Numerical analysis of anisotropic elasto-plastic deformation of porous materials with arbitrarily shaped pores

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The objective of this work is to numerically investigate the anisotropic compressive behavior of porous materials with randomly distributed, arbitrarily shaped pores in various directions. The relative pore volume fraction, the anisotropic aspect ratio and the pore arrangement are taken into account. The direction and anisotropic aspect ratio dependences of Young’s modulus and the initial yield stress are examined. Our results indicate that the anisotropic aspect ratio has a significant effect on the elasto-plastic behaviors of porous materials. Independent of pores distribution, Young’s modulus and the yield stress are found to be symmetric with the transverse direction. However, with increasing the aspect ratio Young’s modulus and the initial yield stress are greatly enlarged in the longitudinal direction of pores than those of other directions while the minimum variations are observed in transverse direction. Moreover, equations for arbitrary porous materials are developed by relating Young’s modulus and the initial yield stress in various directions to those in the transverse direction, which provides a simple and effective method for predicting the deformation of porous materials in arbitrary directions based on that in the transverse direction.

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1. Introduction

Porous materials, offering lightweight, high specific strength and good energy absorption property, are a relatively new and uncommon group of engineering materials [1,2]. Due to the manufacturing process, pores in porous materials are usually longer in the longitudinal direction (LD) than those of normal to it, which makes the porous materials substantially anisotropic [1,3,4–6]. The anisotropy occurs when foaming is performed in a mold, in which the volume expansion generated by gases cause pores to rise in one direction, and the pores become elongated in the direction of rising because they are subjected to viscous forces [1]. Variations of the sizes and shapes of pores or inclusions with direction can lead to the significant direction dependence of their properties [7–10]. As an example, the stiffness and the strength of anisotropic porous materials in LD are much larger than those in transverse direction (TD) [7,11].

The investigation of anisotropic behaviors of porous materials has been an essential problem and extensively analyzed. Gent and Thomas [12] developed a two dimensional (2D) anisotropic model and discussed the anisotropic behaviors of open-cell plastic foams. Huber and Gibson [7] proposed an orthotropic unit-cell model to describe the anisotropy in foams, which was a simple extension of the Gibson and Ashby model [13]. Equations for the ratios of the modulus, of the elastic, plastic and brittle collapse stresses and of the fracture toughness in LD to those of TD are given. Afterward, their model were extended and modified by many researchers [14,15].

Amsterdam et al. [16] studied the anisotropic mechanical properties of open-cell aluminum foams. Their results showed that the stiffness and the plastic collapse stress of the LD specimens are higher than those of the TD specimens, which was attributed to the cell shape anisotropy. Kitazono et al. [17] carried out the uniaxial compressive tests of closed-cell foams. Mu and Yao [15] experimentally investigated the anisotropic compressive behaviors of closed-cell alloy foams. They obtained linear relationships between Young’s modulus ratio, yield strength ratio and anisotropy ratio, and compared their results with those obtained by the Gibson and Ashby model. However, in all of these studies, only the LD and the TD results of the foams were taken into consideration.

Pore morphology is another key parameter that affects the anisotropic behaviors of porous materials. Using the mean-field approximation, Kitazono et al. [14] derived the elasto-plastic properties of isotropic and anisotropic closed-cell foams with an aligned spheroidal pores model. They concluded that the yield stress of closed-cell foams was independent of the loading direction, which
was not consistent with the previous results of Huber and Gibson [7]. Tane et al. [18] extended the mean-field method for predicting yield behaviors of porous materials. Their calculations revealed that the yield stress was virtually independent of the elastic anisotropy of the matrix, but strongly depended on the plastic anisotropy and also on the pore morphology.

