Scalar-kinetic branes

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Abstract – This work tries to find out thick brane solutions in braneworld scenarios described by a real scalar field in the presence of a scalar-kinetic term $F(X, \phi) = X\phi^m$ with a single extra dimension, where $X = \frac{1}{2}\nabla_M \phi \nabla^M \phi$ stands for the standard kinetic term and $m = 0, 1, 2 \ldots$. We mainly consider bent branes, namely de Sitter and anti-de Sitter four-dimensional slices. The solutions of a flat brane are obtained when taking the four-dimensional cosmological constant $\Lambda_4 \to 0$. When the parameter $m = 0$, these solutions turn to those of the standard scenario. The localization and spectrum of graviton on these branes are also analyzed.

Introduction. – In the past decade, the braneworld scenario has attracted a lot of interests for it gives an effective way to solve the hierarchy problem by introducing two 3-branes which are embedded in a five-dimensional anti-de Sitter (AdS$_5$) spacetime [1]. As another attractive property, the Newtonian law of gravity with a correction is also given in this braneworld scenario [2]. The branes in [1,2] are set up artificially, and the thickness of each brane is neglected. However, in superstring theory, there seems to exist a fundamental length scale, so the thin brane model should be modified.

In more realistic models [3–8] branes with thickness were realized and fixed by introducing some bulk fields. In the simplest case a single scalar field with standard dynamics was investigated, and the Lagrange density of the scalar field is given by

$$\mathcal{L} = X - V(\phi), \tag{1}$$

where $X = \frac{1}{2}\nabla_M \phi \nabla^M \phi$ stands for the kinetic term, and $V(\phi)$ is the potential. Therefore in the standard thick braneworld scenario, the five-dimensional action reads

$$S = \int \mathrm{d}^4x \, \mathrm{d}y \sqrt{g} \left(-\frac{1}{4}R + \frac{1}{2}\nabla_M \phi \nabla^M \phi - V(\phi) \right). \tag{2}$$

Recently, there have been many works concerning the thick braneworld scenario under various gravity theories. In addition to the traditional Einstein's gravity theory [4,6,9,10], there are works on the thick brane scenario considered in Weyl geometry theory [11–14], or in $f(R)$

gravity theory [15] (for a review of the $f(R)$ gravity see [16]).

Even under the traditional frame of gravity theory, braneworld scenarios have been developed along different directions. Symmetric and asymmetric single or multiple branes were investigated in [17–21]. Some physicists prefer to consider this scenario in higher dimension and multi-scalar, gauge field or vertex backgrounds as shown in [22–26]. Some other recent works concerned the field localization and resonances on deformed branes [27,28], and on spectra of field fluctuations in braneworld models with broken bulk Lorentz invariance [29].

The majority of works we mentioned above mainly concerned flat branes, namely four-dimensional Minkowski slices which are embedded in the bulk, and the dynamics of the bulk fields are always considered to be standard.

For many aspects, we also want to know what would happen if our spacetime were not flat but curved. The most familiar nonflat spacetime geometries with maximum symmetry are the de Sitter (dS) and anti-de Sitter (AdS) ones. Not only because they are also solutions to Einstein equations, but also because they allow nonzero cosmological constants Λ_4 (for dS $\Lambda_4 > 0$ and for AdS $\Lambda_4 < 0$), thus they shed a light for solving the problem of the evolution of our universe and the puzzle of dark energy.

In a thick braneworld model with nonflat spacetime geometry, Einstein equations are difficult to deal with. However, according to the so-called first-order formalism [4,5,7,30–32], one can reduce the second-order Einstein equations to some first-order ones. With this formalism, some discussions have been given on bent thick

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branes [6,9,10]. For a comprehensive review on thick brane solutions and related topics see ref. [33].

On the other hand, models with nonstandard dynamics of background scalar fields are also investigated by a lot of physicists. k-field theory [31,34–36] is a typical model with nonstandard dynamic terms. In this theory the standard kinetic term is generalized to an arbitrary function of X, as a consequence, the Lagrange density reads

$$\mathcal{L} = F(X) - V(\phi). \qquad (3)$$

k-field theory has already played a very important role in cosmology, because it offers a mechanism for the early-time inflation [37]. In another model inspired and developed by string theorists [38], a nonstandard dynamics of the scalar field was suggested in a way very similar to the Born-Infeld extension of standard electrodynamics [39]. In this case, the Lagrange density of the background scalar field is given by

