Analysis and compensation of reference frequency mismatch in multiple-frequency feedforward active noise and vibration control system

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ABSTRACT

In the field of active noise and vibration control (ANVC), a considerable part of unwelcome noise and vibration is resulted from rotational machines, making the spectrum of response signal multiple-frequency. Narrowband filtered-x least mean square (NFXLMS) is a very popular algorithm to suppress such noise and vibration. It has good performance since a priori-knowledge of fundamental frequency of the noise source (called reference frequency) is adopted. However, if the priori-knowledge is inaccurate, the control performance will be dramatically degraded. This phenomenon is called reference frequency mismatch (RFM). In this paper, a novel narrowband ANVC algorithm with orthogonal pair-wise reference frequency regulator is proposed to compensate for the RFM problem. Firstly, the RFM phenomenon in traditional NFXLMS is closely investigated both analytically and numerically. The results show that RFM changes the parameter estimation problem of the adaptive controller into a parameter tracking problem. Then, adaptive sinusoidal oscillators with output rectification are introduced as the reference frequency regulator to compensate for the RFM problem. The simulation results show that the proposed algorithm can dramatically suppress the multiple-frequency noise and vibration with an improved convergence rate whether or not there is RFM. Finally, case studies using experimental data are conducted under the condition of none, small and large RFM. The shaft radial run-out signal of a rotor test-platform is applied to simulate the primary noise, and an IIR model identified from a real steel structure is applied to simulate the secondary path. The results further verify the robustness and effectiveness of the proposed algorithm.

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1. Introduction

In the fields of aircraft, watercraft, automobile, machinery, etc., the suppression of unwelcome noise and vibration has drawn much attention and effort from researchers and engineers, since over-vibrations of machine may deteriorate working

Abbreviations: ; ALC, adaptive linear combiner; ANVC, active noise and vibration control; FXLMS, Filter-x least mean square algorithm; IC, ICs, initial condition, initial conditions; MF, MFs, mismatch frequency, mismatch frequencies; MFC, mismatch frequency component; NFXLMS, narrowband FXLMS algorithm; PF, PFs, primary frequency, primary frequencies; PFC, primary frequency component; RFM, reference frequency mismatch; RF, RFs, reference frequency, reference frequencies; RFC, reference frequency component

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condition, reduce structural strength, debase the reliability and lower the safety. A considerable part of the unwelcome noise and vibration is resulting from rotational machines, such as engines, propellers, compressors, motors, etc., whose fundamental frequency are usually low (< 200 Hz), time-varying and un-precisely-known [1,2]. Generally, there will be more than one noise source and each source will be more than one harmonic, making the spectrum of response signal multi-frequency [3]. It is hard to reduce such noise and vibration by traditional passive noise and vibration control (PNVC) methods. Fortunately, active noise and vibration control (ANVC) technology which has developed since 1970s is considered as a very promising approach to suppress such noise and vibration [3–10].

The basic idea of ANVC is to generate an equal-but-opposite secondary noise or vibration to counteract the primary one. There are two main topological structures for ANVC, i.e., feedback and feedforward, where the feedforward structure takes priori knowledge (the reference signal) into consideration and will generally have a better performance [11]. The most famous feedforward algorithm for ANVC is the filtered-x least mean square (FXLMS) algorithm, whose reference signals are filtered by a secondary path model to compensate for the influence of the secondary path. Narrowband FXLMS (NFXLMS) is a variant of FXLMS algorithm [4,11], which takes more priori knowledge (i.e., the primary noise is narrowband) into account. Therefore, its controller can be simplified and the performance can be further improved by using non-vibrational/non-acoustic reference sensor to avoid the “feedback effect” [11]. The narrowband assumption is tenable, since a considerable part of unwanted noise and vibration is generated by rotational machines in real application. As a consequence, the study of narrowband noise and vibration control with applications is very active. For example, there is control on finite element (FE) plant [3], control on real-life plant [12] or even control in noise and vibration reshaping application [13].

The most commonly used parallel structure of NFXLMS is shown in Fig. 1. For each frequency component, two orthogonal sinusoids are generated as base signals. The controller is an adaptive linear combiner (ALC) of those two sinusoids. The least mean square (LMS) algorithm estimates the best coefficients of the ALC by minimizing the squared error signal. The sinusoids generator is one of the most important parts of the NFXLMS algorithm. It can be achieved by lookup table technique or digital oscillator, in which digital oscillator method requires fewer computations [11,14]. Among different types of digital oscillators, the “biquad” form (direct form) oscillator requires the least computations (one multiplication) with equal (constant) amplitude output [14]. Since the “biquad” form oscillator has no quadrature output, two oscillators (cosine and sine) are required for NFXLMS algorithm.

Traditional NFXLMS algorithm has very good performance for periodic noise and vibration reduction if the frequency of digital oscillators (called reference frequency) is exactly equal to that of the noise source. Otherwise, the control performance will be dramatically degraded. This phenomenon is called reference frequency mismatch (RFM) [1]. The RFM phenomenon indeed exists in reality, due to aging and fatigue accumulation of the reference sensor.

The influence of RFM on NFXLMS can be explained in frequency domain. The closed-loop transfer function of the
NFXLMS is essentially a notch filter [15,16]. The narrowband noise can be rejected when it falls into the notch. RFM means that the notch is not aligned with the frequency of the noise source. The step-size of the controller determines the bandwidth of the notch. From this point of view, RFM can be mitigated by increasing the step-size for a wider notch. However, since the influence of secondary path, the upper boundary of step-size is lowered [17]. Thus, this method is somewhat inapplicable. Another solution is to introduce the idea of “adaptive notch filter” [18,19], which can adaptively tune the position of the notch. This idea was firstly introduced to NFXLMS by Xiao et al. [1] and several follow-up studies were conducted [20–24].

In time domain, the adaptive notch filter can be realized by adaptive sinusoidal oscillator. The coefficient of the “biquad” oscillator can be adaptively tuned, so that it can lock on to a time-varying input signal, synchronizing its output to both the frequency and phase of the input [25,26]. Xiao et al. [1] also adopted adaptive “biquad” oscillator (he called AR-based filter). However, the second-order FIR filter (they called MPA) they applied restricts the convergence rate of the algorithm, since the eigenvalue spread is very large, especially for the case that sampling frequency is far higher than the primary frequency (pp. 130 of [11]). In this paper, we applied adaptive pair-wise orthogonal oscillators to insure the minimum eigenvalue spread, so the convergence can be improved, especially in low- and multiple-frequency case. Moreover, in NFXLMS algorithm a secondary path model is applied to “align” the phases of reference signals, so that they are in accordance with that of error signal. However, such “phase alignment” operation also introduces amplitude scaling to the reference signals. Hence, in multiple-frequency application, the convergence rate of difference frequency component is different. The overall convergence rate is restricted by the slowest one [3,27,28]. In this study, we further applied output rectifications to the oscillators, so that each component will have a relatively uniform convergence rate.