In the aforementioned investigations, two simplifications were basically adopted: (1) Only the materials behaviors of two directions, LD and TD, were considered for the anisotropic porous materials; (2) The pores in the porous materials were usually simplified to have an ellipsoid shape. However, understanding of the proprieties of the porous materials in arbitrary directions, which is still an open problem, is essential for the extensive application of anisotropic porous materials. Moreover, the pore shape is complex after fabrication and cannot be accurately described by ellipsoids. In this paper, unit cell models with periodic boundary condition are developed and adopted to address the anisotropic compressive behaviors of porous materials. The major concerns are (1) the macroscopic anisotropic compressive behaviors of porous materials in arbitrary directions, and (2) the relations of the modulus and the initial yield stress in arbitrary directions to those of TD. The organization of this paper is as follows: In Section 2, the material models and the numerical process are presented. Then, results of the elasto-plastic deformation in various directions are presented in Section 3 for porous materials with various pore shapes. Section 4 outlines some major conclusions.

2. Formulation of the problem

2.1. Material models

Pore distributions vary in anisotropic porous materials. To well describe the features of material microstructures, it is essential to characterize the macroscopic anisotropic behaviors of the porous materials in arbitrary direction. Thus, models with different pore arrangements and sufficiently large representative volume element (RVE) are necessary, as shown in Fig. 1(a). However, large amounts of computation are involved in this modeling process [19,20]. Therefore, 2D unit cell models are often adopted to study the macroscopic anisotropic behaviors of porous materials. It is necessary to mention that the real microstructure of porous material plotted in Fig. 1(a) is only a diagrammatic sketch that aims to illustrate the complex pore distributions. The RVE applied in this work is not simplified from the real microstructure of the material.

Herein we take the rectangular unit cell model shown in Fig. 1(b) as an example to briefly demonstrate the modeling process. In order to investigate the anisotropy deformation behavior of porous, arbitrary pore shape shown in Fig. 1(c) is modeled to mimic the complex microstructures of anisotropic porous material, which is obtained by applying a compressive load to the spherical pore model with the same pore arrangement as plotted in Fig. 1(b). Firstly, compress the unit cell model with spherical pore, as shown in Fig. 1(b). Then, the node coordinates of

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Fig. 1. The modeling process of unit cell models: (a) porous materials with arbitrary pore distributions; (b) the initial model with spherical pore; and (c) the arbitrarily shaped pore model. F denotes the compressive loading.
the deformed model are used to construct the anisotropic model shown in Fig. 1(c).

The length and the width of the initial model shown in Fig. 1(b) are $L_0$ and $H_0$, respectively. The initial model is under a uniform compressive load $F$. The radius of the pore is $r$. The length and the width of the anisotropic model shown in Fig. 1(c) are denoted by $L$ and $H$, respectively.

Considering the porous geometry shown in Fig. 1(b), the pore volume fraction for the initial model can be calculated by

$$f = \frac{\pi r^2}{L_0 H_0}$$

(1)

To describe the anisotropic characteristics of the porous materials, the ratio of the length in the LD to that in the TD of the unit cell is defined as the anisotropic aspect ratio $R$

$$R = \begin{cases} 
\frac{L}{H}, & \text{for } L_0/H_0 = 1 \\
\frac{L_0}{H_0}, & \text{for } L_0/H_0 \neq 1 
\end{cases}$$

(2)

The arbitrarily shaped pore model shown in Fig. 1(c) can be regarded as the deformed morphology of a square model with spherical pores, which can be obtained by assuming $L_0/H_0 = 1$ in Fig. 1(b). It is worth mentioning that in this work the initial models with spherical pores ($L_0/H_0 = 1$) are only used to generate the anisotropic model, which represents the deformed morphology of a square model with spherical pores. After deformation, the pores of the regular models shown in Fig. 1(a)–(c) are consistent in pore distribution. In contrast, the pores in the random model have different shapes due to the interaction of pores.