$$\mathcal{L} = -V(\phi)\sqrt{1 - 2X}. \qquad (4)$$

Models with Lagrange density given by (4) or its extensions have shown applications in D-brane and k-essence theories [38]. Obviously, when $X \ll 1$, the Lagrange density (4) reduces to

$$\mathcal{L} = -V(\phi) + V(\phi)X + \mathcal{O}(X^2). \qquad (5)$$

In the present paper, we extend the kinetic term of the scalar field X to a scalar-kinetic coupling form $F(X, \phi) = W(\phi)X$, and then the Lagrange density takes the form

$$\mathcal{L} = -V(\phi) + W(\phi)X. \qquad (6)$$

This model can be applied to describe systems with a static background scalar field $\phi(y)$ which has a tiny variation along only the fifth dimension, *i.e.*, $\mathrm{d}\phi(y)/\mathrm{d}y \approx 0$, and so ϕ as well as $V(\phi)$ can be regarded as some constants. It is well known that in the thin brane scenarios there is no bulk scalar field involved, instead, a bulk cosmological constant is introduced. On the other hand, if one takes the scalar potential as a constant in the thick brane frame, then this constant potential can be regarded as a bulk cosmological constant. Thus, when it is considered a small perturbation of the constant scalar potential along the fifth dimension, such systems can be described by (6). In principle, $W(\phi)$ can be any arbitrary function of ϕ, but for simplicity, we focus only on the case $W(\phi) = \phi^m$ with $m = 0, 1, 2 \ldots$. We will give the analytic solutions of our model in flat and dS/AdS 3-brane cases. It turns out that the results of our model are consistent with those of the standard kinetic model studied in [6] when we take $m = 0$.

The investigation is organized as follows: in the next section we will give a brief description of our framework, and give the dynamic equations for general systems governed by the Lagrange denstity $\mathcal{L} = F(X, \phi) - V(\phi)$. Then we come back to the case of scalar-kinetic

extension (6), and take $W(\phi)$ as ϕ^m, in this case the dynamical equations can be solved in a closed form. We assume that the scalar and the warp factor depend only on the extra dimension throughout the following discussions. A particular definition of the mass in the de Sitter and anti-de Sitter spacetimes allows us to discuss the stability of our model as well as the effective gravity on the branes. We also obtain a correction of the Newtonian potential between two massive points lying on the brane. It turns out that the correction is different from the one given by the RS model [2].

The framework and the dynamic equations. – A thick braneworld scenario with non-standard dynamics can be described by a real scalar field ϕ coupled with gravity via the Einstein-Hilbert action, which has the general form

$$S = \int \mathrm{d}^4x\, \mathrm{d}y \sqrt{g} \left(-\frac{1}{4}R + \mathcal{L}(X, \phi) \right). \qquad (7)$$

The brane structures we are interested in here are mainly dS$_4$ and AdS$_4$. The line elements are [40]

$$\mathrm{d}s_5^2 = e^{2A}\big(\mathrm{d}t^2 - e^{2Ht}(\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2)\big) - \mathrm{d}y^2 \qquad (8)$$

and[1]

$$\mathrm{d}s_5^2 = e^{2A}\big(e^{2Hx_3}(\mathrm{d}t^2 - \mathrm{d}x_1^2 - \mathrm{d}x_2^2) - \mathrm{d}x_3^2\big) - \mathrm{d}y^2, \qquad (9)$$

respectively. Here e^{2A} is the warp factor and H is the de Sitter or anti-de Sitter parameter, which is related to the four-dimensional cosmological constant by $\Lambda_4 = \pm 3H^2$; the sign of the proportionality coefficient is determined by the geometry of the spacetime. For de Sitter space $\Lambda_4 = 3H^2 > 0$, and for anti-de Sitter space $\Lambda_4 = -3H^2 < 0$. The case of the Minkowski spacetime ($\Lambda_4 = 0$) is obtained in the limit $H \to 0$.

The scalar-field dynamics is governed by the Lagrange density

$$\mathcal{L} = F(X, \phi) - V(\phi), \qquad (10)$$

which is a little different from the one in [34], where $\mathcal{L} = F(X) - V(\phi)$. This means that the coupling between the scalar field and its kinetic term X is allowed in our model. Explicit examples given below show that a series of analytical solutions can be obtained with this assumption.