All in all, in this paper, we proposed a novel narrowband ANVC algorithm with reference frequency regulator for low- and multiple-frequency noise/vibration suppression. The frequency regulator, which implemented by adaptive pair-wise orthogonal oscillators, is intended to eliminate the RFM in traditional narrowband ANVC algorithm. This paper is structured as follows: Section 2 introduces traditional NFXLMS and analyzes the RFM problem in time domain using matrix representation for multiple-frequency situation. Section 3 further illustrates RFM in frequency domain, derives the RFM compensation algorithms, and compares the performance of different algorithms by simulation. Section 4 carries out case studies using the experimental data and Section 5 briefly concludes this paper.

2. Multiple-frequency NFXLMS

2.1. Algorithm formulation

NFXLMS is a well-known algorithm in ANVC, which is based on the assumption that the primary noise is periodic and the fundamental primary frequency is known. The most commonly used parallel structure of NFXLMS is shown in Fig. 1, where two orthogonal sinusoids are generated by digital oscillator and the coefficients of ALC are tuned by least mean square (LMS)
algorithm. Each parallel control branch in NFXLMS is account for one frequency component.

In the following part, we will demonstrate the multiple-frequency NFXLMS using matrix representation. Assume that (1) a collection of normalized primary frequencies (PFs) is in the vector

\[ \Omega_p = \{ \omega_{pi} \}_{i=1}^{q} = \{ 2\pi f_{pi}/f_s \}_{i=1}^{q}, \quad (i = 1, 2, \ldots, q), \]

where \( F_p \) is a vector contains real PFs, \( f_s \) is the sampling frequency, \( \omega_{pi} \) and \( f_{pi} \) are the normalized and real PF of the \( i \)th component, respectively; (2) the collections of primary amplitudes and primary phases are in vectors

\[ \Phi_p = \{ \phi_{pi} \}_{i=1}^{q}, \quad (i = 1, 2, \ldots, q), \]

where \( p_{pi} \) and \( \phi_{pi} \) are the amplitude and phase of the \( i \)th component of primary noise, and \( q \) is the number of frequency components; (3) the collections of base sinusoids are in vectors

\[ X_{pa}(n) = \{ x_{pa}(n) \}_{i=1}^{q} = \cos(\Omega_p n), \quad X_{pb}(n) = \{ x_{pb}(n) \}_{i=1}^{q} = \sin(\Omega_p n), \quad (i = 1, 2, \ldots, q), \]

where \( x_{pa}(n) \) and \( x_{pb}(n) \) denote cosine and sine signal, respectively, with normalized frequency of \( \omega_{pi} \), the primary noise can be expressed as

\[ d(n) = A_p^T X_{pa}(n) + B_p^T X_{pb}(n), \]

are vectors of primary noise coefficients, and

\[ P_p = \text{diag}[P_p] \]

is a diagonal matrix formed by the vector \( P_p \). NFXLMS requires reference sinusoids whose frequencies are obtained by reference sensors. Assume that a collection of the normalized reference frequencies (RFs) is in the vector

\[ \Omega_r = \{ \omega_{ri} \}_{i=1}^{q} = \{ 2\pi f_{ri}/f_s \}_{i=1}^{q}, \quad (i = 1, 2, \ldots, q), \]

where \( F_r \) is the vector contains actual RFs, \( \omega_{ri} \) and \( f_{ri} \) are the normalized and actual RF of the \( i \)th component. The reference sinusoids can be generated by look-up table method [11] or any kinds of digital sinusoidal oscillators [14]. The “biquad” digital oscillators are considered in Fig. 1, since they require fewer computations. The sine and cosine “biquad” oscillators can be written in the same form, i.e.,

\[ X_{ra}(n) = \{ x_{ra}(n) \}_{i=1}^{q} = \cos(\Omega_n n) = 2 \cos \Omega_r X_{ra}(n-1) - X_{ra}(n-2), \]

\[ X_{rb}(n) = \{ x_{rb}(n) \}_{i=1}^{q} = \sin(\Omega_n n) = 2 \cos \Omega_r X_{rb}(n-1) - X_{rb}(n-2), \]

where

\[ \cos \Omega_r = \text{diag}[\cos \Omega_r] \]

is the diagonal matrix formed by \( \cos \Omega_r \). The initial conditions (ICs) determine the type of oscillator. The ICs of the cosine oscillator are

\[ X_{ra}(0) = 1, \quad X_{ra}(1) = \cos \Omega_r, \]

and the ICs of the sine oscillator are

\[ X_{rb}(0) = 0, \quad X_{rb}(1) = \sin \Omega_r. \]

In NFXLMS, a two coefficients ALC, also called sub-controller, is applied for each parallel branch. The driving signal to actuator is a sum of outputs of all the sub-controllers, i.e.,

\[ y(n) = \text{W}^T(n) X_{ca}(n) + \text{W}^T(n) X_{cb}(n), \]

where

\[ \text{W}(n) = \{ w_{ai}(n) \}_{i=1}^{q}, \quad \text{W}(n) = \{ w_{bi}(n) \}_{i=1}^{q}, \quad (i = 1, 2, \ldots, q) \]

are vectors of sub-controller coefficients. Two types of signals are required for the FXLMS algorithm. One is filtered reference signals, i.e.,
\[ \hat{X}_{id}(n) = X_{id}(n) * s(n), \quad \hat{X}_{ib}(n) = X_{ib}(n) * s(n), \]

where \(^*\) indicates the convolution and \(s(n)\) is the impulse response of the secondary path model. The other one is error signal \(e(n)\), which is an superposition of primary noise and secondary vibration in physical domain, i.e.,

\[ e(n) = d(n) - s(n) * y(n), \]

where \(s(n)\) is the impulse response of real secondary path (the transfer path from ALC output to observation point). In numerical study, \(e(n)\) can be calculated by Eq. (15), but in real-life control, \(e(n)\) is directly obtained by error sensor. According to the steepest descent method, the coefficient updating equations are

\[ W_l(n+1) = W_l(n) + \mu_l e(n) \hat{X}_{il}(n), \quad (l = a, b) \]

where \(\mu_l\) is the step-sizes of ALC. The conventional NFXLMS has a very good performance if the RFs are accurately equal to the PFs [11]. However, if there is an error between them, the behavior of the system will be totally different.

A numerical simulation (S1) is conducted on the secondary path model

\[ S(z) = 0.04 \times \frac{z^{-1} + z^{-2}}{1 - 1.84z^{-1} + 0.96z^{-2}}, \quad (f_s = 2048Hz). \]

(17)

to demonstrate the RFM phenomenon. The errors between PFs and RFs are defined as mismatch frequencies (MFs). Thus, normalized MFs are defined in the vector

\[ \Delta \Omega = [\Delta\omega_i]^T = [\Omega_p - \Omega_i]. \quad (i = 1, 2, ..., q), \]

(18)

and real MFs are defined in the vector

\[ \Delta F = [\Delta f_i]^T = [F_p - F_i]. \quad (i = 1, 2, ..., q). \]

The simulation parameters are listed in Table 1. The results are shown in Fig. 2, in which (a) and (b) illustrates the residual errors and ALC outputs, respectively, under different MFs. It can be seen that traditional NFXLMS can effectively suppress the primary noise when there is no RFM. However, the performance is dramatically degraded when there is RFM, even if the MFs are very small.

