as:

$$\rho = -T^0_0 = \mathcal{L}_{X^{IJ}}\dot\phi^I\dot\phi^J - \mathcal{L}, \tag{12}$$

$$p = T^1_1 = \mathcal{L}. \tag{13}$$

Another important quantity in cosmology is the deceleration parameter:

$$q \equiv -\left(1 + \frac{\dot H}{H^2}\right). \tag{14}$$

1) Also known as the superpotential method. See for example refs. [27, 28] for the application of this method in the study of dark energy.

3 Canonical model and the first-order formalism

In this section, we consider the canonical model, for which the field space metric $\mathcal{G}_{IJ} = \delta_{IJ}$, and the Lagrangian density of the scalar fields takes the form:

$$\bar{\mathcal{L}} = \delta_{IJ}\dot{\phi}^I\dot{\phi}^J/2 - V(\{\phi^I\}), \quad (15)$$
$$\bar{\mathcal{L}}_{X^{IJ}} = \delta_{IJ}. \quad (16)$$

Here an over bar denotes the quantities of the canonical model. In this case,

$$\bar{\rho} = \frac{1}{2}\delta_{IJ}\dot{\phi}^I\dot{\phi}^J + V, \quad (17)$$
$$\bar{p} = \frac{1}{2}\delta_{IJ}\dot{\phi}^I\dot{\phi}^J - V, \quad (18)$$

and the Einstein equations reduce to

$$H^2 = \frac{1}{3}\delta_{IJ}\dot{\phi}^I\dot{\phi}^J + \frac{2}{3}V - \frac{k}{a^2}, \quad (19)$$
$$\dot{H} = \frac{k}{a^2} - \delta_{IJ}\dot{\phi}^I\dot{\phi}^J. \quad (20)$$

To solve these equations, we introduce the following first-order formalism [2)]:

$$\dot{\phi}^I = \delta^{IJ}\frac{\partial W}{\partial\phi^J}, \quad (21)$$
$$H = -W + \alpha k Z, \quad (22)$$

where $\alpha > 0$ is a positive parameter, W and Z are functions of $\{\phi^I\}$, and are referred as the superpotentials. For $n = 1$, it is fairly easy to discern the difference between the above first-order formalism from those in ref. [26] (see also refs. [27, 28]), where

$$\dot{\phi} = \alpha k Z - \frac{\partial W}{\partial\phi}, \quad (23)$$
$$H = W. \quad (24)$$

As analyzed in ref. [26], such a first-order formalism leads difficulties for constructing twinlike models with $k \neq 0$. Besides, it is also not easy to generalize eqs. (23) and (24) to multifield models.

Using the first-order formalism eqs. (21) and (22) and the Einstein eqs. (19) and (20), V, ρ, p, and q can be expressed in terms of W and Z:

$$V = \frac{3}{2}(W - \alpha k Z)^2 + \frac{\delta^{IJ}}{2}\frac{\partial W}{\partial\phi^I}\left(3\alpha k\frac{\partial Z}{\partial\phi^J} - \frac{\partial W}{\partial\phi^J}\right),$$
$$\bar{\rho} = \frac{3}{2}\left[(W - \alpha k Z)^2 + \alpha k\delta^{IJ}\frac{\partial W}{\partial\phi^I}\frac{\partial Z}{\partial\phi^J}\right],$$
$$\bar{p} = -\frac{3}{2}(W - \alpha k Z)^2 - \delta^{IJ}\frac{\partial W}{\partial\phi^I}\left(\frac{3\alpha k}{2}\frac{\partial Z}{\partial\phi^J} - \frac{\partial W}{\partial\phi^J}\right),$$
$$\bar{q} = \frac{\delta^{IJ}}{(W - \alpha k Z)^2}\frac{\partial(W - k\alpha Z)}{\partial\phi^I}\frac{\partial W}{\partial\phi^J} - 1. \quad (25)$$

Note that the superpotentials W and Z are not independent. This can be seen when the first-order eqs. (21) and (22) are substituted into eq. (20). After simplifying, we get

$$a^{-2} = \alpha\delta^{IJ}\frac{\partial Z}{\partial\phi^I}\frac{\partial W}{\partial\phi^J}. \quad (26)$$

Taking the derivative of eq. (26) with respect to t, and using eqs. (21) and (22), the following constraint for the superpotentials is obtained:

$$0 = \delta^{IJ}\left\{2(W - \alpha k Z)\frac{\partial Z}{\partial\phi^I}\frac{\partial W}{\partial\phi^J} - \delta^{KL}\frac{\partial W}{\partial\phi^L}\left(\frac{\partial W}{\partial\phi^J}\frac{\partial^2 Z}{\partial\phi^I\partial\phi^K} + \frac{\partial Z}{\partial\phi^I}\frac{\partial^2 W}{\partial\phi^J\partial\phi^K}\right)\right\}. \quad (27)$$