The equation of motion for the scalar field takes the form [34]

$$-4A'\phi'^2\mathcal{L}_X = (\mathcal{L} - 2\mathcal{L}_X X)', \qquad (11)$$

where the prime denotes the derivative with respect to the extra dimension y, and we take the notation $\mathcal{L}_X = \frac{\partial \mathcal{L}}{\partial X}$. The scalar field is supposed to be a function of y only, *i.e.*, $\phi = \phi(y)$.

In the case of the de Sitter metric (8), we have nonzero components of Christoffel symbols: $\Gamma^\alpha_{\beta 4} = A'\delta^\alpha_\beta$,

---

[1]The AdS$_4$ parameter H is related to the dS$_4$ one by $H(\text{AdS}) = -iH(\text{dS})$, since $x_3(\text{AdS})$ is equivalent to $it(\text{dS})$.

$\Gamma^4_{00} = A' e^{2A}$, $\Gamma^4_{ij} = -\delta_{ij} e^{2(Ht+A)} A'$ and $\Gamma^i_{j0} = \delta^i_j H$, where $i, j = 1, 2, 3$, while Greek letters $\alpha, \beta = 0, 1, 2, 3$. The non-vanishing components of the Ricci tensor are $R_{00} = -3H^2 + 4e^{2A} A'^2 + e^{2A} A''$, $R_{ij} = -\delta_{ij} e^{2Ht} R_{00}$ and $R_{44} = -4(A'^2 + A'')$. The components of the energy-momentum tensor are given by $T_{00} = -e^{2A} \mathcal{L}$, $T_{ij} = \delta_{ij} e^{2A+2Ht} \mathcal{L}$ and $T_{44} = \mathcal{L} - 2X\mathcal{L}_X$. By variating the modified action (7) with respect to the metric (8) we can finally reach to the Einstein equations:

$$A'' + H^2 e^{-2A} = -\frac{2}{3} \phi'^2 \mathcal{L}_X, \qquad (12a)$$

$$A'^2 - H^2 e^{-2A} = \frac{1}{3}(\mathcal{L} - 2X\mathcal{L}_X). \qquad (12b)$$

The equations corresponding to metric (9) can be obtained by replacing H^2 with $-H^2$, thus in the discussions below we always consider the de Sitter brane. Note that when $H = 0$, these equations reduce to the ones in [34]. On the other hand, the Einstein equations for models with a nonvanishing parameter H as well as a standard kinetic term $\mathcal{L}(X, \phi) = X - V(\phi)$ have been given in [6,10]:

$$A'' + H^2 e^{-2A} = -\frac{2}{3} \phi'^2, \qquad (13a)$$

$$A'^2 - H^2 e^{-2A} = \frac{1}{6} \phi'^2 - \frac{1}{3} V(\phi), \qquad (13b)$$

which are consistent with (12). Note that the dynamic equations are not independent, one can easily prove that (11) can be obtained from (12). Thus, solving Einstein equations (12) is enough.

The braneworld scenario. – When referring to the problem of finding a thick braneworld solution, it means that one must find out three quantities: $\phi(y)$, $A(y)$ and $V(\phi)$, which solve the dynamic equations. However, we have only two independent equations. As a result, one of the quantities we are searching for must be given at first. For simplicity, we assume that the warp factor takes the form

$$A = \ln\left(\cos(by)\right). \qquad (14)$$

The warp factor e^{2A} corresponding to eq. (14) has naked singularities at $y = \tilde{y} = \pm\pi/(2b)$, where the scalar ϕ as well as the potential $V(\phi)$ diverge. This feature seems to contradict our assumption that $V(\phi) \approx$ const; however in the area around the brane, $V(\phi)$ varies slowly so that our model is reasonable at least in the inner space. In fact, if we regard the scalar field as a modulus from some compactification manifold, then these singularities can indicate that the compactification manifold shrinks to zero size or extends to an infinitely large size; as a result, the five-dimensional truncation cannot be used here. In addition, such singularities are very similar to those confronted in the AdS flow to the $N = 1$ super Yang-Mills theory [41], which means that such problem of singularity may be solved either by lifting the dimension to ten or by string theory. There are works where singularities

in five dimensions actually correspond to non-singular ten-dimensional geometries [42].

For a dS$_4$ brane we usually expect a horizon at a finite distance $y = \tilde{y}$, so the naked singularity can be regarded as the horizon for the dS$_4$ brane. The argument is not the same for the AdS$_4$ brane. In the case of a flat brane, we have a brane interpolated between AdS$_5$ spaces which have regular horizons infinitely far away from $y = 0$. It is obvious that as $b \to 0$ we have $\tilde{y} \to \infty$. Thus b should be related to the parameter H.