### 2.2. Analysis of RFM

In this section, we will conduct an analytical study of RFM phenomenon in multiple-frequency NFXLMS algorithm in the mean sense. Substitute Eqs. (4) and (12) into (15), the error signal can be express as

\[
\begin{align*}
    e(n) &= A_p^T X_{p}(n) + B_p^T X_{d}(n) - \left[ W_s^T(n) X_{id}(n) + W_s^T(n) X_{ib}(n) \right] s(n).
\end{align*}
\]

(20)

For multiple-frequency noise, the secondary path \(s(n)\) introduces different amplitude ratios and phase differences at different frequencies. Define the amplitude ratios and phase differences of secondary path model at multiple RFs in the following vectors, i.e.,

\[
\begin{align*}
    P_s &= \left[ p_{si} \right]^T = \left[ S(j\omega_i) \right]^T, \\
    \Phi_s &= \left[ \phi_{si} \right]^T = \left[ \angle S(j\omega_i) \right]^T, \quad (i = 1, 2, ..., q),
\end{align*}
\]

(21)

where \(p_{si}\) and \(\phi_{si}\) are the amplitude and phase response of \(S(z)\) at the normalized RF, \(\omega_i\), the error signal can be further expressed as (see in Appendix A for detailed derivation)

\[
\begin{align*}
    e(n) &= A_p^T X_{p}(n) + B_p^T X_{d}(n) - \left[ W_s^T(n) X_{id}(n) + W_s^T(n) X_{ib}(n) \right] \left[ p_{si}^T \right] s(n).
\end{align*}
\]

(22)

### Table 1

Parameters of simulation S1: demonstration of the RFM phenomenon.

<table>
<thead>
<tr>
<th>Case</th>
<th>Samplingfreq. (Hz)</th>
<th>Sampling.time (s)</th>
<th>Primary noise</th>
<th>RFs. (Hz)( F_r )</th>
<th>MFS. (Hz) ( \Delta F )</th>
<th>Step-sizes ( \mu_a = \mu_b = \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>2048</td>
<td>10</td>
<td>[60.90]</td>
<td>[59.89]</td>
<td>[0.5, 0.5]</td>
<td>0.002</td>
</tr>
</tbody>
</table>

\(^*S(z)\) satisfy Eq. (17).
where

\[ \mathbf{\hat{W}}_a(n) = \mathbf{P}_a \cos \mathbf{\Phi}_a \mathbf{W}_a(n) + \mathbf{P}_a \sin \mathbf{\Phi}_a \mathbf{W}_a(n), \]
\[ \mathbf{\hat{W}}_b(n) = \mathbf{P}_b \cos \mathbf{\Phi}_b \mathbf{W}_b(n) - \mathbf{P}_b \sin \mathbf{\Phi}_b \mathbf{W}_b(n), \]

are the vectors of filtered ALC coefficients, and

\[ \mathbf{P}_a = \text{diag}[\mathbf{P}_a], \quad \cos \mathbf{\Phi}_a = \text{diag}[\cos \mathbf{\Phi}_a], \quad \sin \mathbf{\Phi}_a = \text{diag}[\sin \mathbf{\Phi}_a] \]

are diagonal matrices formed by corresponding vectors. Furthermore, the expectation of the squared error signal is (see in Appendix B for detailed derivation)

\[ E[\hat{e}^2(n)] = \frac{1}{2} \left[ \mathbf{A}_p^T \mathbf{A}_p + \mathbf{\hat{W}}_a^T \mathbf{\hat{W}}_a + \mathbf{B}_p^T \mathbf{B}_p + \mathbf{\hat{W}}_b^T \mathbf{\hat{W}}_b \right] - \mathbf{A}_p^T \cos(\Delta \Omega n) \mathbf{\hat{W}}_a + \mathbf{A}_p^T \sin(\Delta \Omega n) \mathbf{\hat{W}}_b - \mathbf{B}_p^T \cos(\Delta \Omega n) \mathbf{\hat{W}}_a - \mathbf{B}_p^T \sin(\Delta \Omega n) \mathbf{\hat{W}}_b, \]

where

\[ \cos(\Delta \Omega n) = \text{diag}[\cos(\Delta \Omega n)], \quad \sin(\Delta \Omega n) = \text{diag}[\sin(\Delta \Omega n)] \]

are diagonal matrices formed by corresponding vectors. According to the steepest descent method, the coefficient updating equations are

\[ \mathbf{W}(n+1) = \mathbf{W}(n) - \mu_I / 2 \partial E[\hat{e}^2(n)] / \partial \mathbf{W}(n), \quad (I = a, b). \]

The gradients can be obtained by substitute Eq. (23) into Eq. (25), i.e.,

\[ \partial E[\hat{e}^2(n)] / \partial \mathbf{W}_a(n) = \mathbf{P}_a^T \mathbf{W}_a(n) - \left[ \cos(\Delta \Omega n) \mathbf{A} + \sin(\Delta \Omega n) \mathbf{B} \right], \]
\[ \partial E[\hat{e}^2(n)] / \partial \mathbf{W}_b(n) = \mathbf{P}_a^T \mathbf{W}_b(n) - \left[ \cos(\Delta \Omega n) \mathbf{B} - \sin(\Delta \Omega n) \mathbf{A} \right], \]

where

\[ \mathbf{A} = \mathbf{P}_a \cos \mathbf{\Phi}_a \mathbf{A}_p - \mathbf{P}_a \sin \mathbf{\Phi}_a \mathbf{B}_p, \]
\[ \mathbf{B} = \mathbf{P}_a \sin \mathbf{\Phi}_a \mathbf{A}_p + \mathbf{P}_a \cos \mathbf{\Phi}_a \mathbf{B}_p. \]

Substitute Eq. (28) into Eq. (27), the updating equations can be further expressed as
\[ W_i(n + 1) = K_i W_i(n) + \mu_i/2 M_i(n), \quad (l = a, b), \]  

(30)

where

\[ K_a = 1 - \mu_a/2 P_a^2, \quad K_b = 1 - \mu_b/2 P_b^2, \]

\[ M_a(n) = \cos(\Delta \Omega n) A + \sin(\Delta \Omega n) B, \]

\[ M_b(n) = \cos(\Delta \Omega n) B - \sin(\Delta \Omega n) A. \]  

(31)

It can be seen from Eq. (31), \( M_a(n) \) and \( M_b(n) \) are sinusoids with frequencies of \( \Delta \Omega \). The transfer function matrix from \( M_a(n) \) to \( W_a(n) \) and that from \( M_b(n) \) to \( W_b(n) \) can be expressed as

\[ H_a(z) = \frac{1}{2} \frac{\mu_a z^{-1}}{1 - K_a z^{-1}}, \quad H_b(z) = \frac{1}{2} \frac{\mu_b z^{-1}}{1 - K_b z^{-1}}. \]  

