

Domain wall brane in squared curvature gravity

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ABSTRACT: We suggest a thick braneworld model in the squared curvature gravity theory. Despite the appearance of higher order derivatives, the localization of gravity and various bulk matter fields is shown to be possible. The existence of the normalizable gravitational zero mode indicates that our four-dimensional gravity is reproduced. In order to localize the chiral fermions on the brane, two types of coupling between the fermions and the brane forming scalar is introduced. The first coupling leads us to a Schrödinger equation with a volcano potential, and the other a Pöschl-Teller potential. In both cases, the zero mode exists only for the left-hand fermions. Several massive KK states of the fermions can be trapped on the brane, either as resonant states or as bound states.

KEYWORDS: Large Extra Dimensions, Field Theories in Higher Dimensions

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1 Introduction

The suggestion that our world might be a four-dimensional domain wall was proposed early in the 1980s [1, 2], but it wasn't until the localization of gravity on either thin [3, 4] or thick [5–9] domain wall was shown to be possible, that the braneworld models attracted renew attentions. More and more studies indicate that the braneworld models might be effective in solving some long existed problems, such as the hierarchy problem and the cosmological constant problem [10–12]. In this paper, we focus mainly on the thick domain wall case. A comprehensive review about thick brane models can be found in [13].

Some questions needed be addressed in the thick braneworld models: the stability of the solution and the possibility of localizing matter fields as well as gravity on the brane. In the frame of Einstein's general relativity, there are a lot of works which devoted to answer these questions, see [5–7, 9] for the localization of gravity, and [14–25] for trapping various kinds of bulk matter fields on both thin and thick branes. For recent developments see [26–34].

However, due to the fact that general relativity is not renormalizable, we are suggested to consider the effects of higher order curvature terms [35]. Such terms also appear in the low-energy effective action of string theory [36]. To evade the appearance of spin-2 ghosts, and the Ostrogradski instability [37], the higher order curvature terms usually are introduced as the Gauss-Bonnet term or an arbitrary function of the curvature, namely, $f(R)$. Both cases were applied to consider various of issues in cosmology and higher energy physics, for details, see [38–40] and references therein.

As to the braneworld scenarios, it is shown in many works that the introduce of the Gauss-Bonnet term usually imposes no impact on the localization of gravity [41–50]. This result might change in $f(R)$ gravity, however, because now some higher order derivatives evolve in. The appearance of higher order derivatives usually renders the constructing of a braneworld models very hard.

For pure gravitational systems which contains no any matter fields, some thin brane-world models have been constructed in a lower order frame [51–54] by using the Barrow-Cotsakis theorem [55]. This theorem states that the fourth order $f(R)$ gravity (higher order frame) is conformally equivalent to a second-order gravity theory (lower order frame). Unfortunately, for thick domain wall models, this method would lead to ambiguous [56] and thus is not a good choice.

Some thick brane solutions were found directly in the higher order frame [57, 58]. In [57], with a background scalar field, the authors found some analytical thick brane solutions in both constant and variant curvature cases. While in [58], by analyzing the existence of the fix points, the authors found some numerical thick $f(R)$ -brane solutions with pure gravity. The trapping of complex scalar field on the brane solutions was shown to be possible in [58]. Both papers considered the case where $f(R) \propto R^n$.

However, the solutions given in [57, 58] are not perfect for the following reasons:

- The solution in M_5 space given in [57] contains singularity, because the warp factor is not smooth at the brane. Meanwhile, even though the solutions found in both dS_5 and AdS_5 spaces are stable under the tensor perturbations, only the solution in dS_5 space supports normalizable zero gravitational mode, see [59]. Therefore, the gravity can be localized only on the brane in dS_5 space. However, the constant curvature space is a rather special case.
- For the case of non-constant curvature, some solutions were suggested in [57, 58]. However, these solutions are not very good: the solution in [57] contains a singular point at where the curvature diverges; while the solution in [58] is given numerically, and we only know its behavior in infinity.
- Besides, the model in [58] is purely gravitational, namely, no matter field were introduced. However, to localize fermions, one usually needs to couple the fermion with a scalar field.

For the reasons above, we aim at constructing a domain wall brane which has the following properties:

- The brane solution is found in a system which contains higher order curvature terms and especially a system contains fourth-order derivatives.
- The brane is smooth, stable and is formed by a background scalar field.
- The gravity can be localized on such brane.
- By introducing the Yukawa coupling between the fermion and the background scalar field, the fermions can be trapped on the brane.

As a toy model, let us focus on the squared curvature gravity, i.e., the higher curvature correction is given by $f(R) \propto R^2$. For this special gravity, a thin braneworld has been constructed in [51] by using the Barrow-Cotsakis theorem.

In the next section, we give a thick domain wall solution in the squared curvature gravity. In section 3, we discuss the localization of four-dimensional gravity. In section 4, the trapping of bulk matter fields with various spins is analyzed, but we mainly focus on the half spin fermions. Our conclusions are listed in the last section.