Model and solutions. With the warp factor given by (14), the solution of Einstein equations (12) can be worked out in a closed form. We consider the following extension of the standard kinetic term X:

$$F(X, \phi) = X\phi^m, \qquad (15)$$

where $m = 0, 1, 2, \ldots$. Einstein equation (12a) turns to

$$\left(b^2 - H^2\right) \sec^2(by) = \frac{2}{3} \phi^m \phi'^2. \qquad (16)$$

The solutions of this equation are slightly varied for different m.

When $m = 4n$, $n = 0, 1, 2, \ldots$, we get $4n + 2$ solutions for eq. (16), we study only one of them:

$$\phi(y) = \left[(2n+1)\beta \, \text{arcsinh}(\tan(by))\right]^{\frac{1}{2n+1}}, \qquad (17)$$

where we have set the integral constant to zero and β is defined as

$$\beta = \frac{1}{b} \sqrt{\frac{3}{2}\left(b^2 - H^2\right)}. \qquad (18)$$

From the other Einstein equation (12b), $V(\phi)$ can be solved as

$$V(\phi) = 3b^2 - \frac{9}{4}\left(b^2 - H^2\right) \cosh^2\left[\frac{\phi^{2n+1}}{(2n+1)\beta}\right]. \qquad (19)$$

Note that $\phi(y)$ and $V(\phi)$ are divergent at the boundary $y = \pm\pi/(2b)$ and $\phi = \phi(\pm\pi/(2b)) = \pm\infty$, respectively.

When $m = 4n + 2$, $n = 0, 1, 2, \ldots$, we will find that

$$\phi(y) = \left[(2n+2)\beta \, \text{arcsinh}(\tan(by))\right]^{\frac{1}{2n+2}}, \qquad (20)$$

$$V(\phi) = 3b^2 - \frac{9}{4}\left(b^2 - H^2\right) \cosh^2\left[\frac{\phi^{2n+2}}{(2n+2)\beta}\right], \qquad (21)$$

where $\phi(y)$ is invalid for negative y.

For odd $m = 2n + 1$, $n = 0, 1, 2, \ldots$, the solution is

$$\phi(y) = \left[\frac{(2n+1)}{2}\beta \, \text{arcsinh}(\tan(by))\right]^{\frac{2}{(2n+1)}}, \qquad (22)$$

$$V(\phi) = 3b^2 - \frac{9}{4}\left(b^2 - H^2\right) \cosh^2\left[\frac{2\phi^{\frac{2n+1}{2}}}{(2n+1)\beta}\right]. \qquad (23)$$

Obviously $V(\phi)$ in (23) is invalid for negative ϕ. One can see that if X is not coupled with ϕ^{4n}, but with

ϕ^{4n+2} or ϕ^{2n+1}, some problems appear. Therefore, taking $F(X,\phi) = X\phi^{4n}$ is the best choice.

In the standard dynamic case ($m = 0$), *i.e.*, $L(X,\phi) = X - V$, the solution is then given by

$$
\begin{aligned}
A &= \ln[\cos(by)], \\
\phi_0(y) &= \beta \arcsinh(\tan(by)), \\
V_0(\phi) &= 3b^2 - \frac{9}{4}(b^2 - H^2)\cosh^2(\phi/\beta).
\end{aligned} \tag{24}
$$

One can find that the above solution is the same one given in [6] if one notices that the expressions below are actually identical:

$$
\arcsinh(\tan(by)) = \ln\left(\frac{1 + \tan(by/2)}{1 - \tan(by/2)}\right). \tag{25}
$$

For a four-dimensional dS slice, the only constraint here is $0 < H^2 < b^2$. On the other hand, when reversing the sign of H^2, we get solutions to the AdS$_4$ braneworld without constraint.

Stability of the model. In our model, modifications of the scalar-field dynamics are introduced, thus it is important to know if these modifications will contribute to destabilize the geometric degrees of freedom of the braneworld model.

As illustrated in [5] the fluctuations of both the metric and the scalar field need to be considered. However, the general treatment is difficult, since the fluctuations of the scalar field will couple with the metric fluctuations, yet, one can continue the discussion in a sector given in [4], where the metric fluctuations decouple from the scalar and satisfy a simple wave equation.