(32)

The transfer functions in Eq. (32) will change the amplitudes and phases of \( M_a(n) \) and \( M_b(n) \), but keep the frequency unchanged. Thus, \( W_a(n) \) and \( W_b(n) \) are also sinusoids with frequencies of \( \Delta \Omega \). Particularly, if \( \Delta \Omega = 0 \), there are

\[ W_a(n + 1) = K_a W_a(n) + \mu_a/2 P_a \left[ \cos \Phi_p A_p - \sin \Phi_p B_p \right], \]

\[ W_b(n + 1) = K_b W_b(n) + \mu_b/2 P_b \left[ \sin \Phi_p A_p + \cos \Phi_p B_p \right]. \]  

(33)

Thus, if \( 0 < K_a < 1, 0 < K_b < 1 \), and when \( n \to \infty \), the vectors of steady state coefficients are

\[ W_a(\infty) = P_a \left[ \cos \Phi_p A_p - \sin \Phi_p B_p \right], \]

\[ W_b(\infty) = P_b \left[ \sin \Phi_p A_p + \cos \Phi_p B_p \right]. \]  

(34)

Three different simulations (S2, S3 and S4) are conducted on the model Eq. (17) to verify the analytical results. The numerical and theoretical ALC coefficients are compared. The theoretical ALC coefficients are calculated by Eq. (30), and the numerical ALC coefficients are collected in the simulation process. The simulation parameters are listed in Table 2.

In simulation S2, the comparisons of the numerical (simulation) and analytical (theory) study of multiple-frequency NFXLMS under different MFs (\([2, 2] \) Hz, \([0.5, 0.5] \) Hz and \([0, 0] \) Hz), same PFs (\([10, 25] \) Hz) and same step-size (0.006) are conducted. Fig. 3 shows the results of S2. Fig. 3(a) is the residual errors under different MFs. It can be seen that under different MFs, (1) the theory analysis and the simulation agrees well; (2) the ALC coefficient oscillates in mismatch frequency (no oscillation when the mismatch frequency is zero).

In simulation S3, the comparisons of the numerical (simulation) and analytical (theory) study of multiple-frequency NFXLMS under different step-sizes (0.001, 0.006 and 0.016), same PFs (\([10, 25] \) Hz) and same MFs (\([0.5, 0.5] \) Hz) are conducted. Fig. 4 shows the results of S3. Fig. 4(a) shows the residual errors under different step-sizes. It can be seen that larger step-size will result in a better suppression performance. Similarly, Fig. 4(b) shows the numerical and theoretical coefficients of 10 Hz PFC when the step-size is 0.001, 0.006 and 0.016, respectively. Similarly, Fig. 4(c) shows that of 25 Hz PFC when the step-size is 0.001, 0.006 and 0.016, respectively. It can be seen that under different step-sizes, (1) the theory analysis and the simulation agrees well; (2) larger step-size will result in larger amplitudes of ALC coefficients.

In simulation S4, the comparisons of the numerical (simulation) and analytical (theory) study of multiple-frequency NFXLMS under different PFs (\([50, 65] \) Hz, \([30, 45] \) Hz and \([10, 25] \) Hz), same MFs (\([1, 1] \) Hz) and same step-size (0.006) are

### Table 2

<table>
<thead>
<tr>
<th>Case</th>
<th>Samplingfreq. (Hz)</th>
<th>Pfs. (Hz)</th>
<th>Mag. (dB)</th>
<th>Phase (deg.)</th>
<th>RFs. (Hz)</th>
<th>Mfs. (Hz)</th>
<th>Step-sizes $\mu_a = \mu_b = \mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2</td>
<td>2048</td>
<td>2</td>
<td>([10,25] ) T</td>
<td>([2, 2] ) T</td>
<td>([45,60] ) T</td>
<td>([8,23] ) T</td>
<td>([2, 2] ) T</td>
</tr>
<tr>
<td>S3</td>
<td>2048</td>
<td>2</td>
<td>([10,25] ) T</td>
<td>([2, 2] ) T</td>
<td>([45,60] ) T</td>
<td>([10,25] ) T</td>
<td>([0, 0] ) T</td>
</tr>
<tr>
<td>S4</td>
<td>2048</td>
<td>2</td>
<td>([50,65] ) T</td>
<td>([2, 2] ) T</td>
<td>([45,60] ) T</td>
<td>([49,64] ) T</td>
<td>([1, 1] ) T</td>
</tr>
</tbody>
</table>

*($S$) satisfy Eq. (17).*
conducted. Fig. 5 shows the results of S4. Fig. 5(a) shows the residual error under different PFs. It can be seen that the residual errors are nearly the same under different PFs. Fig. 5(b) shows the numerical and theoretical ALC coefficients when the PFC is 10 Hz, 30 Hz and 50 Hz, respectively. Fig. 5(c) shows the numerical and theoretical ALC coefficients when the PFC is 10 Hz, 30 Hz and 50 Hz, respectively.
is 25 Hz, 45 Hz and 65 Hz, respectively. It can be seen that under different PFs, (1) the theory analysis and the simulation agree well; (2) the frequency of coefficients keeps the same (as the mismatch frequency).

Summarily, we can conclude from the above analytical study and simulations that when there is RFM, (1) larger MFs will result in worse control performance (Fig. 3); (2) larger step-size will result in a better control performance (Fig. 4); (3) the PFs do not affect the control performance (Fig. 5); (4) the parameter estimation problem (Eq. (34)) of ALC (controller) becomes tracking problem (Eq. (30)); (5) the ALC coefficients oscillate in the mismatch frequencies.

3. Compensation of RFM

3.1. Multiple-notch filter

From the above analysis, the influence of RFM can be, to some extent, mitigated by setting larger step-sizes of ALC. However, in NFXLMS algorithm, the maximum step-size is restricted because of the influence of secondary path. Another solution is to adaptively tune the RFs, so that MFs can be eliminated. To better verify the feasibility of this solution, a z-domain analysis of traditional NFXLMS is conducted. The closed-loop transfer function of the multiple-frequency NFXLMS is actually a multiple-notch filter [11]. The sketch of the system in Fig. 1 can be reduced to Fig. 6. The oscillators and the ALC of each parallel branch can be represented by an open-loop transfer function $G_i(z)$ ($i = 1, 2, \ldots, q$). The analytical equation of the open-loop transfer function can be obtained by the z-transform analysis [15] or impulse response analysis [29]. The open-loop transfer function of the $i$th parallel branch is

![Fig. 6. Simplified diagram of multiple-frequency NFXLMS system.](image-url)
where $E(z)$ is the $z$-transform of residual error $e(n)$, $Y_i(z)$ is the $z$-transform of the output of the $i$th sub-controller, and $\omega_ri$ is the $i$th normalized RF. Therefore, the closed-loop transfer function of the system can be expressed as

$$H(z) = \frac{E(z)}{D(z)} = \frac{1}{1 + S(z) \sum_{i=1}^{q} G(z)},$$

where $D(z)$ is the $z$-transform of the primary noise $d(n)$ and $S(z)$ is the $z$-transform of the secondary path $s(n)$.