In order to predict the deformation behavior of porous materials in arbitrary directions, the periodic boundary condition adopted by Arabnejad et al. [21], Zhang et al. [22], Xia et al. [23] and Fan et al. [24] is employed to the boundaries of the unit cell model. To apply the periodic boundary conditions to the unit cell, nodes at the boundaries of the unit cell must be allocated in pair at the opposite boundaries, as shown in Fig. 2(a). Consequently, the displacements of boundary nodes should satisfy [22,24]

$$\begin{align*}
\vec{x}_i/C_0 &= \vec{x}_i/C_0 \\
\vec{y}_i/C_0 &= \vec{y}_i/C_0 \\
\vec{x}_j/C_0 &= \vec{x}_j/C_0 \\
\vec{y}_j/C_0 &= \vec{y}_j/C_0
\end{align*}$$

(3)

$$\begin{align*}
\vec{u}_i &= \vec{u}_i \\
\vec{v}_i &= \vec{v}_i
\end{align*}$$

(4)

To simplify the description of deformation behavior of the unit cell model with arbitrarily shaped pore (Fig. 1(c)) under an assumed uniaxial loading in $\theta$ direction, a coordinate system can be established with its origin being fixed at the vertex $a$ and its positive $x$ axis lying along the loading direction, as shown in...
Fig. 2(c). As a result, the overall deformation of the unit cell can be described by the displacements of vertexes a, b and c under periodic boundary conditions.

The displacement loading can be expressed as follows [25]:

\[ u_i = \varepsilon_{ij} X_j + \alpha \]  

(7)

where \( u_i \) is the displacement loading, \( X_j \) (j=x,y) is the coordinates, \( \alpha \) is a constant, \( \varepsilon_{ij} \) (i,j=1,2) is the local strain of the porous materials.

Eq. (7) is the realistic displacements at vertexes b and c, which are implicitly included in the periodic boundary conditions. Based on the coordinate system defined in Fig. 2(c), Eq. (7) eventually becomes

\[
\begin{cases}
  u_b = \bar{x} L \cos \theta \\
  u_c = \bar{x} H \sin \theta
\end{cases}
\]  

(8)

where \( \bar{x} \) is the macroscopic strain.

The loading process is referenced to the work of Zhang et al. [22]. So the detail is omitted in this paper. Based on the volume-average method, the macroscopic stress and strain can be calculated from the outputted stress and strain of each element

\[
\bar{\sigma}_y = \frac{1}{V} \int_V \sigma_{yy} dv
\]  

(9)

\[
\bar{\varepsilon}_y = \frac{1}{V} \int_V \varepsilon_{yy} dv
\]  

(10)

where \( V \) the volume of unit cell model, \( \sigma_{yy} \) and \( \varepsilon_{yy} \) the local stress and strain, \( \bar{\sigma}_y \) and \( \bar{\varepsilon}_y \) the macroscopic stress and strain of the porous materials, respectively.

Then, after obtaining the macroscopic stress and strain, the extension in x direction corresponding to the macroscopic strain \( \varepsilon \) can be accomplished by enforcing the periodic boundary conditions.

2.2. Numerical calculations

In this work, to reduce the computation cost required for arbitrary pore distributions simulations, four frequently used 2D pore arrangement models shown in Fig. 3 are considered, in which...
Fig. 3(a)–(c) are models with regular pores while Fig. 3(d) is the model with arbitrarily shaped pores arranged randomly (designated "the random model"). The basic procedure for modeling and analysis of porous materials consists of the following steps. Firstly, generate the initial model with spherical pores and apply external compressive load to the model. Then, output the node information of the initial model and construct the anisotropic model based on the information of node coordinates of the deformed model. Thirdly, mesh the anisotropic model. Next, apply corresponding periodic boundary conditions and extent load in arbitrary directions to the anisotropic model. Finally, calculate the macroscopic stress and strain based on Eqs. (9) and (10). Herein, the initial random model is created by using the random function in ANSYS through controlling the pore radius and volume fraction. In this work, two sizes of pore radius are applied. Then the generated model is modified to include physical details such as the distance between pores. Note that these models presented in Fig. 3 are only schematic geometries. Actually, several anisotropic aspect ratios are studied for each pore arrangement.