Following [9], we consider the gravity perturbations

$$
\mathrm{d}s^2 = e^{2A(z)}\left[(g_{\mu\nu} + h_{\mu\nu})\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2\right], \tag{26}
$$

where $g_{\mu\nu} = g_{\mu\nu}(x)$ represents the four-dimensional AdS, Minkowski, or dS metric, $h_{\mu\nu} = h_{\mu\nu}(x,z)$ is the metric perturbation which satisfies the transverse-traceless (TT) condition, *i.e.*,

$$
h^\lambda_{\ \lambda} = 0 = h_{\mu\nu;\lambda}g^{\nu\lambda}. \tag{27}
$$

Notice we have used a coordinate transformation

$$
\begin{aligned}
z(y) &= \int e^{-A(y)}\mathrm{d}y = \int \sec(by)\mathrm{d}y \\
&= \frac{1}{b}\arcsinh(\tan(by))
\end{aligned} \tag{28}
$$

to rewrite the metrics (8) and (9) into a conformal form. Then the main equation for $h_{\mu\nu}$ follows [8]:

$$
h''_{\mu\nu} + 3A'h'_{\mu\nu} - \Box h_{\mu\nu} - 2\alpha^2 h_{\mu\nu} = 0. \tag{29}
$$

Here the prime denotes the derivative with respect to the extra dimension z. We define the mass by the last two terms of the equation above as

$$
-m^2 h_{\mu\nu} = \Box h_{\mu\nu} - 2\alpha^2 h_{\mu\nu}. \tag{30}
$$

Then by introducing a polarization tensor $\epsilon_{\mu\nu}(x^\alpha)$ which depends only on the four-dimensional coordinates x^α and satisfies the TT condition (27), and taking the decomposition of $h_{\mu\nu}$

$$
h_{\mu\nu} = e^{-3A(z)/2}\epsilon_{\mu\nu}(x^\alpha)\psi(z), \tag{31}
$$

one obtains a Schrödinger-like equation for $\psi(z)$:

$$
\left[-\frac{\mathrm{d}^2}{\mathrm{d}z^2} + V_{Sch}(z)\right]\psi(z) = m^2\psi(z) \tag{32}
$$

with

$$
V_{Sch}(z) = \frac{9}{4}A'^2(z) + \frac{3}{2}A''(z). \tag{33}
$$

In the z-coordinate, $A(z) = -\ln[\cosh(bz)]$, thus

$$
V_{Sch} = \frac{9}{4}b^2 - \frac{15}{4}b^2\operatorname{sech}^2(bz). \tag{34}
$$

This is the well-known Pöschl-Teller potential which appears in many works [6,9,10,14,19,43]. The potential here supports two bound states: $\psi_0(z) \sim \cosh^{-3/2}(bz)$ and $\psi_1(z) \sim \sinh(bz)\cosh^{-3/2}(bz)$ with the eigenvalues $m_0^2 = 0$ and $m_1^2 = 2b^2 < (9/4)b^2$, respectively. Furthermore, there is also a continuum of states which asymptote to plane waves when far away from the brane.

The spectrum of the KK modes is essential for us to discuss the effective gravity on the brane. We have shown that with the mass defined by (30), there are a normalizable zero mode which will give rise to 4D gravity on the 3-brane, and a separate and bound massive mode as well as a continuum of massive KK modes. We will show that it is the continuous KK modes that give a correction to the Newtonian potential, while the bound massive mode has no contribution to the correction.

However, in some other papers such as [6], the mass is defined differently, as a consequence, there is no zero mode for the AdS brane case; while a tachyonic state appears in the case of dS brane, which may show a sign of instability. If we follow the definition of mass in [6], then the correction of the 4D gravity in the case of de Sitter and AdS case can no longer be treated under the traditional methods. For all of these reasons, we take the mass defined by (30).