The amplitude-frequency diagram of the closed-loop transfer function is illustrated in Fig. 7, where Fig. 7(b) is a zoom-in of Fig. 7(a). The secondary path model is the same as Eq. (17) and the RFs are $[30,60,90]^T$ Hz. It can be seen that the closed-loop transfer function of the multiple-frequency NFXLMS has multiple notches. The notch frequencies (i.e., the position of the notch) are determined by the RFs. If the RFs are mismatched with the PFs, there will be no notch (noise rejection) effect. Increase the step-size will enlarge the bandwidth of the notch, which indicates that the noise can be partly reduced by using large step-size (seen in Fig. 4). However, this method is somewhat inapplicable. One reason is that the step-size of the FXLMS is restricted owing to the influence of secondary path [17]. Another reason is that large step-size will magnify the frequency component that beyond the notch (seen in Fig. 7(b)), resulting in a magnification of the broadband noise.

### 3.2. Modified-I: add RF regulator

Another straightforward idea is to drive the notches approaching to the real PFs. Since the positions of the notches are determined by RFs, this method can be implemented by adaptive sinusoidal oscillators in time domain. Substitute Eqs. (12) and (8) into Eq. (15), the error signal can be express as

$$e(n) = d(n) - \left[2W_0^T(n)\cos\Omega_1X_1(n-1) + W_2^T(n)\cos\Omega_2X_2(n-2) + W_3^T(n)\cos\Omega_3X_3(n-3)\right]s(n),$$

where the secondary path $s(n)$ can be replaced by its model, $\hat{s}(n)$. It can be seen that the residual error is a non-linear function of the RFs due to the nonlinear function $\cos \Omega$. However, since the normalized RF $\Omega$ is defined in the interval $[0, \pi]$ and the non-linear function $\cos \Omega$ is a monotone function in its domain of definition, the search direction will not be affected when using optimization algorithm. Thus, the RFs can be tuned to minimize the $e(n)$ (called RF regulator). According to the steepest descent method, the updating equation of RF regulator can be expressed as

$$\Omega_i(n + 1) = \Omega_i(n) + \mu \frac{\partial e^2(n)}{\partial \Omega_i},$$

where $\mu$ is the step-size of RF regulator and the gradient is
It can be seen from Eqs. (38)–(40) that the error signal, the coefficients of ALC and the one-step delayed oscillator outputs are needed for the updating of RFs. These modifications to NFXLMS (we call modified-I algorithm) are similar to that of Xiao et al. [1]. However, the difference is that we apply ALC with two orthogonal oscillators (instead of a two-order FIR filter with one oscillator) for a better convergence. The eigenvalues spread of a NFXLMS system determines its convergence rate. By using two-order FIR filter with one oscillator, the eigenvalues spread is (seen in pp. 130 of [11])

\[ \rho_i = \frac{\lambda_{\text{max},i}}{\lambda_{\text{min},i}} = \left( \frac{1 + \cos \omega_{ri}}{1 - \cos \omega_{ri}} \right) \left( \frac{1}{\cos(2\pi f_i) / f_s} \right)^2 (i = 1, 2, ..., q). \]

It can be seen from Eq. (41) that a small \( f_{ri} \) (low-frequency) or a high sampling frequency \( f_s \) will result in a large eigenvalue spread. Thus, if the primary noise contains multiple frequency components, the sampling frequency must be high enough to cover the highest frequency component, leading to an extremely large eigenvalue spread of the lowest frequency component. In this paper, we apply ALC with two orthogonal oscillators, the eigenvalues spread is (find supports in the text of pp. 122 of [11])

\[ \rho_i = \frac{\lambda_{\text{max},i}}{\lambda_{\text{min},i}} = 1, \quad (i = 1, 2, ..., q). \]

It is well known that the LMS algorithm converges fastest when eigenvalue spread approaches 1. It can be seen from Eq. (42) that the eigenvalue spread does not affected by the reference frequency and sampling frequency. Thus, modified-I algorithm is superior in convergence performance, especially in the application of low- and multiple- noise and vibration control.

3.3. Modified-II: rectification

The convergence of the modified-I algorithm can be further improved from the viewpoint of multiple-frequency control. According to Eq. (39) and Eq. (16), there are two factors that affect the overall convergence of the modified-I algorithm: (1) the secondary path model, \( \hat{s}(n) \), and (2) the non-linear relationship \( \sin(\Omega_r) \).

The secondary path model will simultaneously affect the convergence of ALC and RF regulator, since the oscillator outputs are filtered by it. Fig. 8(a) illustrates the influence of secondary path model (Eq. (17)) on multiple-frequency NFXLMS system. It shows that \( \hat{s}(n) \) will introduce different amplitude scaling to the oscillator outputs at different frequencies, making the tuning rate of different frequency component uneven. The overall convergence will be restricted by the

Fig. 8. The factors to influence the convergence of modified-I method: (a) influence of secondary path model, \( \hat{s}(z) \); (b) influence of the nonlinear relationship, \( \sin(\Omega_r) \).
component with smallest amplitude. To remove the influence of the secondary path model, we introduce rectifications to the adaptive oscillators. Assume that a collection of amplitude responses of secondary path model at RFs is in the vector
\[
P_s = \begin{bmatrix} P_{s1} & P_{s2} & \cdots & P_{s43} \end{bmatrix},
\]
the rectification vector can be expressed as
\[
P_{rec} = \begin{bmatrix} P_{rec1} & P_{rec2} & \cdots & P_{rec43} \end{bmatrix} = \max\left[ P_s \right] P_s.
\]

Therefore, the rectified oscillator outputs can be expressed as
\[
X_{a,rec}(n) = P_{rec}(n)X_a(n),
\]
\[
X_{b,rec}(n) = P_{rec}(n)X_b(n).
\]

where
\[
P_{rec}(n) = \text{diag}\left[ P_{rec}(n) \right].
\]

The secondary factor, \(\sin(\Omega_r)\), only affects the convergence of RF regulator. Fig. 8(b) shows the influence of \(\sin(\Omega_r)\) on multiple-frequency NFXLMS system. It can be seen that (1) \(\sin(\Omega_r)\) is always positive in its range of definition (from 0 to half sampling frequency); (2) it has the maximum value at a quarter of sampling frequency. Since the values of \(\sin(\Omega_r)\) at difference frequency is different, the overall convergence will be restricted by the component with smallest value. To remove the influence of \(\sin(\Omega_r)\), we introduce an rectification to the adaptive law of the RF regulator. Assume that the rectification vector is defined as
\[
\Omega_{rec}(n) = \max\left[ \sin(\Omega_r(n)) \right] \sin(\Omega_r(n))
\]
and substitute Eq. (47) and Eq. (45) into Eq. (39), the updating equation of RF regulator becomes
\[
\Omega_{r}(n+1) = \Omega_{r}(n) + \mu_x \Omega_{rec}(n) \left[ Y_{rec}(n) \hat{s}(n) \right] e(n),
\]
\[
= \mu_x \max\left[ \sin(\Omega_r(n)) \right] Y_{rec}(n) \hat{s}(n),
\]

where
\[
R_{rec}(n) = \text{diag}\left[ R_{rec}(n) \right],
\]
\[
Y_{rec}(n) = \left[ W_a(n)X_{a,rec}(n-1) + W_b(n)X_{b,rec}(n-1) \right].
\]

