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Tensor perturbations of $f(R)$ -branes

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ABSTRACT

We analyze the tensor perturbations of flat thick domain wall branes in $f(R)$ gravity. Our results indicate that under the transverse and traceless gauge, the metric perturbations decouple from the perturbation of the scalar field. Besides, the perturbed equation reduces to the familiar Klein–Gordon equation for massless spin-2 particles only when the bulk curvature is a constant or when $f(R) = R$. As an application of our results, we consider the possibility of localizing gravity on some flat thick branes. The stability of these brane solutions is also shortly discussed.

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1. Introduction

The trapping of various matter fields on four-dimensional domain walls has been successfully realized by using both non-gravitational or gravitational methods [1,2]. Recently, domain walls in five-dimensional space–time (called the bulk) attract renew attentions from the physical community after Randall and Sundrum [3,4] pointed out that massless four-dimensional graviton can be realized on a thin wall (called the brane) if the extra dimension is large and warped properly. Randall and Sundrum assume that all the observable matter fields are confined on the brane while gravitons can propagate in the bulk. For this reason, their model is also known as the (thin) braneworld scenario, or the RS model for short. The braneworld scenario is very important, because it, to some extent, solves some of the long existed problems such as the hierarchy problem, and the cosmological constant problem, etc., for reviews see [5,6].

However, the thin braneworld model suffers a drawback: the whole theory is singular at the location of the brane, for example, the bulk curvature diverges at there and a junction condition must be introduced. Besides, in the original braneworld scenario we are not clear how does our world (as a brane) formed. Such problems are solved in the so called thick braneworld models, in which gravity couples with a background scalar field [7–11]. The formation of the domain wall is ascribed to the non-linear property of the gravitational system.

In general relativity, four-dimensional gravity has been successfully realized in some thick braneworld models [7–10]. The trap-

ping of various kinds of matter fields on single or multi branes are also discussed for both thin and thick branes [12–31].

However, for some reasons, we have to take the contributions from the higher order curvature terms into consideration. On one hand, the adding of higher order curvature terms might render general relativity renormalizable [32]. On the other hand, higher-order curvature invariants also appear in the effective low-energy gravitational action of more fundamental theories, such as the string theory [33]. To prevent the theory from the spin-2 ghosts, and the well-known Ostrogradski instability [34], the higher order curvature terms are usually introduced as the Gauss–Bonnet term or an arbitrary function of the curvature, namely, $f(R)$. Both of these two modified gravity theories were applied in discussing a wide range of issues in cosmology and higher energy physics, for details, see [35–37] and references therein.

It was shown that the introduce of the Gauss–Bonnet term usually imposes no impact on the localization of gravity on the brane [38–47]. However, in $f(R)$ gravity things are more complex, since the dynamical equations are of fourth-order. Based on the fact that $f(R)$ gravity is conformally equivalent to a second-order gravity theory [48], some thin braneworld models have been constructed in the lower order frame [49–52] by introducing a conformal transformation. We hope this method still valid for the discussions of thick branes. However, as stated in [53], the method used in Refs. [49–52] would lead to ambiguous when a background scalar field is introduced.

For this reason, the thick brane solutions usually were found directly in the higher order frame [54,55]. In [54], with a background scalar field, the authors offered us some analytical thick brane solutions in both constant curvature spaces and more general space–time. While in [55], the authors numerically discussed some thick brane solutions with pure gravity. In fact, the authors

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of [55] identified the contribution of higher order curvature term $f(R) = -\alpha R^n$ ($\alpha > 0$ and $1 < n < 2$) with an effective “matter” source. The solutions were obtained by analyzing the existence of the fix points. Besides, the trapping of complex scalar field on the brane solutions was also discussed in [55].

An ideal thick brane solution should be smooth, stable and possible to localize gravity and various kinds of matter fields. Whether the four-dimensional gravity can be reproduced on the brane solutions in [54,55] is still unclear. In order to address the issue of localizing gravity on thick $f(R)$ -brane, we analyze the tensor perturbations for a particular model. Finally, we apply our results on some of the solutions found in [54].

We organize this Letter as the follows: In the next section, we set up our model and give the dynamical equations. In Section 3 we discuss the tensor perturbations of the model. The localization of four-dimensional gravity on some thick branes is investigated in Section 4. We focus mainly on the solutions for which the bulk curvatures are constants. For such solutions the perturbed equations reduce to the Klein–Gordon equation for massless spin-2 particles. Our summary is given in Section 5.