The models plotted in Fig. 3(a), (b) and (d) are obtained by deforming a square model with spherical pores, that is \( L_0/H_0 = 1 \), while the model of Fig. 3(c) is obtained by deforming a rectangle unit model with pores aligned in hexagon, so \( L_0/H_0 = 1.732 \).

The numerical calculations are performed by using the commercial finite element code ANSYS. Periodic boundary conditions described in Section 2.1 are applied to the numerical models, in which nodes at the boundaries of unit cell are allocated in pair at opposite boundaries. The finite element meshes for the four kinds of anisotropic porous presented in Fig. 3 are shown in Fig. 4. In this work, the 6-node triangle element of the finite element code ANSYS are used. Finite element meshes are dedicatedly designed by considering mesh convergence.

In the present analysis, aluminum alloy porous materials are considered as an example. The matrix is assumed to be linear hardening with a plastic modulus of \( E_p \), and it is also assumed to be incompressible, isotropic and obeys von Misses yield criterion

\[
\sigma = \begin{cases} 
    E_s \varepsilon, & \text{for } \varepsilon \leq \varepsilon_0 \\
    \sigma_s + E_p (\varepsilon - \varepsilon_0), & \text{for } \varepsilon > \varepsilon_0
\end{cases}
\]

(11)

where \( E_s \) and \( E_p \) are elastic modulus and plastic modulus, respectively. The material parameters of the matrix are listed in Table 1 [26].

![Table 1](image)

Table 1

<table>
<thead>
<tr>
<th>Material parameters of the matrix</th>
<th></th>
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<tbody>
<tr>
<td>( E_s )</td>
<td>( E_p )</td>
</tr>
<tr>
<td>72.7 GPa</td>
<td>3.635 GPa</td>
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![Fig. 4](image)
3. Results and discussion

In this section, the anisotropic elasto-plastic deformation of porous materials under compressive load is discussed in detail. The effects of pore volume fraction and anisotropic aspect ratio on the stress–strain responses of anisotropic porous materials are addressed. In addition, the effect of pore arrangement on the elasto-plastic deformation of porous materials in various directions is also studied through different unit cell models shown in Fig. 3.

3.1. Effect of pore volume fraction

The unit cell model shown in Fig. 3(a) is used to investigate the effect of the pore volume fraction on the behavior of anisotropic porous materials. Herein, we choose the pore volume fraction calculated from Eq. (1) to vary from 20% to 50%, corresponding pore radii from 0.2524 to 0.399. To illustrate the effect of the pore volume fraction on the anisotropic deformation behavior of porous material, it is preferred to fix the aspect ratio $R$ at a constant value. Without loss of generality, the anisotropic aspect ratio $R$ is selected to be 1.633, which is achieved by applying a suitable compressive load to each square unit cell model.

A systematic exploration of how the anisotropic deformation of porous materials depends on the pore volume fraction allow different stress–strain curves to be plotted, as shown in Fig. 5 for 0°, 30°, 45° and 90° directions. In this work, the macroscopic stress and strain are obtained by using Eqs. (3) and (4) according to the volume-averaged method. It is clear that the higher the pore volume fraction, the lower the stress–strain curves. Furthermore, with the increase of the pore volume fraction, Young’s modulus and the flow stress decrease faster in 0° direction (LD) than those of in other directions (especially than those in 90° direction, which is the TD). Herein, Young’s modulus is defined as the ratio of stress to strain in elastic range of uniaxial stress–strain curves, while the initial yield stress is defined as the intersection of the extrapolations of the linear elastic and stress plateau lines [27]. What is more, the loading direction significantly affects the stress–strain response of porous with different pore volume fractions, which is discussed in detail in Section 3.3.

3.2. Effect of anisotropic aspect ratio

Again, we take the unit cell model in Fig. 3(a) as an example to address the effect of anisotropic aspect ratio on the stress–strain response in each loading direction by changing $R$. Numerical calculations are performed with four different anisotropic aspect ratios of 1.0, 1.354, 1.533 and 2.010. The pore volume fraction is fixed at 50% for all the results presented in this section.