2 Model and solution

We start with the action

$$S = \int d^5x \sqrt{-g} \left(\frac{1}{2\kappa_5^2} f(R) - \frac{1}{2} \partial^M \phi \partial_M \phi - V(\phi) \right), \quad (2.1)$$

where $\kappa_5^2 = 8\pi G_5$ is the gravitational coupling constant with G_5 the five-dimensional Newtonian constant. We have introduced a background scalar field ϕ , of which the self-interacting is described by the potential $V(\phi)$. We prefer to use Roman letters to denote the bulk coordinates, i.e., $M, N = 0, 1, 2, 3, 4$ and the Greek letters represent the brane coordinates, i.e. $\mu, \nu = 0, 1, 2, 3$.

Our discussions will be limited on the static flat braneworld scenario, for which the metric is given by

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (2.2)$$

with $y = x^4$ the extra dimension. Meanwhile, the scalar field, $\phi = \phi(y)$, is independent of the brane coordinates. For system (2.1)–(2.2), the Einstein equations read

$$f(R) + 2f_R (4A'^2 + A'') - 6f'_R A' - 2f''_R = \kappa_5^2 (\phi'^2 + 2V), \quad (2.3)$$

and

$$-8f_R (A'' + A'^2) + 8f'_R A' - f(R) = \kappa_5^2 (\phi'^2 - 2V), \quad (2.4)$$

where the primes represent derivatives with respect to the coordinate y and $f_R \equiv df(R)/dR$. The scalar field is described by the following equation:

$$4A'\phi' + \phi'' - \frac{\partial V}{\partial \phi} = 0. \quad (2.5)$$

We consider a toy model, in which $f(R) = R + \gamma R^2$. Then we would have $f_R = 1 + 2\gamma R$, $f'_R = 2\gamma R'$, $f''_R = 2\gamma R''$, $R' = -(8A''' + 40A'A'')$, $R'' = -(8A'''' + 40A''^2 + 40A'A''')$. When $\gamma = 0$, we come back to the case of general relativity. Obviously, if $\gamma \neq 0$, the Einstein equations are fourth-order differential equations. In order to find a domain wall solution, let us consider the following ϕ^4 model:

$$V(\phi) = \lambda^{(5)} (\phi^2 - v^2)^2 + \Lambda_5, \quad (2.6)$$

where $\lambda^{(5)} > 0$ is the self coupling constant of the scalar field, and Λ_5 is another constant. Obviously, at $\phi = \pm v$ the scalar potential $V(\phi)$ takes its minimum value, i.e., $V(\phi = \pm v) = \Lambda_5$.

It is easy to proof that eq. (2.5) supports the following solution:

$$\phi(y) = v \tanh\left(\sqrt{\frac{2\lambda^{(5)}}{3}}vy\right), \tag{2.7}$$

$$e^{A(y)} = \cosh^{-1}\left(\sqrt{\frac{2\lambda^{(5)}}{3}}vy\right). \tag{2.8}$$

However, the Einstein equations (2.3)–(2.4) strongly constrain the possible values of the parameters. Specifically, the Einstein equations are satisfied only when the parameters take the following values:

$$\lambda^{(5)} = \frac{3}{784} \frac{\kappa_5^2}{\gamma}, \quad v = 7\sqrt{\frac{3}{29\kappa_5^2}}, \quad \Lambda_5 = -\frac{477}{6728} \frac{1}{\gamma\kappa_5^2}. \tag{2.9}$$

Defining a new parameter by

$$k = \sqrt{\frac{3}{232\gamma}}, \tag{2.10}$$

we can rewrite (2.7)–(2.8) in a more simpler form:

$$\phi(y) = v \tanh(ky), \quad e^{A(y)} = \cosh(ky)^{-1}. \tag{2.11}$$

For this reason, in the discussions below, we always use the parameter k rather than γ , to reflect the influence from the squared curvature term.

As shown in figure 1, the scalar field is a kink which satisfies $\phi(0) = 0$, $\phi(\pm\infty) = \pm v$. $V(\phi)$ takes its minimum at $\phi = \pm v$. We also see how the energy density $\rho = T_{00} = e^{2A}(\frac{1}{2}\phi'^2 + V(\phi))$ distributes along the fifth dimension. Obviously, ρ peaks at $y = 0$, where the brane locates at. It is not difficult to verify that the geometry of the space-time becomes anti-de Sitter at $y = \pm\infty$, where the bulk curvature $R = -20k^2$ and the corresponding cosmological constant is nothing but the parameter Λ_5 . The value of Λ_5 can also be obtained from eq. (2.3) by taking the limit $y \rightarrow \infty$.

3 Tensor perturbations and the localization of gravity

To discuss the localization of massless four-dimensional gravity, we need to consider the tensor perturbations:

$$ds^2 = e^{2A(y)}(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + dy^2, \tag{3.1}$$

where $h_{\mu\nu} = h_{\mu\nu}(x^\rho, y)$ depend on all the coordinates and satisfy the transverse-traceless condition

$$\eta^{\mu\nu}h_{\mu\nu} = 0 = \partial_\mu h^\mu{}_\nu. \tag{3.2}$$

The perturbation of the scalar field is denoted as $\delta\phi = \tilde{\phi}(x^\mu, y)$.