2. The model

We start with the action

$$S = \int d^4x dy \sqrt{-g} \left(\frac{1}{2\kappa_5^2} f(R) - \frac{1}{2} \partial^M \phi \partial_M \phi - V(\phi) \right), \quad (1)$$

where ϕ is a background scalar field which generates the brane. $V(\phi)$ describes a self-interacting potential for the scalar field. The gravitational coupling constant $\kappa_5^2 = 8\pi G_5$ with G_5 the five-dimensional Newtonian constant. Indices $M, N, \dots = 0, 1, 2, 3, 5$ and $\mu, \nu, \dots = 0, 1, 2, 3$ are always applied to denote the bulk and the brane coordinates, respectively.

For simplicity, we consider the static flat braneworld scenarios with the metric

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (2)$$

and the scalar field is assumed to be a function of the extra dimension $y = x^4$, i.e., $\phi = \phi(y)$. For system (1)–(2), Einstein equations are

$$f(R) + 2f_R(4A'^2 + A'') - 6f'_R A' - 2f''_R = \kappa_5^2(\phi'^2 + 2V), \quad (3)$$

and

$$-8f_R(A'' + A'^2) + 8f'_R A' - f(R) = \kappa_5^2(\phi'^2 - 2V), \quad (4)$$

where the primes represent derivatives with respect to the coordinate y and $f_R = df(R)/dR$. The equation of motion for the scalar field is given by

$$4A'\phi' + \phi'' - \frac{\partial V}{\partial \phi} = 0. \quad (5)$$

Obviously, these equations contain higher-order derivatives of the coordinates. Usually, it is very hard to solve these equations analytically, needless to say to analyze the feedbacks of these equations for the perturbations from both the metric and the scalar field, because, in general, the perturbed equations are coupling equations with higher-order derivative terms.

3. Tensor perturbations

We consider the following metric perturbations:

$$ds^2 = e^{2A(y)} (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu + dy^2, \quad (6)$$

or, in another form

$$g_{MN} = \bar{g}_{MN} + \Delta g_{MN}, \quad (7)$$

with

$$\bar{g}_{MN} = \begin{pmatrix} e^{2A} \eta_{\mu\nu} & 0 \\ 0 & 1 \end{pmatrix}, \quad \Delta g_{MN} = \begin{pmatrix} e^{2A} h_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \quad (8)$$

the background metric and the metric perturbations, respectively. Here $h_{\mu\nu} = h_{\mu\nu}(x^\rho, y)$ depend on all the coordinates. Obviously, $\Delta g_{5M} = 0$, which means we consider only tensor perturbations. According to the relation $g^{NP} g_{PM} = \delta_M^N$, one obtains the inverse of Δg_{MN} , i.e., Δg^{MN} . We keep only the first order term, i.e., $\Delta g^{MN(1)}$ and denote it as

$$\delta g^{MN} = \begin{pmatrix} -e^{-2A} h^{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix}, \quad (9)$$

where $h^{\mu\nu} = \eta^{\mu\lambda} \eta^{\nu\rho} h_{\lambda\rho}$ is raised by $\eta^{\mu\nu}$. We always use δX to denote the first order contribution of the perturbations to an arbitrary quantity X . The perturbation of the scalar field is assumed to be of first order and is denoted by $\delta\phi = \tilde{\phi}(x^\mu, y)$.

Denoting $a(y) \equiv e^{A(y)}$, we immediately obtain the following relations:

$$\begin{aligned} \delta R_{\mu\nu} &= -\frac{1}{2} (\square^{(4)} h_{\mu\nu} + \partial_\mu \partial_\nu h - \partial_\nu \partial_\sigma h_\mu^\sigma - \partial_\mu \partial_\sigma h_\nu^\sigma) - 2aa'h'_{\mu\nu} \\ &\quad - 3h_{\mu\nu}a'^2 - ah_{\mu\nu}a'' - \frac{a^2 h''_{\mu\nu}}{2} - \frac{a\eta_{\mu\nu} a' h'}{2}, \\ \delta R_{\mu 5} &= \frac{1}{2} \partial_y (\partial_\lambda h_\mu^\lambda - \partial_\mu h), \quad \delta R_{55} = -\frac{1}{2} \left(\frac{2a'h'}{a} + h'' \right), \\ \delta R &= \delta(g^{\mu\nu} R_{\mu\nu}) = -\frac{\square^{(4)} h}{a^2} + \frac{\partial_\mu \partial_\nu h^{\mu\nu}}{a^2} - \frac{a'}{a} 5h' - h''. \end{aligned} \quad (10)$$