The stress–strain responses in directions 0°, 30°, 45° and 90° are plotted in Fig. 6 for four different anisotropic aspect ratios. It is readily seen that quite different deformation behaviors in different directions are induced by different anisotropic aspect ratios. The larger the anisotropic aspect ratio, the higher the stress–strain curve. With increasing the anisotropic aspect ratio the strengths increase rapidly in all 0°, 30° and 45° directions. In contrast, as the anisotropic aspect ratios changes, the variation of stress–strain response in 90° direction (TD) is relatively insignificant compared with those of other directions.

3.3. Effect of pore arrangement

The effect of pore arrangement on the overall stress–strain response is also important for the understanding of the mechanical
Fig. 6. Effect of anisotropic aspect ratio $R$ on the stress–strain curves in different directions (a) 0°, (b) 30°, (c) 45° and (d) 90°. The value of pore volume fraction for this plot is $f = 50\%$.

Fig. 7. Stress–strain curves in different directions for different anisotropic aspect ratios $R$ with $f = 50\%$ (the square unit model shown in Fig. 3 (a)).
behaviors of the anisotropic porous materials. In this section, different unit cell models, e.g. regular/random cells with single and/or multi-pore of different distributions are adopted to study the influence of pore arrangement.

Based on the information in Fig. 7, we can compare the stress–strain responses of porous materials in various directions, in which the pore volume fraction $f$ is fixed at a constant value of 50%. It is readily seen that in the case of $R=1$, same deformation behaviors are observed in 0° and 90° directions. Moreover, Young’s modulus and the flow stresses in 0° and 90° directions are the largest and those in 45° are the smallest in all directions. In the case of $R=1$, the Young’s modulus and the flow stress are symmetric with 45°, as shown in Fig. 7(c), see Fig. 8 for more detail. In contrast, quite different deformations are obtained for anisotropic aspect ratio $R \neq 1$, the flow stress in 90° is not the largest anymore. The stress–strain response shows that Young’s modulus and the flow stress lose their symmetry with 45°. More detailed discussions are necessary and are presented as follows.

3.3.1. Regular model—pores aligned in regular (with reference to Fig. 3(a))

Herein, the normalized Young’s modulus $E/E_S$ and the normalized initial yield stress $\sigma/\sigma_S$ are defined as Young’s modulus and the initial yield stress of the porous to their corresponding parts of the matrix (designated by the lower script “S”), respectively.

We discuss the variations of $E/E_S$ and $\sigma/\sigma_S$ by changing the anisotropic aspect ratio $R$ for the rectangular unit cell model of Fig. 3(a) ($L_0/H_0 = 1$), in which pores are aligned in regular in the matrix. In the present calculations, the pore volume fraction is fixed to be $f = 50\%$, and the aspect ratio $R$ takes four different values, i.e. 1.0, 1.354, 1.633 and 2.020. Loading is applied at an interval of 5°.

Variations of $E/E_S$ and $\sigma/\sigma_S$ in different directions are plotted in Fig. 8 for different anisotropic aspect ratios $R$. It is seen that Young’s modulus and the yield stress are very sensitive to directions. The peak values of Young’s modulus and yield stress always appear in 0° direction, which is perpendicular to the compression direction. It is important to note that neither the minimum value of $E/E_S$ nor the minimum value of $\sigma/\sigma_S$ is exactly in 45° direction. Results also indicate that with increasing the anisotropic aspect ratio, $E/E_S$ and $\sigma/\sigma_S$ increase rapidly in 0° direction, while grows relatively slowly in 90° direction. For the initial model of $R = 1$, $E/E_S$ and $\sigma/\sigma_S$ are symmetric with 45°, 90° and 0°. However, in the case of $R \neq 1$, the results are only symmetric with 0° and 90°. In other words, in the case of $R \neq 1$, $E/E_S$ and $\sigma/\sigma_S$ lose their symmetries with respect to 45° direction as they present for $R = 1$. For all the cases mentioned above, there is a period of 180° for $E/E_S$ and $\sigma/\sigma_S$.