Now let us turn to the discussion of the effective gravity on the brane. For simplicity, we consider two point-like sources with mass M_1 and M_2, both confined at the origin of the fifth dimension, *i.e.*, $z = 0$. This assumption is justified when the thickness of the brane is small as compared with the bulk curvature. Then the effective potential is given by [7]

$$
U(r) = G_N\frac{M_1 M_2}{r} + \frac{M_1 M_2}{M_*^3}\int_{m_0}^\infty \mathrm{d}m\frac{e^{-mr}}{r}|\psi_m(0)|^2, \tag{35}
$$

where $G_N = M_4^{-2}$ is the four-dimensional coupling constant, *i.e.*, Newton's gravitational constant, M_* is the fundamental five-dimensional Planck scale, and $m_0 = 3b/2$

is the minimal eigenvalue where the continuous KK modes start at. It is the massless zero mode which offers the four-dimensional Newtonial interaction potential, and all the massive KK modes give a correction to the standard Newtonian potential; however since the wave function of the first excited state $\psi_1(z)$ vanishes at $z = 0$, we start the integral from $m_0 = 3b/2$. With a coordinate transformation $l = bz$, the Schrödinger-like equation (32) turns into

$$-\psi''(l) - \frac{15}{4}\mathrm{sech}^2(l)\psi(l) = M^2\psi(l), \qquad (36)$$

where the prime denotes the derivative with respect to the coordinate l, and $M = \sqrt{\frac{m^2}{b^2} - \frac{9}{4}}$. The solution of this equation is given by a linear combination of the Legendre functions:

$$\psi_m(l) = C_1 \ \mathrm{P}\left(\frac{3}{2}, iM, \tanh(l)\right) + C_2 \ \mathrm{Q}\left(\frac{3}{2}, iM, \tanh(l)\right), \tag{37}$$

where C_1, C_2 are M-dependent parameters:

$$
\begin{aligned}
C_1 =\ & \frac{\Gamma(1 - iM)}{2} \\
& + \frac{\cosh(M\pi)^2\Gamma\left(-\frac{3}{2} - iM\right)\Gamma\left(\frac{5}{2} - iM\right)\Gamma(1 + iM)}{2\pi}, \\
C_2 =\ & \frac{M\Gamma\left(-\frac{3}{2} - iM\right)\Gamma\left(\frac{5}{2} - iM\right)\Gamma(iM)\sinh[2M\pi]}{2\pi^2}.
\end{aligned}
$$

Thus, by substituting $\psi_m(0)$ into eq. (35), we obtain the correction of the Newtonian potential. It was found [12] that the correction of the Newtonian potential given by a Pöschl-Teller potential is

$$\Delta U(r) \propto \frac{e^{-3br/2}}{M_*^3}\frac{M_1 M_2}{r^2}, \qquad (38)$$

which is different to the correction given by a volcano-like potential [2] where $\Delta U(r) \propto 1/r^3$, for more details see [12].

Conclusions. – In this work, we have studied a brane model in which the standard kinetic term X is extended to scalar-kinetic coupling terms. Such extension can be regarded as a result of trying to combine features of the k-field theory and the Born-Infeld extension of the kinetic term under the condition $X \ll 1$. It is well known that when $X \ll 1$ we have $\phi \approx \mathrm{const}$ and $V(\phi) \approx \mathrm{const}$, while the constant scalar potential can be identified with the bulk cosmological constant. If we introduce a tiny perturbation to $V(\phi)$, then the scalar-kinetic coupling can be applied to describe such system.

Another assumption of our model is the form of the warp factor which is given by (14). The warp factor implies a naked singularity at $y = \tilde{y} = \pm\pi/(2b)$ where the scalar field is divergent. However, such singularity can be solved by lifting the dimensions of spacetime to ten, or by using string theory. The parameter b relates to the de Sitter parameter H which controls the thickness of the brane: a finite and positive b leads to finite $\tilde{y}$ which implies a finite boundary or horizon of the de Sitter spacetime. As $b \to 0$ the horizon appears at $y \to \pm\infty$.

On the stability of the metric fluctuations we took the definition of mass as in [8,9]. Then fluctuations of metric lead to a Schrödinger equation with the so-called Pöschl-Teller potential. This potential supports two bound states along with a continuum of states which asymptote to plane waves at infinity. With this mass spectrum, we analyzed the effective gravity on the brane. It is the zero mode which gives rise to the standard Newtonian potential, and a correction is given by all the massive modes. Although we have another bound state $\psi_1(z)$ with $m = \sqrt{2}b$, we showed that $\psi_1(z)$ has no contribution to the effective gravity, since $\psi_1(0) = 0$. Thus the continuous massive modes (separated from the zero mode by a mass gap $\Delta m = 3b/2$) offer the Newtonian potential a correction: $\Delta U(r) \propto \frac{e^{-3br/2}}{M_*^3}\frac{M_1 M_2}{r^2}$, which is much different to the one given by a volcano-like potential.

This scalar-kinetic field may have some applications in inflation, we hope to provide some related works in the future.

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