Here $\square^{(4)} = \eta^{\mu\nu} \partial_\mu \partial_\nu$, is the four-dimensional d'Alembert operator, and $h = \eta^{\mu\nu} h_{\mu\nu}$ is the trace of the tensor perturbations.

Obviously, if

$$h = 0 = \partial_\mu h^{\mu\nu}, \quad (11)$$

only $\delta R_{\mu\nu}$ remains non-zero. The condition (11) is called the transverse-traceless gauge which can largely simplify the perturbed equations.

In $f(R)$ gravity, the Einstein equations are

$$R_{MN} f_R - \frac{1}{2} g_{MN} f(R) + (g_{MN} \square^{(5)} - \nabla_M \nabla_N) f_R = \kappa_5^2 T_{MN}, \quad (12)$$

with $\square^{(5)} = g^{MN} \nabla_M \nabla_N$ the five-dimensional d'Alembert operator. As the perturbations are considered, the feedback from Einstein equations reads

$$\begin{aligned} \delta R_{MN} f_R + R_{MN} f_{RR} \delta R - \frac{1}{2} \delta g_{MN} f(R) - \frac{1}{2} g_{MN} f_R \delta R \\ + \delta(g_{MN} \square^{(5)} f_R) - \delta(\nabla_M \nabla_N f_R) = \kappa_5^2 \delta T_{MN}. \end{aligned} \quad (13)$$

We need only to calculate the second line of the above equation. Note that

$$\begin{aligned} \nabla_M \nabla_N f_R &= (\partial_M \partial_N - \Gamma_{MN}^P \partial_P) f_R, \\ g_{MN} \square^{(5)} f_R &= g_{MN} g^{AB} (\nabla_A \nabla_B f_R), \end{aligned} \quad (14)$$

we have

$$\delta(\nabla_M \nabla_N f_R) = (\partial_M \partial_N - \Gamma_{MN}^P \partial_P) (f_{RR} \delta R) - \delta \Gamma_{MN}^P \partial_P f_R, \quad (15)$$

$$\begin{aligned} \delta(g_{MN} \square^{(5)} f_R) &= \delta g_{MN} \square^{(5)} f_R + g_{MN} \delta g^{AB} (\nabla_A \nabla_B f_R) \\ &\quad + g_{MN} g^{AB} \delta(\nabla_A \nabla_B f_R). \end{aligned} \quad (16)$$

The fluctuations of the energy-momentum

$$T_{MN} = \nabla_M \phi \nabla_N \phi - \frac{1}{2} g_{MN} g^{AB} \nabla_A \phi \nabla_B \phi - g_{MN} V(\phi) \quad (17)$$

are given by

$$\begin{aligned} \delta T_{\mu\nu} &= -\frac{a^2}{2} \phi'^2 h_{\mu\nu} - a^2 \eta_{\mu\nu} \phi' \tilde{\phi}' - a^2 V h_{\mu\nu} - a^2 \eta_{\mu\nu} \frac{\partial V}{\partial \phi} \tilde{\phi}, \\ \delta T_{5\mu} &= \phi' \partial_\mu \tilde{\phi}, \quad \delta T_{55} = \phi' \tilde{\phi}' - \frac{\partial V}{\partial \phi} \tilde{\phi}. \end{aligned} \quad (18)$$

Note that under the transverse and traceless gauge, $\delta R = 0$, and Eq. (16) reduce to

$$\begin{aligned} \delta(\nabla_M \nabla_N f_R) &= -\delta_M^\rho \delta_N^\sigma \delta \Gamma_{\rho\sigma}^5 f_R', \\ \delta(g_{MN} \square^{(5)} f_R) &= \delta_M^\rho \delta_N^\sigma \delta g_{\rho\sigma} \square^{(5)} f_R. \end{aligned} \quad (19)$$