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**Fig. 8.** Normalized Young’s modulus (a) and normalized initial yield stress (b) for regular model in Fig. 3 (a) with $f = 50\%$.

**Fig. 9.** Normalized Young’s modulus (a) and normalized initial yield stress (b) for regular model shown in Fig. 3 (b) with $f = 40\%$. 
3.3.2. Regular model—pores aligned in stripes (with reference to Fig. 3(b))

Here, the unit cell models with multi-pores ($L_0 = H_0 = 1$), as shown in Fig. 3(b), are employed to study the anisotropy behaviors of porous material with pores align in stripes. Again, the normalized Young’s modulus $E/E_S$ and the normalized yield stress $\sigma/\sigma_S$ of porous materials are calculated and compared for different directions and aspect ratios $R$. The anisotropic aspect ratios are assumed to be 1.0, 1.3998 and 1.6588.

The variations of $E/E_S$ and $\sigma/\sigma_S$ are plotted in Fig. 9. Note that the deformation behaviors of porous materials with pores aligned in stripes (Fig. 3(b)) are quite different from those of regular aligned pores (Fig. 3(a)). In the case of $R = 1$, the peak values of $E/E_S$ and $\sigma/\sigma_S$ appear in $45^\circ$ direction for pores aligned in stripes (Fig. 3(b)), but exhibit the minimum $E/E_S$ and $\sigma/\sigma_S$ values in $45^\circ$ direction for regular aligned pores (Fig. 3(a)). Correspondingly, in the case of $R = 1$, the minimum values of $E/E_S$ and $\sigma/\sigma_S$ are observed in $0^\circ$ and $90^\circ$ directions for pore arrangement of Fig. 3(b), but the maximum values for regular aligned pores in Fig. 3(a). For the case of $R \neq 1$, the angles of the maximum $E/E_S$ and $\sigma/\sigma_S$ for pores aligned in stripes (Fig. 3(b)) decrease with increasing the aspect ratio and finally their maximum values are observed in $0^\circ$ direction. While for regular aligned pores shown in Fig. 3(a), the minimum values of $E/E_S$ and $\sigma/\sigma_S$ are found nearby $45^\circ$ direction. For regular aligned pores shown in Fig. 3(a), with the increase of anisotropic aspect ratio, both $E/E_S$ and $\sigma/\sigma_S$ increase rapidly in $0^\circ$ direction while maintain relatively constant values in $90^\circ$ direction; in contrast, for the model represented by Fig. 3(b), $E/E_S$ and $\sigma/\sigma_S$ increase in $0^\circ$ direction and decrease in $90^\circ$ direction.

3.3.3. Regular model—pores aligned in hexagon (with reference to Fig. 3(c))

Another regular model studied here is the rectangular unit cell model with pores hexagonally arranged, as shown in Fig. 3(c). We still focus on the variations of $E/E_S$ and $\sigma/\sigma_S$ in various directions. The anisotropic aspect ratio $R$ of the unit cell with pores aligned in hexagon is calculated by Eq. (2), and the value is $L_0/H_0 = 1.732$.

Variations of $E/E_S$ and $\sigma/\sigma_S$ are plotted in Fig. 10 for different anisotropic aspect ratios. Comparable to these results of regular models presented in Figs. 8 and 9, as direction changes, the variation of $E/E_S$ also have a period of $180^\circ$. However, different from that of above two regular models, the yield stress of the porous materials with pores hexagonally arranged has a period of...
direction dependence of

...direction, and decrease slowly in 90–130

...aspect ratio increases, $E/E_5$ and $\sigma/\sigma_5$ for porous materials with irregular pore arrangement is consistent with those of regular models. However, for irregular model, $E/E_5$ and $\sigma/\sigma_5$ lose their isotropic characteristics.