In [59], we have demonstrated that the scalar perturbation $\tilde{\phi}$ decouples from the tensor perturbations $h_{\mu\nu}$ under the transverse-traceless condition (3.2), and the perturbed Einstein equations are given by

$$\left(\partial_y^2 + 4A'\partial_y + e^{-2A}\square^{(4)}\right) h_{\mu\nu} = -\frac{f'_R}{f_R}\partial_y h_{\mu\nu}, \quad (3.3)$$

or in a more simpler form

$$\square^{(5)} h_{\mu\nu} = \frac{f'_R}{f_R}\partial_y h_{\mu\nu}. \quad (3.4)$$

Here $\square^{(4)} = \eta^{\mu\nu}\partial_\mu\partial_\nu$ and $\square^{(5)} = g^{MN}\nabla_M\nabla_N$ are the four-dimensional and five-dimensional d'Alembert operators, respectively.

Obviously, for $R = \text{const.}$ or $f(R) = R$, which simply gives the theory of general relativity, eq. (3.4) reduces to the five-dimensional Klein-Gorden equation for the massless spin-2 gravitons, and the reproduction of brane gravity has been discussed in [5–7]. However, for an arbitrary form of $f(R)$ with variant R , the issue of localization of gravity should be given a further consideration.

Following the method given in [5–7], we introduce a coordinate transformation $dz = e^{-A(y)}dy$, under which the metric (2.2) is conformally flat:

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2). \quad (3.5)$$

It is very easy to proof that in the new coordinates system, $y = \text{arcsinh}(kz)/k$ and therefore the warp factor takes the simple form:

$$e^{2A(z)} = (1 + k^2 z^2)^{-1}. \quad (3.6)$$

Then the perturbed equation (3.4) reads

$$\left[\partial_z^2 + \left(3\frac{\partial_z a}{a} + \frac{\partial_z f_R}{f_R}\right)\partial_z + \square^{(4)}\right] h_{\mu\nu} = 0. \quad (3.7)$$

Here we have denoted $a(z) = e^{A(z)}$ to write the final result looks more symmetrical.

By doing the decomposition $h_{\mu\nu}(x^\rho, z) = (a^{-3/2}f_R^{-1/2})\epsilon_{\mu\nu}(x^\rho)\psi(z)$, with $\epsilon_{\mu\nu}(x^\rho)$ satisfying the transverse-traceless condition $\eta^{\mu\nu}\epsilon_{\mu\nu} = 0 = \partial_\mu\epsilon^\mu_\nu$, we obtain a Schrodinger like equation for $\psi(z)$:

$$[\partial_z^2 - W(z)]\psi(z) = -m^2\psi(z), \quad (3.8)$$

with the potential given by [59]

$$W(z) = \frac{3a'^2}{4a^2} + \frac{3a''}{2a} + \frac{3a'f'_R}{2af_R} - \frac{1f_R'^2}{4f_R^2} + \frac{1f_R''}{2f_R}. \quad (3.9)$$

One can also factorize the Schrodinger equation (3.8) as

$$\left[\left(\partial_z + \left(\frac{3}{2}\frac{\partial_z a}{a} + \frac{1}{2}\frac{\partial_z f_R}{f_R}\right)\right)\left(\partial_z - \left(\frac{3}{2}\frac{\partial_z a}{a} + \frac{1}{2}\frac{\partial_z f_R}{f_R}\right)\right)\right]\psi(z) = -m^2\psi(z) \quad (3.10)$$