Therefore, under this gauge, we have

$$\begin{aligned} \delta(g_{MN} \square^{(5)} f_R) - \delta(\nabla_M \nabla_N f_R) \\ = \delta_M^\rho \delta_N^\sigma a^2 \left[h_{\rho\sigma} \left(3 \frac{a'}{a} f_R' + f_R'' \right) - \frac{1}{2} f_R' h'_{\rho\sigma} \right]. \end{aligned} \quad (20)$$

The perturbed Einstein equations (13) reduce to

$$\begin{aligned} \delta R_{MN} f_R - \frac{1}{2} \delta g_{MN} f(R) \\ + \delta_M^\rho \delta_N^\sigma a^2 \left[h_{\rho\sigma} \left(3 \frac{a'}{a} f_R' + f_R'' \right) - \frac{1}{2} h'_{\rho\sigma} f_R' \right] = \kappa_5^2 \delta T_{MN}. \end{aligned} \quad (21)$$

By plugging (10) and (18) into (21), we obtain the (μ, ν) components of the perturbed Einstein equations

$$\begin{aligned} \left(-\frac{1}{2} \square^{(4)} h_{\mu\nu} - 3h_{\mu\nu} a'^2 - 2aa' h'_{\mu\nu} - ah_{\mu\nu} a'' - \frac{a^2 h''_{\mu\nu}}{2} \right) f_R \\ - \frac{1}{2} a^2 h_{\mu\nu} f(R) + a^2 \left[h_{\mu\nu} \left(3 \frac{a'}{a} f_R' + f_R'' \right) - \frac{1}{2} h'_{\mu\nu} f_R' \right] \\ = \kappa_5^2 \left(-\frac{a^2}{2} \phi'^2 h_{\mu\nu} - a^2 \eta_{\mu\nu} \phi' \tilde{\phi}' - a^2 V h_{\mu\nu} - a^2 \eta_{\mu\nu} \frac{\partial V}{\partial \phi} \tilde{\phi} \right). \end{aligned} \quad (22)$$

Note that the (μ, μ) components of Einstein equations (12) is

$$\begin{aligned} f(R) + 2f_R \left[3 \left(\frac{a'}{a} \right)^2 + \frac{a''}{a} \right] - 6f_R' \frac{a'}{a} - 2f_R'' \\ = \kappa_5^2 (\phi'^2 + 2V), \end{aligned} \quad (23)$$

which is exactly Eq. (3). We can simplify Eq. (22) as

$$\begin{aligned} \left(-\frac{1}{2} \square^{(4)} h_{\mu\nu} - 2aa' h'_{\mu\nu} - \frac{a^2 h''_{\mu\nu}}{2} \right) f_R - \frac{1}{2} a^2 h'_{\mu\nu} f_R' \\ = \kappa_5^2 \left(-a^2 \eta_{\mu\nu} \phi' \tilde{\phi}' - a^2 \eta_{\mu\nu} \frac{\partial V}{\partial \phi} \tilde{\phi} \right). \end{aligned} \quad (24)$$

Contracting the above equation with $\eta^{\mu\nu}$ one can proof that $\phi' \tilde{\phi}' + \frac{\partial V}{\partial \phi} \tilde{\phi} = 0$. Therefore, the (μ, ν) components of the perturbed Einstein equations read

$$\left(a^{-2} \square^{(4)} h_{\mu\nu} + 4 \frac{a'}{a} h'_{\mu\nu} + h''_{\mu\nu} \right) f_R + h'_{\mu\nu} f_R' = 0, \quad (25)$$

or, equivalently,

$$\square^{(5)} h_{\mu\nu} = \frac{f_R'}{f_R} \partial_y h_{\mu\nu}. \quad (26)$$

$f(R)$ 引力中张量涨落满足的方程

With the coordinate transformation

$$dz = a^{-1} dy, \quad (27)$$

we can rewrite the perturbed Eq. (26) as

$$\left[\partial_z^2 + \left(3 \frac{\partial_z a}{a} + \frac{\partial_z f_R}{f_R} \right) \partial_z + \square^{(4)} \right] h_{\mu\nu} = 0. \quad \text{KK分解} \quad (28)$$