A first-order formalism makes it easy to find analytical solutions for cosmological models. Some examples can be found in refs. [27, 28] for the dark energy. However, the purpose of this paper is to construct twinlike models for the canonical model $\bar{\mathcal{L}}$, i.e., to find models whose Lagrangian $\mathcal{L}$ contains noncanonical kinetic terms but share the same field configuration $\phi(t)$, scale factor $a(t)$, energy density ρ, pressure p, and acceleration parameter q with the canonical model.

4 Twinlike models for the canonical model

In this section, we use the aforementioned first-order formalism to construct twinlike models for the canonical model. We explicitly show that for a canonical model, infinite noncanonical models exist, which have the same background solution and properties as the canonical one. For simplicity, we display only three types of twinlike Lagrangians.

4.1 Type-1 model

The Lagrangian of the first type of twinlike model reads:

$$\mathcal{L} = \mathcal{L}(X, \phi^I), \quad (28)$$

where $X = \delta_{IJ}X^{IJ} = \frac{1}{2}\delta_{IJ}\dot{\phi}^I\dot{\phi}^J$. Using the first-order eq. (21), we obtain the following on-shell condition:

$$X = \frac{1}{2}\delta_{IJ}\frac{\partial W}{\partial\phi^I}\frac{\partial W}{\partial\phi^J}. \quad (29)$$

2) This is a multi-field generalization of the first-order formalism in ref. [27], where the authors first established the same first-order formalism for cosmological models with a single scalar. The first-order formalism method has also been used in multi-field cosmological models with $k = 0$ in refs. [29, 30].

To become a twinlike model of $\bar{\mathcal{L}}$, the noncanonical model has to satisfy the following on-shell equations:

$$\phi| = \bar{\phi}, \quad p| = \bar{p}, \quad \rho| = \bar{\rho}, \quad a| = \bar{a}, \quad q| = \bar{q}. \tag{30}$$

The symbol | here means taking the on-shell condition eq. (29). Obviously, $\phi| = \bar{\phi}$ is already satisfied, because both ϕ and $\bar{\phi}$ satisfy the on-shell condition eq. (21), or equivalently, eq. (29).

Thus, we only need to check the other four equations, let us start with $p| = \bar{p}$. From eqs. (13) and (15) we know that $p| = \bar{p}$ is equivalent to

$$\begin{aligned}\mathcal{L}| &= \delta_{IJ}\dot{\phi}^I\dot{\phi}^J/2 - V(\{\phi^I\}) \\ &= X - V.\end{aligned} \tag{31}$$

Then, from eqs. (12) and (16), we know that $\rho| = \bar{\rho}$ is equivalent to

$$\mathcal{L}_{X^{IJ}}| = \delta_{IJ}. \tag{32}$$

Since $\mathcal{L}_{X^{IJ}} = \frac{\partial \mathcal{L}}{\partial X}\frac{\partial X}{\partial X^{IJ}} = \mathcal{L}_X\delta_{IJ}$, the on-shell equation for the energy density is simply

$$\mathcal{L}_X| = 1. \tag{33}$$

From eqs. (10) and (11), we know that once $\phi| = \bar{\phi}, \rho| = \bar{\rho}$ and $p| = \bar{p}$ are satisfied, the last two equations $a| = \bar{a}$ and $q| = \bar{q}$ will be automatically satisfied, because both a and $\bar{a}$ can be solved by introducing the first-order eq. (22) along with the constraint eq. (27). Finally, the deceleration parameter is defined only by the scale factor, thus, $q| = \bar{q}$, if $a| = \bar{a}$.

Now, we are ready to construct the first type of twinlike model for $\bar{\mathcal{L}}$. One of the simple Lagrangian that satisfies all the on-shell eq. (30) is

$$\mathcal{L} = X - V + \sum_{i=2}^{+\infty} U_i\left(X - \frac{\delta_{IJ}}{2}\frac{\partial W}{\partial \phi^I}\frac{\partial W}{\partial \phi^J}\right)^i, \tag{34}$$

where $U_i = U_i(G_{IJ}, X^{IJ}, \phi^K)$ are arbitrary functions.