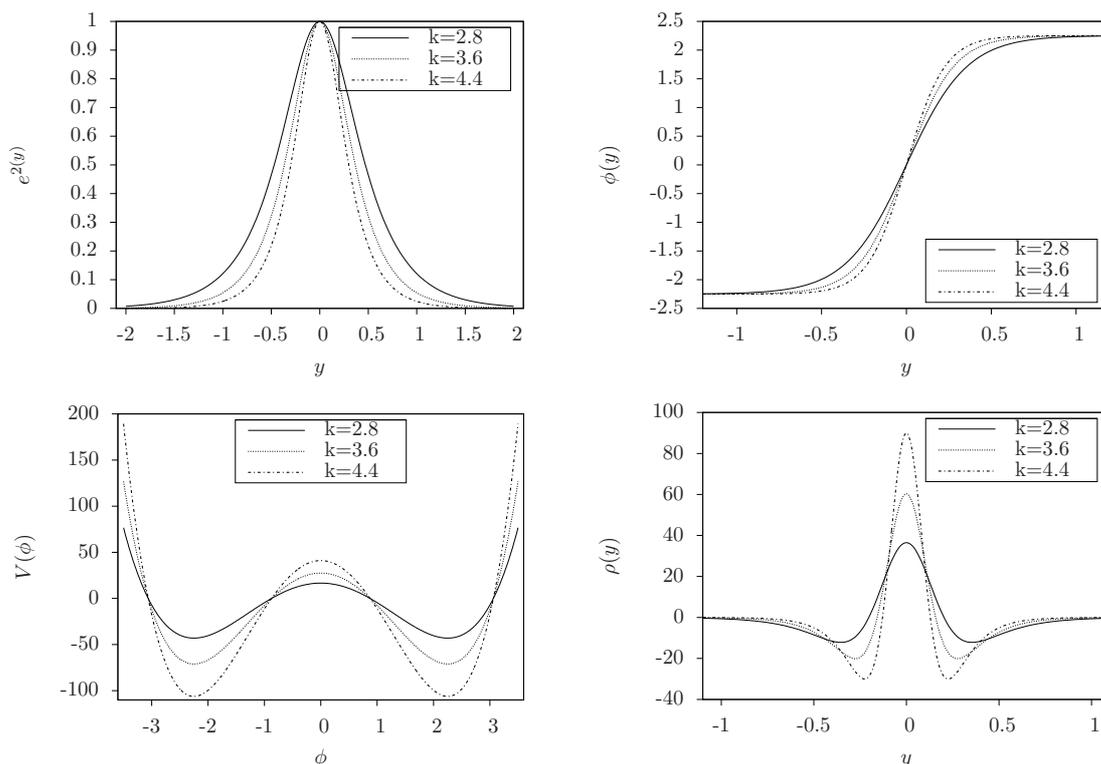


Figure 1. The dependence of the domain wall properties on the parameter k .

to conclude that there is no gravitational mode with $m^2 < 0$ and therefore the solution in (2.7)–(2.9) is stable. For eq. (3.8), the zero mode (for which $m^2 = 0$) reads

$$\psi^{(0)}(z) = N_0 a^{3/2}(z) f_R^{1/2}(z), \tag{3.11}$$

where N_0 represents the normalization constant.

By plugging (3.6) into (3.9), one obtains

$$W(z) = \frac{15k^2 (-14 + 37k^2 z^2 + 28k^4 z^4 + 4k^6 z^6)}{4(5 + 7k^2 z^2 + 2k^4 z^4)^2}, \tag{3.12}$$

the famous volcano potential [3, 5–7]. This potential supports a normalizable zero mode (see figure 3)

$$\psi^{(0)}(z) = \sqrt{\frac{k}{8}} \frac{\sqrt{5 + 2k^2 z^2}}{(1 + k^2 z^2)^{5/4}}, \tag{3.13}$$

as well as a series of continuous massive KK modes.

Starting from $m^2 > 0$, the continuum of KK modes might lead to a correction to the Newtonian potential on the brane. As depicted in figure 2, $W(z) \sim \frac{15}{4z^2}$ as $|z| \gg 1$. It is well known (see, for example [6, 9]) that if the potential $W(z) \sim \alpha(\alpha + 1)/z^2$ as $|z| \gg 1$, then the KK modes on the brane take the form $\psi^{(m)}(0) \sim m^{\alpha-1}$, and for two massive objects

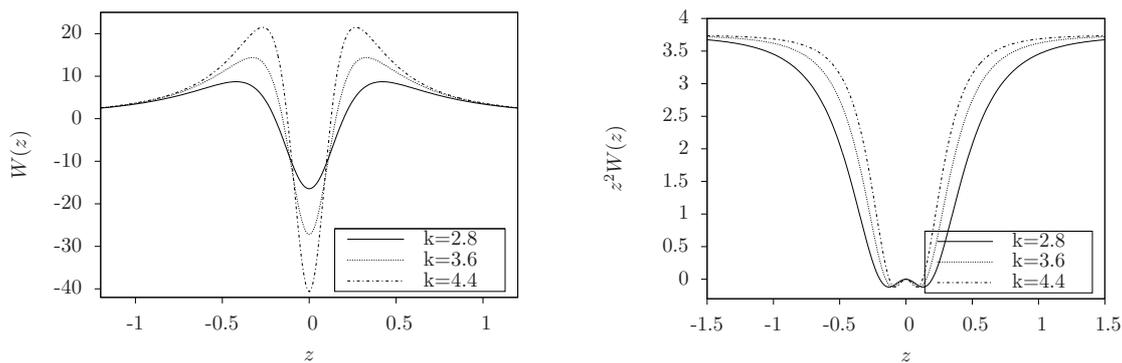


Figure 2. Plots of $W(z)$ and $z^2W(z)$.

at a distance r from each other, the correction for the Newtonian potential is $\Delta U \propto 1/r^{2\alpha}$. For our model $\alpha = 3/2$, thus $|\psi^{(m)}(0)|^2 \sim m$ for small masses. Thus the correction to the Newtonian potential is $\Delta U \propto 1/r^3$, which is same to the one given by the RS model [3].

Note that for KK modes with $0 < m^2 < W_{\max}$, there might exist some resonant modes due to the quantum tunneling effect [5, 6]. Here W_{\max} is the maximum of the potential $W(z)$. To discuss this issue, let us redefine the variables as

$$\bar{z} = kz, \quad \bar{m}^2 = m^2/k^2. \tag{3.14}$$

As a consequence, one can rewrite equation (3.8) as

$$\left[\partial_{\bar{z}}^2 - \frac{15(-14 + 37\bar{z}^2 + 28\bar{z}^4 + 4\bar{z}^6)}{4(5 + 7\bar{z}^2 + 2\bar{z}^4)^2} \right] \psi(\bar{z}) = -\bar{m}^2 \psi(\bar{z}), \tag{3.15}$$

which is parameter independent. This equation is hardly to be solved analytically, but we can apply some numerical methods to analyze the resonant structure.