As a summarization, the algorithm with further rectification can be expressed as
\[
X_a(n) = 2 \cos(\Omega_r(n))X_a(n-1) - X_a(n-2),
\]
\[
X_b(n) = 2 \cos(\Omega_r(n))X_b(n-1) - X_b(n-2),
\]
\[
y(n) = W_a(n)X_a(n) + W_b(n)X_b(n),
\]
\[
e(n) = d(n) - y(n) + s(n),
\]
\[
W_a(n+1) = W_a(n) + \mu_x e(n) \left[ X_{a,rec}(n) \hat{s}(n) \right],
\]
\[
W_b(n+1) = W_b(n) + \mu_b e(n) \left[ X_{b,rec}(n) \hat{s}(n) \right],
\]
\[
\Omega_{r}(n+1) = \Omega_{r}(n) + \mu_x \max\left[ \sin(\Omega_r(n)) \right] Y_{rec}(n) \hat{s}(n),
\]

We call Eq. (50) as modified-II algorithm, which can eliminate the uneven of convergence rate between different frequency component. The detailed diagram of the modified algorithms is shown in Fig. 9. It can be seen that two types modifications are made to the traditional NFXLMS: (1) make the sinusoidal oscillator adaptive to account for the RFM problem (modified-I algorithm); (2) further rectify the oscillator outputs and adaptive law of RF regulator for a faster convergence (modified-II algorithm). A comparison of the modified (both I and II) algorithms and the traditional NFXLMS algorithm will be conducted in the next section.

3.4. Comparison

According to the analysis above, we have traditional, modified-I and modified-II NFXLMS algorithm to deal with the multiple-frequency ANVC problem. A numerical simulation (S5) is conducted to compare the performance of these algorithms. In simulation S5, the PFs are \([30,60,90]^T\) Hz, the MFs are \([1, 1, 1]^T\) Hz, and other parameters can be found in Table 3.
Fig. 10 shows the results of S5. Fig. 10(a) shows the residual error with traditional, modified-I and modified-II control. It can be seen that when there is RFM, (1) the traditional algorithm is invalid while the modified algorithms work; (2) the modified-II algorithm converge faster than modified-I. Fig. 10(b) and (c) show the convergence process of RF regulator with modified-I and modified-II control, respectively. It can be seen that the modified-I algorithm has slower convergence of RF regulator, especially for the 30 Hz component. Fig. 10(d), (e) and (f) show the convergence process of ALC coefficients of 30 Hz, 60 Hz and 90 Hz component, respectively. It can be seen that (1) the coefficients of the traditional algorithm keep varying; (2) the coefficients of modified algorithm can finally converge to constants; (3) the modified-II algorithm has a faster and more uniform (between components) convergence property than modified-I. This simulation indicates that the convergence rates of different components are very different in multiple-frequency control, and the modifications for uniform convergence rate are meaningful in improving the overall convergence.

4. Case study using experimental data

4.1. Experimental data

We obtained a shaft radial run-out signal of a rotor test-platform to simulate the primary noise. Fig. 11(a) and (b) shows the time domain and frequency domain of the obtained signal, respectively. The sampling frequency is 2048 Hz and the length of the data is 10 s. The primary noise contains three prominent components whose frequencies are \( 30, 60, 90 \) kHz. In real application, the secondary path \( S(z) \) is the transfer function from the ALC output to the error signal input, including the amplifier, the actuator, the controlled plant, the sensor and the relevant electric circuits. We use a steel structure

Table 3
Parameters of simulation S5: algorithm comparison using simulated data.

<table>
<thead>
<tr>
<th>Case</th>
<th>( f_s ) (Hz)</th>
<th>( t_s ) (s)</th>
<th>Primary noise</th>
<th>RFs. (Hz): ( \Delta F )</th>
<th>MFs. (Hz): ( \Delta F )</th>
<th>Step-sizes ( \mu_a = \mu_b = \mu )</th>
<th>Step-sizes ( \mu_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS</td>
<td>2048</td>
<td>15</td>
<td>([30,60,90]^T)</td>
<td>([2,2,2]^T)</td>
<td>([45]^T)</td>
<td>([29,59,89]^T)</td>
<td>([1,1,1]^T)</td>
</tr>
</tbody>
</table>

*S(z) satisfy Eq. (17).

Fig. 10 shows the results of S5. Fig. 10(a) shows the residual error with traditional, modified-I and modified-II control. It can be seen that when there is RFM, (1) the traditional algorithm is invalid while the modified algorithms work; (2) the modified-II algorithm converge faster than modified-I. Fig. 10(b) and (c) show the convergence process of RF regulator with modified-I and modified-II control, respectively. It can be seen that the modified-I algorithm has slower convergence of RF regulator, especially for the 30 Hz component. Fig. 10(d), (e) and (f) show the convergence process of ALC coefficients of 30 Hz, 60 Hz and 90 Hz component, respectively. It can be seen that (1) the coefficients of the traditional algorithm keep varying; (2) the coefficients of modified algorithm can finally converge to constants; (3) the modified-II algorithm has a faster and more uniform (between components) convergence property than modified-I. This simulation indicates that the convergence rates of different components are very different in multiple-frequency control, and the modifications for uniform convergence rate are meaningful in improving the overall convergence.

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Fig. 10. Multiple-frequency ANVC using traditional, modified-I and modified-II method when there is RFM. (a) Residual errors; (b) and (c) are convergence process of RF regulator of modified-I and modified-II method, respectively. (d), (e) and (f) are ALC coefficients of 30 Hz, 60 Hz and 90 Hz PFC, respectively.

Fig. 11. Experimental data: (a1) and (a2) are time and frequency domain data of the primary noise; (b1) and (b2) are amplitude-frequency and phase-frequency plot, respectively, of real and identified frequency response function; (c) FRF of IIR Filter; (d) Amplitude and phase of each harmonic.
as the plant for the secondary path identification. Since both the input and output are voltage, the unit of the transfer
function is 1. In detail, multiple sets of random excitation signal were generated and the relevant response signals were
obtained. An IIR filter with 50 forward coefficients and 50 backward coefficients are applied to identify the system. Fig. 11
(c) and (d) shows the amplitude-frequency and phase-frequency diagram, respectively, of the experimental and the iden-
tified FRF. It shows that the IIR filter can precisely model the secondary path.