Consider the decomposition $h_{\mu\nu}(x^\rho, z) = (a^{-3/2} f_R^{-1/2}) \epsilon_{\mu\nu}(x^\rho) \psi(z)$, and ask $\epsilon_{\mu\nu}(x^\rho)$ satisfies the transverse and traceless condition $\eta^{\mu\nu} \epsilon_{\mu\nu} = 0 = \partial_\mu \epsilon_\nu{}^\mu$, we would have a Schrödinger equation for $\psi(z)$:

$$\left[\partial_z^2 - W(z) \right] \psi(z) = -m^2 \psi(z), \quad (29)$$

with the potential $W(z)$ given by **势函数的表达式**

$$W(z) = \frac{3}{4} \frac{a'^2}{a^2} + \frac{3}{2} \frac{a''}{a} + \frac{3}{2} \frac{a' f_R'}{a f_R} - \frac{1}{4} \frac{f_R'^2}{f_R^2} + \frac{1}{2} \frac{f_R''}{f_R}. \quad (30)$$

To be more explicit, one can also factorize the Schrödinger equation (29) as

$$\begin{aligned} \left[\left(\partial_z + \left(\frac{3}{2} \frac{\partial_z a}{a} + \frac{1}{2} \frac{\partial_z f_R}{f_R} \right) \right) \left(\partial_z - \left(\frac{3}{2} \frac{\partial_z a}{a} + \frac{1}{2} \frac{\partial_z f_R}{f_R} \right) \right) \right] \psi(z) \\ = -m^2 \psi(z), \end{aligned} \quad (31)$$

which indicates that there is no gravitational mode with $m^2 < 0$. Therefore any solution of the system (1)–(2) is stable under the tensor perturbations. The zero mode (if exists) takes the form

$$\psi^{(0)}(z) = N_0 a^{3/2}(z) f_R^{1/2}(z), \quad \text{零模} \quad (32)$$

with N_0 the normalization constant.

These results indicate that as the transverse and traceless gauge is taken, the perturbation of the scalar field decouples from the metric perturbations. In the case of general relativity, Eq. (26) reduce to the five-dimensional Klein-Gorden equation for the massless spin-2 gravitons. However, for an arbitrary form of $f(R)$ and non-constant curvature R , the equation for $h_{\mu\nu}$ is largely different from the massless Klein-Gorden equation. Fortunately, the perturbed equation always remains second order due to the introducing of the transverse and traceless gauge. Now let us see a simple application of our results on some solutions given previously in [54].

4. Applications

In [54], the authors gave us brane solutions for both constant and variant curvature cases. However, the solution for later case contains a singular point, and therefore is not regularized. So we would like to consider only the constant curvature case. The corresponding Einstein equations in this case reduce to some second ones:

$$f(R) + 2f_R \left(3 \left(\frac{a'}{a} \right)^2 + \frac{a''}{a} \right) = \kappa_5^2 (\phi'^2 + 2V), \quad (33a)$$

$$-8f_R \frac{a''}{a} - f(R) = \kappa_5^2 (\phi'^2 - 2V). \quad (33b)$$

Constraining R as a constant, the solution for warp factor is uniquely determined [54]:

$$a(y) = \begin{cases} (\frac{5}{2}ky)^{2/5}, & \text{for } R = 0, & (a) \\ \cos^{2/5}(\frac{5}{2}ky), & \text{for } R = 20k^2 > 0, & (b) \\ \cosh^{2/5}(\frac{5}{2}ky), & \text{for } R = -20k^2 < 0. & (c) \end{cases} \quad (34)$$

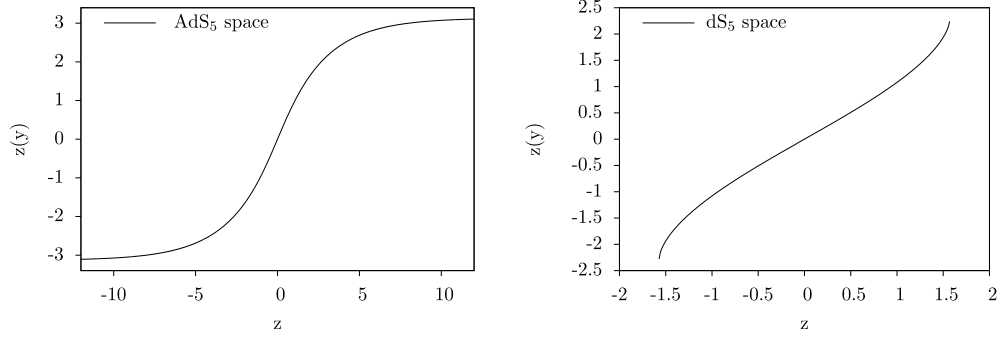


Fig. 1. Figure of $z(y)$ for AdS_5 space (left panel) and for dS_5 space (right panel) with $k = 2/5$.