To start with, let us impose some initial conditions to simplify the numerical calculation [30, 60–62]:

$$\psi_{\text{even}} = c_1, \quad \psi'_{\text{even}} = 0; \quad \psi_{\text{odd}} = 0, \quad \psi'_{\text{odd}} = c_2. \tag{3.16}$$

Here we have denoted ψ_{even} , ψ_{odd} as the even and odd parity modes of $\psi(\bar{z})$, respectively. The prime represents the derivative with respect to \bar{z} . In principle, c_1 and c_2 are arbitrary constants, here we simply take $c_1 = c_2 = 1$. The massive modes $\psi(\bar{z})$ here are analogous to the wave function in the quantum mechanics. Therefore we can interpret $|\psi(\bar{z})|^2 d\bar{z}$ (after normalizing $\psi(\bar{z})$), as the probability for finding the massive KK modes within $(\bar{z}, \bar{z} + d\bar{z})$. However, the massive modes are non-normalizable because they oscillate strongly when $\bar{z} \gg 1$. Following the method given in [30, 60–62], we define the function

$$P_G(\bar{m}^2) = \frac{\int_{-z_b}^{z_b} |\psi(\bar{z})|^2 d\bar{z}}{\int_{-z_{\max}}^{z_{\max}} |\psi(\bar{z})|^2 d\bar{z}} \tag{3.17}$$

as the relative probability for finding a massive KK mode in a narrow range $-z_b < z < z_b$ (as compared to a wider interval $-z_{\max} < z < z_{\max}$) with mass square \bar{m}^2 . Here $2z_b$ is

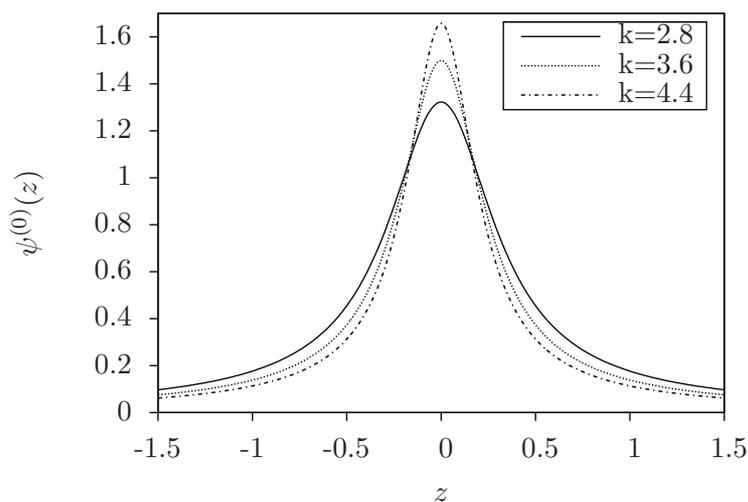


Figure 3. The shapes of the gravitational zero mode $\psi^{(0)}(z)$.

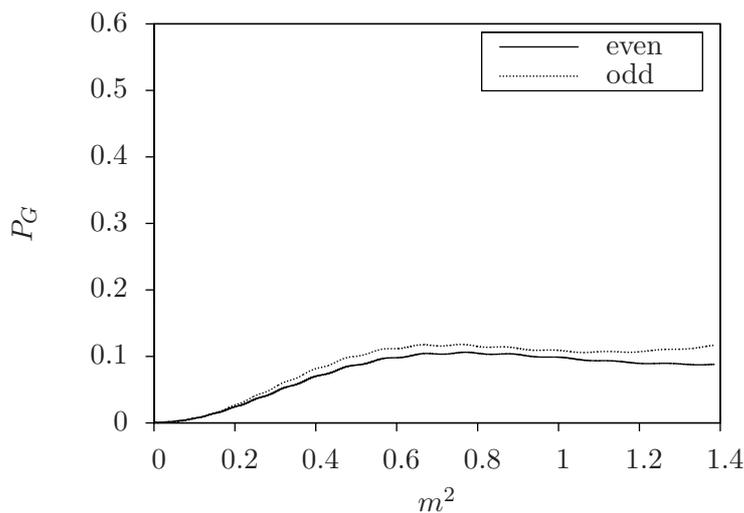


Figure 4. The relative probability P_G for various values of \bar{m}^2 .

about the width of the thick brane and $z_{\max} = 10z_b$. For an eigenvalue $\bar{m}^2 \gg W_{\max}$, the corresponding KK mode $\psi(\bar{z})$ can be approximately identified as a plane wave, for which the probability $P_G(\bar{m}^2)$ tends to 0.1 .

According to the numerical calculation of $P_G(\bar{m}^2)$, see figure 4, there exists no resonant state for the Schrödinger equation (3.15). Obviously, as the eigenvalue \bar{m}^2 approaches to the maximum of the scaled potential $\bar{W}_{\max}(\bar{z}) = 1.1086$, the relative probability P_G tends to 0.1, as expected.

The analysis above indicates that the solution (2.7)–(2.9) is stable under tensor perturbations, and the four-dimensional massless gravity can be localized on the brane. Besides, there exists no resonant gravitational KK modes. Thus the massive gravitons can not be quasi-localized on the brane. The continuous KK spectrum contributes a correction to the brane Newtonian potential, i.e., $\Delta U \propto r^{-3}$. Now let us move on to the issue of trapping matter fields.