Obviously, under the on-shell eq. (29), $\mathcal{L}| = \bar{\mathcal{L}}$ and $\mathcal{L}_X| = 1$; therefore, $\mathcal{L}$ describes a twinlike model for $\bar{\mathcal{L}}$. However, $\mathcal{L}$ and $\bar{\mathcal{L}}$ are essentially two different models. For example, $\mathcal{L}_{XX}| = 2U_2(G_{IJ}, X^{IJ}, \phi^K) \neq \bar{\mathcal{L}}_{XX} = 0$. Such difference appears when the linear perturbations are considered (see the discussion of refs. [21,22]).

4.2 Type-2 model

In this type of Lagrangain

$$\mathcal{L} = \mathcal{L}(\tilde{X}, X, \phi^I), \tag{35}$$

where

$$\tilde{X} \equiv \frac{1}{2}G_{IJ}(\phi^K)\frac{\partial W}{\partial \phi^I}\frac{\partial W}{\partial \phi^J}. \tag{36}$$

In this case, the symbol | would represent two on-shell conditions for ϕ^I:

$$X = \frac{1}{2}\delta_{IJ}\frac{\partial W}{\partial \phi^I}\frac{\partial W}{\partial \phi^J}, \quad \tilde{X} = \frac{1}{2}G_{IJ}\frac{\partial W}{\partial \phi^I}\frac{\partial W}{\partial \phi^J}. \tag{37}$$

Obviously, the condition $p| = \bar{p}$ still requires $\mathcal{L}| = X - V$, but now the condition $\rho| = \bar{\rho}$ imposes one more equation:

$$\mathcal{L}_{\tilde{X}}| = 0, \tag{38}$$

in addition to eq. (33). A Lagrangian that satisfies all the on-shell equations is

$$\begin{aligned}\mathcal{L} =& X - V + \sum_{i=2}^{+\infty} c_i\left(X - \frac{\delta_{IJ}}{2}\frac{\partial W}{\partial \phi^I}\frac{\partial W}{\partial \phi^J}\right)^i \\ &+ \sum_{i=2}^{+\infty} d_i\left(\tilde{X} - \frac{G_{IJ}}{2}\frac{\partial W}{\partial \phi^I}\frac{\partial W}{\partial \phi^J}\right)^i,\end{aligned} \tag{39}$$

where both c_i and d_i are arbitrary functions of G_{IJ}, X^{IJ} and ϕ^I. Once the superpotential W is specified, $\mathcal{L}$ can be expressed in terms of $\tilde{X}$, X and ϕ^I.

4.3 Type-3 model

Now, we present the last type of twinlike model, whose Lagrangian reads

$$\mathcal{L} = \mathcal{L}(X^{IJ}, G_{IJ}(\phi^K), X, \phi^I). \tag{40}$$

The on-shell symbol | in this case represents the following conditions:

$$X = \frac{1}{2}\delta_{IJ}\frac{\partial W}{\partial \phi^I}\frac{\partial W}{\partial \phi^J}, \quad X^{IJ} = \frac{1}{2}\frac{\partial W}{\partial \phi^I}\frac{\partial W}{\partial \phi^J}. \tag{41}$$

To be a twinlike model of $\bar{\mathcal{L}}$, the Lagrangian $\mathcal{L}$ must satisfy the following on-shell equations:

$$\mathcal{L}| = X - V, \quad \mathcal{L}_X| = 1, \quad \mathcal{L}_{X^{IJ}}| = 0. \tag{42}$$

One of the possible Lagrangian is

$$\begin{aligned}\mathcal{L} =& X - V + \sum_{i=2}^{+\infty} c_i\left(X - \frac{\delta^{IJ}}{2}\frac{\partial W}{\partial \phi^I}\frac{\partial W}{\partial \phi^J}\right)^i \\ &+ \sum_{i=2}^{+\infty} d_i\left(X^{IJ} - 2\frac{\partial W}{\partial \phi^I}\frac{\partial W}{\partial \phi^J}\right)^i.\end{aligned} \tag{43}$$

Once again, c_i and d_i are arbitrary functions of G_{IJ}, X^{IJ} and ϕ^I. There might be some residual field indices $I, J, \cdots$ in the second line of eq. (43), which can be contracted by constructing suitable coefficient $d_i(G_{IJ}, X^{IJ}, \phi^I)$ with appropriate indices.

5 Explicit solutions

In this section, we explore the manner in which a first-order formalism can be used to reproduce some important cosmological inflation models that have been reported in literature.