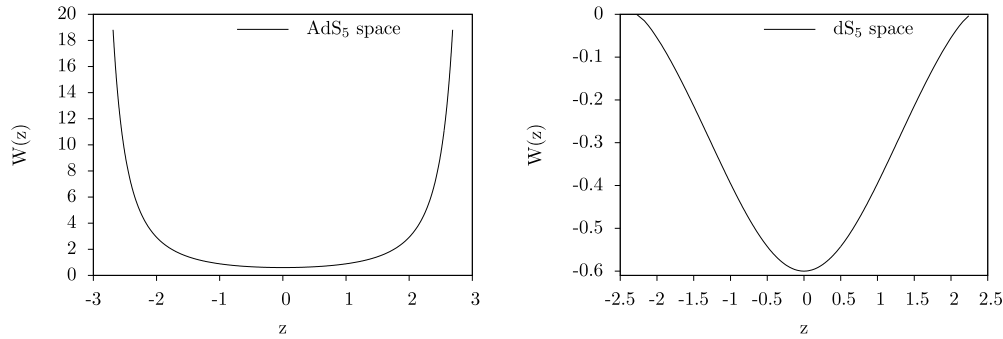


Fig. 2. Figure of $W(z)$ for AdS_5 space (left panel) and for dS_5 space (right panel) with $k = 2/5$.

Note that the warp factor given in (34a) is not smooth, it contains a cusp at $y = 0$. This cusp of the warp factor leads to at most a δ -function in the second-order Einstein gravity, which can be explained as the appearance of a thin brane. However, in the fourth-order $f(R)$ gravity, such cusp will introduce derivatives of $\delta(y)$, such as $\delta'(y)$ and $\delta''(y)$, and cross terms of them. It is still a problem of how to deal with such terms in brane theory. For this reason, we consider only solutions for dS_5 space where $R > 0$; and for AdS_5 space where R is a negative constant.

One can easily proof that for dS_5 and AdS_5 spaces, Eqs. (33) support the following non-trivial solutions:

- For dS_5 and $f_R > 0$

$$\phi = \pm \sqrt{\frac{6f_R}{5\kappa_5^2}} \operatorname{arctanh}(\sin(5ky/2)), \tag{35}$$

$$V(\phi) = V_1 - \frac{9k^2 f_R}{4\kappa_5^2} \sinh^2\left(\sqrt{\frac{5\kappa_5^2}{6f_R}} \phi\right), \tag{36}$$

with $V_1 = \frac{2f(R) - 25k^2 f_R}{4\kappa_5^2}$. We consider the interval $-\frac{\pi}{5k} \leq y \leq \frac{\pi}{5k}$, within which the warp factor (34b) is regularized.

- For AdS_5 , and $f_R < 0$

$$\phi = \pm \sqrt{\frac{6|f_R|}{5\kappa_5^2}} \operatorname{arctan}(\sinh(5ky/2)), \tag{37}$$

$$V(\phi) = V_2 + \frac{9k^2 |f_R|}{4\kappa_5^2} \sin^2\left(\sqrt{\frac{5\kappa_5^2}{6|f_R|}} \phi\right), \tag{38}$$

with $V_2 \equiv \frac{f(R)}{2\kappa_5^2} - \frac{25k^2 |f_R|}{4\kappa_5^2}$.

For the warp factors given in (34b) and (34c), the coordinate transformation (27) cannot be integrated out analytically. However,

we can find the numerical relation between z and y , see Fig. 1. Note that for the case of dS_5 space,

$$z(y) \leq \int_0^{\frac{\pi}{5k}} \cos^{-\frac{2}{5}}\left(\frac{5}{2}ky\right) dy = \frac{\sqrt{\pi}\Gamma(\frac{3}{10})}{5k\Gamma(\frac{4}{5})}, \tag{39}$$

where $\Gamma(\beta) = \int_0^\infty t^{\beta-1} e^{-t} dt$ is the Euler gamma function. Therefore, $z(y)$ is a bounded function of y .