4 Trapping of matter fields

An interesting but also vitally important question in brane world scenarios is how to localize various bulk matter fields on the brane by a natural mechanism. In many braneworld models the massless scalar fields can be trapped on the branes [26, 33, 63]. Usually, Spin-1 Abelian vector fields can not be localized on five-dimensional flat branes. But, it can be localized on the RS brane in higher dimensional case [64] or on the thick dS brane [65] and Weyl thick brane [26, 28].

As far as our solution (3.6) is concerned, the scalar fields can be localized on the brane, while the vector fields can not. This conclusion can be achieved by following the discussions given in [28, 63].

In addition to the scalar and vector fields, the localization of Kalb-Ramond fields and gauge fields on single- or multi-branes were also investigated [31, 66–72]. But we are not going to discuss these issues. In stead, we would like to focus on the localization of the spin-1/2 fermions.

According to the works in the past, fermions can not be localized on branes in five and six dimensions without introducing the scalar-fermion coupling [20, 24, 29, 30, 62, 73–77]. This is the reason why we insist on introducing a background scalar field.

We proceed with the action

$$S_\Psi = \int d^4x dy \sqrt{-g} \left(\bar{\Psi} \Gamma^M (\partial_M + \omega_M) \Psi - \eta \bar{\Psi} F(\phi) \Psi \right), \quad (4.1)$$

where $F(\phi)$ determines the coupling type and η tells the strength. As in refs. [27, 28, 32], we denote $\Gamma^M = e^{-A}(\gamma^\mu, \gamma^5)$ as the five-dimensional gamma matrices with γ^μ and γ^5 the usual four-dimensional Dirac gamma matrices. ω_M are the spin connections, of which the non-vanished components read $\omega_\mu = \frac{1}{2}(\partial_z A)\gamma_\mu\gamma_5$. With the action in (4.1), we obtain the following Dirac equation [27, 29, 75, 77]:

$$\left\{ \gamma^\mu \partial_\mu + \gamma^5 (\partial_z + 2\partial_z A) - \eta e^A F(\phi) \right\} \Psi = 0, \quad (4.2)$$

with $\gamma^\mu \partial_\mu$ the 4-dimensional Dirac operator.

It is convenient to decompose the full 5-dimensional spinor by

$$\Psi(x, z) = \sum_n \psi_{L_n}(x) f_{L_n}(z) + \sum_n \psi_{R_n}(x) f_{R_n}(z). \quad (4.3)$$

Here $\psi_{L_n}(x)$ and $\psi_{R_n}(x)$ are the left-hand and right-hand components of a 4-dimensional Dirac field, which satisfy the four-dimensional Dirac equations

$$\gamma^\mu \partial_\mu \psi_{L_n}(x) = m_n \psi_{R_n}(x), \quad \gamma^\mu \partial_\mu \psi_{R_n}(x) = m_n \psi_{L_n}(x). \quad (4.4)$$

Now, we are left with the following coupled equations

$$[\partial_z + \eta e^A F(\phi)] f_{Ln}(z) = m_n f_{Rn}(z), \tag{4.5a}$$

$$[\partial_z - \eta e^A F(\phi)] f_{Rn}(z) = -m_n f_{Ln}(z). \tag{4.5b}$$

Provided that eqs. (4.5a)–(4.5b), along with the following orthonormal conditions

$$\int_{-\infty}^{\infty} e^{4A} f_{Lm}(z) f_{Rn}(z) dz = \delta_{RL} \delta_{mn}, \tag{4.6}$$

are satisfied, we can recast action (4.1) into the standard four-dimensional actions for the massless and a series of massive chiral fermions. By redefining $\tilde{f}_{Ln} = e^{2A} f_{Ln}$, we can rewrite eqs. (4.5) as two Schrödinger equations

$$(-\partial_z^2 + V_L(z)) \tilde{f}_{Ln} = m_n^2 \tilde{f}_{Ln}, \tag{4.7a}$$

$$(-\partial_z^2 + V_R(z)) \tilde{f}_{Rn} = m_n^2 \tilde{f}_{Rn}. \tag{4.7b}$$

The effective potential for \tilde{f}_{Ln} has been denoted as

$$V_L(y) = e^{2A} \eta^2 F^2(\phi) - e^A \eta \partial_z F(\phi) - (\partial_z A) e^A \eta F(\phi). \tag{4.8}$$

For \tilde{f}_{Rn} , the corresponding potential $V_R(y)$ can be obtained from eq. (4.8) by simply changing $\eta \rightarrow -\eta$. Note that in z -coordinate (defined by eq. (3.5))

$$\phi(z) = \pm \frac{\sqrt{2\lambda^{(5)}} v^2 z}{\sqrt{3 + 2\lambda^{(5)} v^2 z^2}}, \tag{4.9}$$

$$A(z) = -\frac{1}{2} \ln \left(1 + \frac{2}{3} \lambda^{(5)} v^2 z^2 \right). \tag{4.10}$$

Thus if we want to obtain Z_2 symmetric potentials $V_L(y)$ and $V_R(y)$, we have to constrain $F(\phi)$ to be an odd function of ϕ .