5.1 Single field

We first consider the case with $n = 1$, and

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{44}$$

The first-order equations are

$$\dot{\phi} = \frac{\partial W}{\partial \phi}, \tag{45}$$

$$H = -W + \alpha k Z. \tag{46}$$

In case $k \neq 0$, W and Z should satisfy the following constraint:

$$2(W - \alpha k Z)Z_\phi = \left(W_\phi Z_{\phi\phi} + Z_\phi W_{\phi\phi}\right), \tag{47}$$

where $W_\phi \equiv \frac{\partial W}{\partial \phi}$, $W_{\phi\phi} \equiv \frac{\partial^2 W}{\partial \phi^2}$, and so on. The scalar potential reads

$$V = \frac{3}{2}(W - \alpha k Z)^2 + \frac{1}{2}W_\phi\left(3\alpha k Z_\phi - W_\phi\right). \tag{48}$$

Consider, for example,

$$W = m\phi, \quad \frac{\partial W}{\partial \phi} = m, \tag{49}$$

where m is a constant, then the constraint eq. (47) reduces to

$$2(m\phi - \alpha k Z)Z_\phi = m Z_{\phi\phi}. \tag{50}$$

For $k \neq 0$

$$Z = \frac{m}{\alpha k}\phi. \tag{51}$$

Therefore, the scalar potential is

$$V = \frac{3}{2}m^2\phi^2 - \frac{1}{2}m^2, \tag{52}$$

for $k = 0$, and

$$V = m^2, \tag{53}$$

for $k \neq 0$. An inflation model of such potential with $k = 0$ was discussed in the textbook [1]; so we will not repeat the discussions here. However, when $k \neq 0$, eq. (22) reads $H = 0$; therefore it predicts an unfavorable static universe.

Now that we have the explicit expression of W, we can express the Lagrangian of the twinlike models in terms of the field. For type-1 model the result is

$$\mathcal{L} = X - V + \sum_{i=2}^{+\infty} U_i\left(X - \frac{m^2}{2}\right)^i, \tag{54}$$

where $X = \frac{1}{2}\dot{\phi}^2$ and U_i are some arbitrary functions of X and ϕ. For the single field system here, a similar result can be seen in the three models.

5.2 Multi-field models

Using the first-order formalism in sect. 3 we can also reproduce some multi-field inflation models. For example, when $k = 0$ the superpotential

$$W \propto w_0 \exp(\lambda_I \phi^I) \tag{55}$$

would lead to the following scalar potential:

$$V \propto \exp(\lambda_I \phi^I). \tag{56}$$

Here λ_I are n constant coefficients. This potential describes the generalized assisted inflation model [31, 32] (see refs. [33, 34] for the original assisted inflation model). While, by using

$$W \propto \mathbf{e}^{-\phi_1} f(\phi_2), \tag{57}$$

the so-called soft inflation model can be obtained [35].

Once the superpotential W is specified, it is easy to use eqs. (34), (39) and (43) to express the Lagrangian of the corresponding twinlike models.

The construction of models with $k \neq 0$ is difficult because of the complexity of the constraint eq. (27), but it is always possible in principle. By taking a paticular form of W, it is possible to find the solution of Z. After W and Z are obtained, one can immediately write down the scalar potential V.

6 Summary and discussion

In this study, we considered the construction of cosmological twinlike models for cases with n canonical scalar fields for arbitrary spatial curvature. Using a new first-order formalism, we showed a possibility of establishing different types of twinlike models for a given canonical model with arbitrary integer n, regardless of a vanished spatial curvature. This furthers the work of ref. [26], which failed to construct cosmological twinlike models for $k \neq 0$. In fact, the first-order formalism used by the authors of ref. [26] prohibits the existence of the twinlike models for $k \neq 0$. This study can also be regarded as an extension of ref. [25] from Minkowski space to cosmology.

As explicit applications, we reproduced inflation models both for $n = 1$ and $n > 1$. For $n = 1$ case, a linear superpotential $W \propto \phi$ can reproduce a quadratic scalar potential $V \propto \phi^2$ for $k = 0$, and a constant potential for $k \neq 0$.

The former describes a typical inflation model, whereas the later describes a disfavored static universe. For $n > 2$ case, we showed that when $k = 0$, it is possible to reproduce the generalized assisted inflation and the soft inflation models by choosing particular superpotentials. The explicit Lagrangians for the corresponding twinlike models can be easily obtained by simply substituting the superpotentials into eqs. (34), (39) and (43).

We did not offer explicit models in the cases where $n > 2$ and $k \neq 0$, which deserves for a further considerations. Despite twinlike models possess the same background revolution, they usually have rather different behavior when linear perturbation is considered [17]. Therefore, twinlike model can be used in model reconstruction [36]. It is interesting to see the differences in twinlike models when compared with the recent observational data [37-45]. In addition to inflation, the first-order formalism used in this study could also be useful in studies relating to other cosmological issues such as dark energy.

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