### 4.2. Control on observational model

In this section, we conduct three different case studies (S6, S7 and S8) based on observational model and real primary
noise (Fig. 11), in which the situation of no RFM (S6), small RFM (S7) and large RFM (S8) are considered. In each case, the
performance of traditional, modified-I and modified-II NFXLMS algorithm are compared. In all cases, sampling frequency
($f_s$), sampling time ($t_s$), step-size of ALC ($\mu_a, \mu_b$) and step-size of RF regulator ($\mu_c$) are the same (seen in Table 4), the only
difference is the initial values of the RFs. The PFs of the observational primary noise are estimated in advance. They are
$[\sim 32.4, \sim 64.8, \sim 97.2]^T$ Hz.

<table>
<thead>
<tr>
<th>Case</th>
<th>$f_s$(Hz)</th>
<th>$t_s$(s)</th>
<th>PFs. (Hz)$F_p$</th>
<th>RFs. (Hz)$F_r$</th>
<th>MFs. (Hz)$\Delta F$</th>
<th>Step-sizes$\mu_a = \mu_b = \mu$</th>
<th>Step-sizes $\mu_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S6</td>
<td>2048</td>
<td>10</td>
<td>$[\sim 32.4, \sim 64.8, \sim 97.2]^T$</td>
<td>$[32.4, 64.8, 97.2]^T$</td>
<td>$[\sim 0, \sim 0, \sim 0]^T$</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>S7</td>
<td>2048</td>
<td>10</td>
<td>$[31.9, 64.3, 96.7]^T$</td>
<td>$[\sim 5, \sim 5, \sim 5]^T$</td>
<td>10</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>S8</td>
<td>2048</td>
<td>10</td>
<td>$[29.4, 61.8, 94.2]^T$</td>
<td>$[\sim 3, \sim 3, \sim 3]^T$</td>
<td>10</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

*S(z) and d(n) satisfy Fig. 11.*

Fig. 12. Control results using experimental data, no RFM: (a), (c) and (e) are time domain residual errors; (b), (d) and (f) are frequency domain residual errors (the tenth second); (g) and (h) are the ALC coefficients of modified-II method; (e) convergence process of MFs of modified-II method.

as the plant for the secondary path identification. Since both the input and output are voltage, the unit of the transfer
function is 1. In detail, multiple sets of random excitation signal were generated and the relevant response signals were
obtained. An IIR filter with 50 forward coefficients and 50 backward coefficients are applied to identify the system. Fig. 11
(c) and (d) shows the amplitude-frequency and phase-frequency diagram, respectively, of the experimental and the iden-
tified FRF. It shows that the IIR filter can precisely model the secondary path.

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performance of traditional, modified-I and modified-II NFXLMS algorithm are compared. In all cases, sampling frequency
($f_s$), sampling time ($t_s$), step-size of ALC ($\mu_a, \mu_b$) and step-size of RF regulator ($\mu_c$) are the same (seen in Table 4), the only
difference is the initial values of the RFs. The PFs of the observational primary noise are estimated in advance. They are
$[\sim 32.4, \sim 64.8, \sim 97.2]^T$ Hz.
In case S6, we set the initial value of RFs to be \([32.4, 64.8, 97.2] \text{Hz}\) to simulate the "no RFM" situation. Fig. 12 illustrates the control results. Fig. 12(a) and (b) shows time and frequency domain, respectively, of the residual error when there are no control and traditional control. Fig. 12(c) and (d) shows time and frequency domain, respectively, of the residual error when there are no control and modified-I control. Fig. 12(e) and (f) shows time and frequency domain, respectively, of the residual error when there are no control and modified-II control. It can be seen that when there is no RFM, (1) the three methods can suppress the periodic components of the primary noise with almost the same performance; (2) the modified-I control has a little bit residual \(\frac{1}{C_2}\) component. This is because that the convergence of the \(\frac{1}{C_2}\) component is slowest owing to the influence of the secondary path model.

Fig. 12(g) and (h) show the ALC coefficients of the modified-II method. It can be seen that they can quickly converge. The vibration of the coefficient after convergence is owing to the uneven amplitude of the real primary noise. Fig. 12(i) shows the convergence process of MFs, i.e., RFs–PFs, of modified-II method, it can be seen that the MFs keep almost the same around zero.

In case S7, we set the initial value of RFs to be \([31.9, 64.3, 96.7] \text{Hz}\) to simulate the "small RFM" situation (MFs are \([\sim 0.5, \sim 0.5, \sim 0.5] \text{Hz}\)). Fig. 13 illustrates the control performance. The content of each sub-figure is the same as Fig. 12. It can be seen from Fig. 13(a)–(f) that with small RFM, (1) the performance of the traditional method is badly degraded; (2) both modified-I and modified-II method have control effect; (3) there is still a little bit residual \(\frac{1}{C_2}\) component in modified-I method. Fig. 13(g), (h) and (i) show the convergence process of ALC coefficients and MFs of modified-II method.

In case S8, we set the initial value of RFs to be \([29.4, 61.8, 94.2] \text{Hz}\) to simulate the "large RFM" situation (MFs is \([\sim 3, \sim 3, \sim 3] \text{Hz}\)). Fig. 14 illustrates the control performance. The content of each sub-figure is the same as Fig. 12 and Fig. 13. It can be seen from Fig. 14(a)–(f) that with large RFM, (1) the traditional method almost has no noise reduction effect; (2) both modified-I and modified-II method have control effect. However, the residual \(\frac{1}{C_2}\) component is even larger than S9 in modified-I method. This is because that larger RFM will result in a slower convergence. The modified-II method still works well.

The methods considered in this paper are summarized in Table 5. With an accurate knowledge of primary frequency, the traditional method can adapt the amplitude and phase of orthogonal oscillators output to contract the primary noise.
Without accurate knowledge of the reference frequency, the modified method (both I and II) can adapt the frequency, amplitude and phase of the orthogonal oscillators output to contract the primary noise. Specifically, the modified-II has an improved convergence compared with modified-II. When there is “no RFM”, the traditional and modified-II methods work well, but modified-I method has a fair performance due to the uneven convergence. When there is “small RFM” and “large RFM”, the traditional method has a bad performance, but modified methods still work. Especially for the modified-II method, it always has a superior performance.

5. Conclusions

In this paper, a novel narrowband ANVC algorithm with reference frequency (RF) regulator for low- and multiple-frequency noise/vibration suppression is proposed, in which the RF regulator is intended to eliminate the reference frequency
mismatch (RFM) in traditional narrowband ANVC algorithm. Firstly, the RFM phenomenon is closely investigated both analytically and numerically. It concludes that in conventional algorithm (1) RFM will degrade the control performance (larger RFM will result in worse control performance); (2) larger step-sizes can, to some extent, mitigate the influence of the RFM; (3) the primary frequencies do not affect control performance. Then, adaptive orthogonal pair-wise oscillators are introduced as the RF regulator to enable the adjustment of RFs (modified-I). Further modifications of oscillator outputs and adaptive law of RF regulator are made to improve the convergence from the viewpoint of multiple-frequency control (modified-II). Finally, case studies using experimental data are conducted, in which signal from a rotor test-platform is applied to simulate the primary noise and IIR model identified from a real steal structure is applied to simulate the secondary path. The results show that (1) both modified-I and modified-II method can compensate for the RFM problem; (2) the modified-II method has faster convergence, no matter when there is no, small or large RFM.