Now, our task is to solve the Schrödinger-like equation (4.7) with different forms of $F(\phi)$ which determines the shapes of the potentials V_L and V_R . In braneworld models we mainly interested in two types of effective potentials.

The first one is the so called volcano potential, which has been discussed in section 3 (see eq. (3.12)). For such potential we usually get a unique bound state, i.e., the zero mode, and a gapless continuum of KK modes. Sometimes, we might obtain several quasi-localized massive states (the resonant states). In section 3, we got no sign which indicates the existence of a resonant state. But as we will state below, such resonant states might exists for particular $F(\phi)$.

The other one is the well known Pöschl-Teller potential, which approaches to a positive constant rather than zero as $z \rightarrow \pm\infty$. This potential is very interesting, because in addition to the zero mode, there might be some massive KK states. Besides, the continuous KK modes are separated from the bound states by a mass gap.

To proceed on, let us consider the following two cases, the first one leads to a volcano potential, while the other one a Pöschl-Teller potential. Due to the fact that left- and right-handed fermions usually share similar resonant and massive spectrum [62], we only discuss the left-handed fermions.

4.1 Massive fermions as resonant states

By taking $F(\phi) = \sqrt{\frac{3}{2\lambda^{(5)}}}\phi$ we obtain a Schrödinger equation with the following potential

$$\begin{aligned}
 V_L(z) &= \frac{3v^2\eta (v^2 z^2 (3\eta + 2\lambda^{(5)}) - 3)}{(3 + 2v^2 z^2 \lambda^{(5)})^2} \\
 &= \frac{147\eta (147\eta z^2 + 29 (k^2 z^2 - 1) \kappa_5^2)}{841 (1 + k^2 z^2)^2 \kappa_5^4},
 \end{aligned}
 \tag{4.11}$$

which satisfies $V_L(0) = -v^2\eta$ and $V_L(\infty) = 0$. That means only the left-hand fermion zero mode could be trapped on the brane for positive coupling constant η . The zero mode, according to the discussions in [27, 77], takes the form

$$\begin{aligned}
 \tilde{\alpha}_{L0}(z) &\propto \exp \left[-\eta \int_0^z dz' e^{A(z')} \phi(z') \right] \\
 &= \left(3 + 2\lambda^{(5)} v^2 z^2 \right)^{-\frac{\beta}{2}},
 \end{aligned}
 \tag{4.12}$$

where $\beta = \frac{3}{2} \frac{\eta}{\lambda^{(5)}}$ is a dimensionless constant. It is easy to prove that $\tilde{\alpha}_{L0}(z)$ is normalizable if and only if $\eta > \eta_0 \equiv \lambda^{(5)}/3$. Since usually the fermions are massive, we hope to find a way to trap the massive fermions. For volcano potential, the massive bound state is absent. However, there might be some resonant states, which have large relative probabilities near the location of the brane. The existence of such resonant states offer us an effective way to trap massive fermions. This matter trapping mechanism is referred to as the quasi-localization. Mimic to the case of gravity, we define

$$P_{L,R}(m^2) = \frac{\int_{-z_b}^{z_b} |\tilde{f}_{L,R}(z)|^2 dz}{\int_{-z_{\max}}^{z_{\max}} |\tilde{f}_{L,R}(z)|^2 dz},
 \tag{4.13}$$

as the relative probability for finding a left- (or right-) hand fermion resonant state $\tilde{f}_L(z)$ (or $\tilde{f}_R(z)$) with mass m . Sharp peaks in figure of $P_{L,R}(m^2)$ can be identified as the resonant states. The resonant states are unstable, their lifetime can be estimated by $\tau \sim \Gamma^{-1}$, with $\Gamma = \delta m$ the width of the half height of the resonant peak. As shown in figure 5, for larger value of η , there are more resonant states. The mass, width and lifetime of the resonant states are listed in table 1.

4.2 Massive fermions as bound states

A coupling given by

$$F(\phi) = k^2 \sqrt{\frac{v^2}{v^2 - \phi^2}} \tanh \left[\frac{\sqrt{\frac{3}{2}} k \phi}{\sqrt{v^2 \lambda^{(5)} (v^2 - \phi^2)}} \right]
 \tag{4.14}$$