For constant curvature spaces $f'_R = 0$, the perturbed Einstein equations reduce to

$$\square^{(5)} h_{\mu\nu} = 0. \tag{40}$$

This is the familiar Klein–Gordon equation one obtains in general relativity [8,9]. To analyze the possibility of localizing four-dimensional gravity on the brane, we simply solve the Shrödinger equation (29) with the following potential:

$$W(z) = \frac{3}{4} \frac{a'^2}{a^2} + \frac{3}{2} \frac{a''}{a}. \tag{41}$$

The corresponding zero mode takes the form

$$\psi^{(0)}(z) \propto a^{3/2}(z). \tag{42}$$

As shown in Fig. 2, for the case of AdS_5 space–time, the potential $W(z)$ is positive everywhere and diverge at $z = \pm\infty$. Such potential supports only bounded and discrete KK states. There is no continuum spectrum. Thus, the solution itself is stable under tensor perturbations. However, the zero mode dose not exist, as a consequence, the four-dimensional massless graviton cannot be localized on the brane.

The potential $W(z)$ for dS_5 space, as shown in Fig. 2, is a bounded function. Both the potential $W(z)$ and the zero mode (see Fig. 3) vanish at the boundary $z(y = \pm\frac{\pi}{5k}) = \pm\frac{\sqrt{\pi}\Gamma(\frac{3}{10})}{5k\Gamma(\frac{4}{5})}$. That means the solution in the case of dS_5 is stable; in addition, the four-dimensional graviton can be localized on the brane.

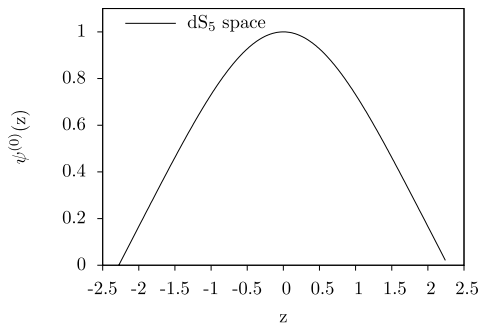


Fig. 3. The zero mode $\psi^{(0)}(z)$ for the case of dS_5 space with $k = 2/5$.

5. Conclusions

To sum up, we considered the fluctuations from both the metric and the scalar field around the flat thick $f(R)$ -branes. It turns out that the perturbation from the scalar field decouples from the tensor part of the metric perturbations when the transverse and traceless gauge is considered. The propagation of the metric perturbations in the bulk is not described by the simple massless Klein–Gordon equation any more, except $f(R) = R$ or R is a constant. As an application of our results, we studied the stability of some simple solutions given previously. These solutions were found by constraining the bulk curvatures as constants. Among these solutions, the one for $R = 0$ is problematic because at the location of the brane the metric poses a cusp, which would lead to the problem of divergence in the fourth-order $f(R)$ gravity. The analysis of the solution in case of AdS_5 indicates that the solution is stable, there are infinite discrete massive KK states. While for the case of dS_5 , the solution is stable, and the normalizable zero mode does exist. Therefore, the four-dimensional massless graviton can be localized on dS_5 brane.

The results we obtained in this Letter are valid for any solution of the system (1)–(2). In the present Letter, we confined our discussions only on the simplest case, i.e., $R = \text{const}$. However, the constant curvature spaces are rather special, because in this case the equations of motion of the tensor perturbations are independent of the form of $f(R)$. Thus it is natural to ask what would be different if the curvature is variant. To answer this question we have to firstly find a well behaved thick $f(R)$ -brane solution which is regularized, stable, and analytical (rather than numerical).¹ We also hope that the localization of gravity and the trapping of bulk matters are guaranteed by these solutions. Unfortunately, there is, to our knowledge, no such solution has ever been reported. In one of our recent works [56], we have found that at least in squared curvature gravity (where $f(R) \propto R^2$), there exists a thick domain wall solution which possess nearly all the properties we are searching for. It seems that for the case of variant curvature, the thick $f(R)$ -brane solutions contain more interesting features.

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¹ In fact, some numerical thick $f(R)$ -brane solutions have been found in [55], where R is a function of the fifth dimension y . However, it is very hard for us to analyze these numerical solutions.