Acknowledgments

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Appendix A

For slow adaptation, the position of secondary path and ALC coefficients can be exchanged, so Eq. (20) can be express as

\[ e(n) = A_p^\top X_{pd}(n) + B_p^\top X_{pa}(n) - \begin{bmatrix} W_2(n)\hat{X}_{r2}(n) + W_0(n)\hat{X}_{r0}(n) \end{bmatrix} \quad \text{(A.1)} \]

where

\[ \hat{X}_{r2}(n) = X_{r2}(n) + s(n), \quad \hat{X}_{r0}(n) = X_{r0}(n) + s(n) \quad \text{(A.2)} \]

are the filtered reference signals. The convolution in (A.2) introduces amplitude ratios and phase differences for sinusoidal signals (defined in Eq. (21)), so the first equation of (A.2) can be deduced as

\[
\hat{X}_{r2}(n) = \begin{bmatrix} p_{11} \cos(\omega_1 n + \phi_{11}) \\ \vdots \\ p_{pq} \cos(\omega_q n + \phi_{pq}) \end{bmatrix} = \begin{bmatrix} p_{11} \cos(\omega_1 n) \cos \phi_{11} - p_{11} \sin(\omega_1 n) \sin \phi_{11} \\ \vdots \\ p_{pq} \cos(\omega_q n) \cos \phi_{pq} - p_{pq} \sin(\omega_q n) \sin \phi_{pq} \end{bmatrix}
\]

\[
= \begin{bmatrix} p_{11} \\ \vdots \\ p_{pq} \end{bmatrix} \begin{bmatrix} \cos \phi_{11} \\ \vdots \\ \cos \phi_{pq} \end{bmatrix} - \begin{bmatrix} p_{11} \\ \vdots \\ p_{pq} \end{bmatrix} \begin{bmatrix} \sin \phi_{11} \\ \vdots \\ \sin \phi_{pq} \end{bmatrix}
\]

\[ = \begin{bmatrix} P_s \cos \Phi \hat{X}_{r2}(n) - P_s \sin \Phi \hat{X}_{r0}(n) \end{bmatrix}. \quad \text{(A.3)} \]

Similarly, the second equation of (A.2) can be represented as

\[
\hat{X}_{r0}(n) = P_s \sin \Phi \hat{X}_{r2}(n) + P_s \cos \Phi \hat{X}_{r0}(n). \quad \text{(A.4)}
\]

Substitute (A.3) and (A.4) into (A.1) and readily leads to Eqs. (22) and (23).

Appendix B

For any \( \omega \), there are

\[
\mathbb{E}[\cos^2(\omega n)] = \frac{1}{2}, \quad \mathbb{E}[\sin^2(\omega n)] = \frac{1}{2}, \quad \mathbb{E}[\cos(\omega n)\sin(\omega n)] = 0. \quad \text{(B.1)}
\]

where \( \mathbb{E}[\cdot] \) denotes expectation. For any \( \omega_a \neq \omega_b \), define \( \Delta \omega = \omega_a - \omega_b \) and \( \Sigma \omega = \omega_a + \omega_b \), there are
\[ E[\cos(\omega_n)\cos(\omega_{0n})] = \frac{1}{2}E[\cos(\Delta \omega n)] = 0, \]
\[ E[\sin(\omega_n)\sin(\omega_{0n})] = -\frac{1}{2}E[\cos(\Delta \omega n)] = 0, \]
\[ E[\cos(\omega_n)\sin(\omega_{0n})] = \frac{1}{2}E[\sin(\Delta \omega n)] = 0, \]
\[ E[\sin(\omega_n)\cos(\omega_{0n})] = \frac{1}{2}E[\sin(\Delta \omega n)] = 0. \]  
(B.2)

However, if \( \Delta \omega \) is sufficiently small, the variation of \( \cos(\Delta \omega n) \) and \( \sin(\Delta \omega n) \) is so slow that it can be viewed as a deterministic process. Thus, for small \( \Delta \omega \) there are
\[ E[\cos(\omega_n)\cos(\omega_{0n})] \approx \frac{1}{2} \cos(\Delta \omega n), \quad E[\sin(\omega_n)\sin(\omega_{0n})] \approx \frac{1}{2} \cos(\Delta \omega n), \]
\[ E[\cos(\omega_n)\sin(\omega_{0n})] \approx -\frac{1}{2} \sin(\Delta \omega n), \quad E[\sin(\omega_n)\cos(\omega_{0n})] \approx \frac{1}{2} \sin(\Delta \omega n). \]  
(B.3)

From Eq. (22), the error signal can be expanded as
\[ e(n) = \sum_{i=1}^{q} \left[ a_p \cos(\omega_p n) + b_p \sin(\omega_p n) - \hat{\omega}_a \cos(\omega_r n) - \hat{\omega}_b \sin(\omega_r n) \right]. \]  
(B.4)

Assume that the RFM is small, i.e., the difference between \( \omega_p \) and \( \omega_r \) is small, the expectation of the relevant items obeys Eq. (B.3). Assume that the difference between \( \omega_p \) and \( \omega_{0p} \), \( \omega_p \) and \( \omega_r \), and \( \omega_r \) and \( \omega_i \) (\( i \neq j \)) are large, then the expectation of the relevant items obeys Eq. (B.2). Thus, the expectation of the squared error signal can be expressed as
\[ E[e^2(n)] = \frac{1}{2} \sum_{i=0}^{q} \left[ a_p^2 + \hat{\omega}_a^2 + b_p^2 + \hat{\omega}_b^2 \right] \]
\[ - \sum_{i=0}^{q} \left[ a_p \hat{\omega}_a \cos(\Delta \omega n) - a_p \hat{\omega}_b \sin(\Delta \omega n) - b_p \hat{\omega}_a \sin(\Delta \omega n) - b_p \hat{\omega}_b \cos(\Delta \omega n) \right] \]
\[ = \frac{1}{2} \left[ A_p^2 + \hat{\omega}_a^2 + B_p^2 + \hat{\omega}_b^2 \right] - A_P \cos(\Delta \Omega n) \hat{\omega}_a \]
\[ + A_p^2 \sin(\Delta \Omega n) \hat{\omega}_b - B_p^2 \sin(\Delta \Omega n) \hat{\omega}_b. \]  
(B.5)

Then, Eq. (25) is readily obtained.

References

[21] J. Sun, F. Ma, B. Huang, L. Wen, A narrowband active noise control system with frequency mismatch compensation, in: Proceedings of the 2014 Asia-