would lead us to the Pöschl-Teller potential

$$V_L(z) = k^2 [(k\eta)^2 - k\eta(1 + k\eta)\text{sech}^2(kz)],
 \tag{4.15}$$

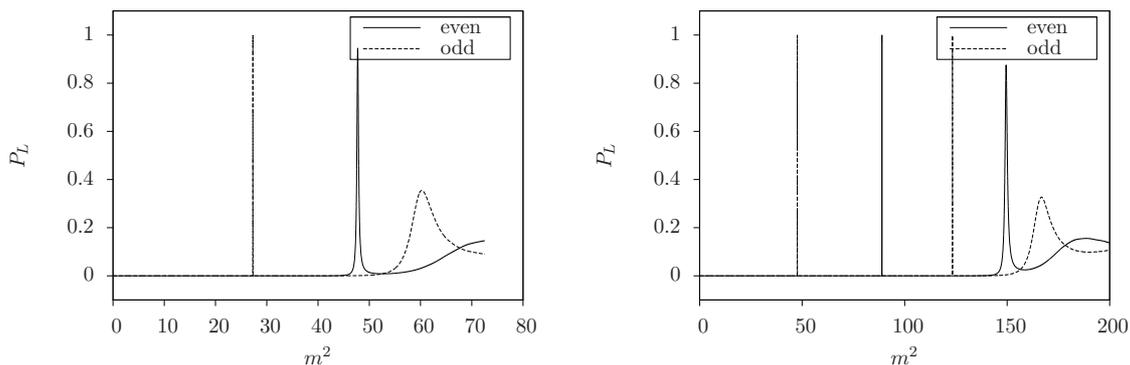


Figure 5. The figures of P_L with $k = 1$, $\kappa_5^2 = 1$, $\eta = 3$ (the left panel) and $\eta = 5$ (the right panel).

η	V_L^{\max}	n	m^2	m	Γ	τ
3	58.033367	1	27.28253	5.22327	0.000095	10525.6787
		2	47.73727	6.90922	0.0275038	36.3586
5	160.820587	1	47.62055	6.90076	2.43773×10^{-10}	4102176995.31678
		2	88.91114	9.42927	0.000005757	173695.37532
		3	123.32123	11.10501	0.0017165	582.58201
		4	149.47803	12.22612	0.0540928	18.48673

Table 1. The mass, width, and lifetime of resonances of the left-handed fermions (shown in figure 5). The parameters are $k = 1$, $\kappa_5^2 = 1$. V_L^{\max} is the maximum of the height of the potential V_L , and n is the order of resonant states corresponding m^2 from small to large.

for which $V_L(0) = -k^3\eta$, and $V_L(\infty) = k^4\eta^2$ is a positive constant. Again, the zero mode exists only when $\eta > 0$:

$$\tilde{\alpha}_{L0}(z) = N_0 \cosh^{-k\eta}(kz). \tag{4.16}$$

$\tilde{\alpha}_{L0}(z)$ is normalizable, provide that $k\eta > 1/2$. For simplicity, let us take $k\eta = 3/2$,¹ then the potential (4.15) reduces to

$$V_L(z) = \frac{9}{4}k^2 - \frac{15}{4}k^2 \operatorname{sech}(kz)^2. \tag{4.17}$$

As we have studied in [29, 78], such potential supports two bound states:

$$\tilde{\alpha}_{L0}(z) \propto \cosh^{3/2}(kz) \quad \text{with} \quad m_0^2 = 0, \tag{4.18}$$

$$\tilde{\alpha}_{L1}(z) \propto \sinh(kz) \cosh^{3/2}(kz) \quad \text{with} \quad m_1^2 = 2k^2. \tag{4.19}$$

We got only one massive bound state mode, i.e., $\tilde{\alpha}_{L1}(z)$. However, as the product $k\eta$ increases, more and more massive bound states appear. The number of the bound states

¹In fact, when $k\eta = 1, 2, \dots$ is a positive integer, the quantum system possesses very peculiar property, namely, the system is reflectionless (and such property turns out to be intimately related to nonlinear integrable KdV system). Such peculiar property of Poschl-Teller system V_L is reflected in the presence of a hidden nonlinear bosonized supersymmetry of order $2k\eta + 1$ in it, see [79, 80]. We would like to thank Mikhail S. Plyushchay for reminding us of their interesting works.

equals to the integer part of $k\eta$ [26]. In addition to the bound states, there is also a continuum of KK states which start from $m^2 = k^4\eta^2$ and asymptote to plane waves as $z \gg 1$.

5 Conclusions and prospect

In this paper, we study a thick domain wall solution in squared curvature gravity. Different from the Gauss-Bonnet gravity, the dynamical equation in our model contains higher order derivatives. As hoped, the solution itself is smooth and regular, and is stable under the gravitational tensor perturbations. The localization of gravity and spin-1/2 fermions is shown to be possible for our solution. For gravity, only the zero mode can be localized. While for the left-handed fermions, in addition to the zero mode, some massive KK modes can also be localized as bound states (with infinite life), or be quasi-localized as resonant states (with finite life). The number of massive KK modes depends on the thickness of the brane $k \sim \gamma^{-1}$, and the strength of the scalar-fermion coupling, i.e., η . The KK spectrum of the right-handed fermions is similar to the one of the left-handed fermions, except the absent of the zero mode.

A new feature of our model is that the curvature square term might influence the thick of the wall via the relation $k \sim \gamma^{-1}$. As the temperature effects is considered, the coefficient γ might be a function of temperature T , i.e., $\gamma = \gamma(T)$. Then, the thickness of the brane can be tuned by temperature. Some massive KK states of the fermions might appear or disappear, for k is another factor which influences the number of the KK states of the fermions. Besides, the Mexican hat potential (2.6) indicates that there might be a spontaneous symmetry breaking, which leads to phase transition and finally the formation of our four-dimensional domain wall. The quantitative calculation is a non-trivial task, we hope some related works can be reported in the near future.

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