3.4. Relationships between $E_\theta/E_90$ and $\sigma_\theta/\sigma_90$ and anisotropic aspect ratio

Gibson and Ashby [13] proposed the ratios of the elasto-plastic parameters in LD to those in TD, which was extended in many investigations afterwards, i.e.

$$\frac{E_\theta}{E_90} = \frac{2R^2}{1+1/R^3}$$  \hspace{1cm} (12)$$

$$\frac{\sigma_\theta}{\sigma_90} = \frac{2R}{1+1/R}$$  \hspace{1cm} (13)$$

However, the aforementioned work only investigated the properties of porous materials in the directions of LD and TD. The anisotropic deformation behavior of porous materials in arbitrary direction is still an open problem.

Based on the random model with more actual pore shapes, we adopt the periodic boundary condition to numerically analyze the anisotropic compressive behaviors of the porous materials in arbitrary directions. Herein, two non-dimensional parameters $E_\theta/E_90$ and $\sigma_\theta/\sigma_90$ are defined, which respectively represent the ratios of Young’s modulus and yield stress in arbitrary directions to those of TD. The normalized Young’s modulus and yield stress for FEM results (dot) and numerical fitting results (real line) are plotted in Fig. 12. The periodic observations of $E_\theta/E_90$ and $\sigma_\theta/\sigma_90$ for porous materials with the irregular pore arrangement are consistent with $E/E_5$ and $\sigma/\sigma_5$, which are symmetric with 0° and 90° direction. From Fig. 12, it is clear that the proposed relationships for $E_\theta/E_90$ and $\sigma_\theta/\sigma_90$ can predict the deformation behavior of porous materials in arbitrary directions. Based on the large amounts of computations and the previous works, we obtain the relationship between $E_\theta/E_90$ and the anisotropic aspect ratio $R$

$$\frac{E_\theta}{E_90} = \frac{2R^2}{1+1/R^3} \cos^2\theta + \sin^2\theta$$  \hspace{1cm} (14)$$

and the relationship between $\sigma_\theta/\sigma_90$ and the anisotropic aspect ratio $R$

$$\frac{\sigma_\theta}{\sigma_90} = \frac{2R}{1+1/R} \cos^2\theta + \sin^2\theta$$  \hspace{1cm} (15)$$

Using Eqs. (14) and (15), one can calculate Young’s modulus and the initial yield stress of the porous materials in arbitrary directions based on the results of TD. It is obvious that Eqs. (14) and (15) reduce to those of isotropic model if the anisotropic aspect ratio $R = 1$, i.e. if $R = 1$, then $E_\theta/E_90 = 1$ and $\sigma_\theta/\sigma_90 = 1$.

4. Conclusion

In this work, anisotropic deformation of porous material is numerically investigated by considering the unit cell models with pores both regularly and randomly distributed and arbitrarily shaped, respectively. The overall stress–strain curves, the normalized Young’s moduli and the normalized initial yield stresses in various directions are obtained for porous materials. The effects of the relative pore volume fraction, the anisotropic aspect ratio and the pore distribution on the anisotropic deformation behaviors of porous materials are discussed in detail.

Numerical results for four kinds of porous materials reveal that pore distribution plays a key role in the elasto-plastic behavior of porous materials, which can result in different trends for the normalized Young’s modulus and normalized initial yield stress
with increasing the anisotropic aspect ratio. As the anisotropic aspect ratio increases, Young’s modulus and the initial yield stress can be greatly enhanced in the longitudinal direction; in contrast, the smallest differences are observed in the transverse direction.

Based on large amount of numerical calculations for porous materials with various pore arrangements, the ratios of Young’s modulus and the initial yield stress in arbitrary directions to those of transverse direction are formulated, which makes it possible to predict Young’s modulus and the initial yield stress of the porous materials in arbitrary directions. Numerical results show that the proposed formulation can be used to give pretty good approximations to the deformation behavior of porous materials in arbitrary directions.

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