

EXPLOSIVE SOLUTIONS OF PARABOLIC STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS WITH LÉVY NOISE

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(Communicated by Manuel Del pino)

ABSTRACT. In this paper, we study the explosive solutions to a class of parabolic stochastic semilinear differential equations driven by a Lévy type noise. The sufficient conditions are presented to guarantee the existence of a unique positive solution of the stochastic partial differential equation under investigation. Moreover, we show that positive solutions will blow up in finite time in mean L^p -norm sense, provided that the initial data, the nonlinear term and the multiplicative noise satisfies some conditions. Several examples are presented to illustrate the theory. Finally, we establish a global existence theorem based on a Lyapunov functional and prove that a stochastic Allen-Cahn equation driven by Lévy noise has a global solution.

1. Introduction. Fujita [13] considered the initial-boundary problem for a semilinear parabolic equation

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta u + u^{1+\alpha}, & t > 0, x \in \mathbb{R}^d, \\ u(x, 0) = a(x), & x \in \mathbb{R}^d, \end{cases} \quad (1)$$

Fujita showed that there does not exist a global solution for any nontrivial nonnegative initial data when $0 < d\alpha < 2$, and there exists a global solution for sufficiently small initial data when $d\alpha > 2$. Hayakawa [17] proved that (1) has no global solution for any nontrivial nonnegative initial data in the critical case $d\alpha = 2$ if the dimension d equals 1 or 2.

Fujita [14] studied the initial-boundary problem for a semilinear parabolic equation in domain $D \subset \mathbb{R}^d$:

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta u + f(u), & t > 0, x \in D, \\ u(x, 0) = a(x), & x \in D, \\ u(x, t) = 0, & x \in \partial D, \end{cases} \quad (2)$$

2010 *Mathematics Subject Classification.* Primary: 60H15, 60J75.

Key words and phrases. Stochastic reaction-diffusion equation, positive solution, blow-up of solutions, Lévy noise.

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Denote λ_0 as the smallest eigenvalue of $-\Delta$ and the corresponding eigenfunction $\phi_0 > 0$, satisfies $\int_D \phi_0(x)dx = 1$ and

$$\begin{cases} -\Delta\phi_0 = \lambda_0\phi_0 & \text{in } D, \\ \phi_0|_{\partial D} = 0. \end{cases}$$

Assume that f satisfies the following

(f.1) f is locally Lipschitz continuous.

(f.2) $f(0) \geq 0$ and $f(r) > 0$ for $r > 0$.

(f.3) $1/f$ is integrable at $t = +\infty$.

(f.4) f is convex in $[0, \infty)$.

(f.5) $f(r) - \lambda_0 r > 0$ for $r > \int_D a_0 \phi_0 dx$, where $a_0 = \exp(-k|x|^2)$, $k > 0$, $x \in \mathbb{R}^d$.

Fujita [14] showed that if D is bounded, $a(x) \geq 0$ in D and f satisfies (f.1)-(f.5), then the solution of (2) blows up in finite time. As a special case that $f(r) = r^{1+\alpha}$ ($\alpha > 0$), the solution of (2) blows up in finite time if

$$\int_D a(x)\phi_0(x)dx \geq \lambda_0^{1/\alpha}.$$

We refer to [15] about the many developments on solutions of nonlinear parabolic equations may blow up in finite time.

In recent years, stochastic partial differential equations have attracted the attention of many researchers. It is of interest to study the non-existence of global solutions to parabolic stochastic partial differential equations perturbed by random noise as follows:

$$\begin{cases} du = \Delta u + f(u) + \sigma(u)dW_t, & t > 0, x \in D, \\ u(x, 0) = g(x), & x \in D, \\ u(x, t) = 0, & x \in \partial D. \end{cases} \quad (3)$$

When $f(u) \equiv 0$, $\sigma(u) = u^\gamma$ ($\gamma \geq 1$), Mueller [20] considered the equation

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta u + u^\gamma \dot{W}, & \gamma \geq 1, t > 0, 0 \leq x \leq J, \\ u(t, 0) = u(t, J) = 0, \end{cases} \quad (4)$$

where $\dot{W} = \dot{W}(t, x)$ is 2-parameter white noise and $u(x, 0)$ is nonnegative and continuous. The conclusion is that for $1 \leq \gamma < \frac{3}{2}$, u exists for all time. Mueller [21] showed that when $u(x, 0)$ is a continuous nonnegative function on $[0, J]$, vanishing at the endpoints, but not identically zero, then there is a positive probability that the solution u of (4) blows up in finite time if $\gamma > 3/2$. When $\sigma(u) \equiv 1$ and the Laplacian operator Δ is replaced by the infinitesimal generator of a C_0 semigroup, Prato and Zabczyk [9] considered the stochastic semilinear equation

$$\begin{cases} du = (Au + F(u))dt + dW \\ u(0) = \xi, \end{cases} \quad (5)$$

where A is the generator of semigroup $S(t) = e^{At}$ on a Banach space E , and F is a mapping from E into E . W is a Wiener process defined on a probability space $(\Omega, \mathcal{F}_t, P)$. ξ is an \mathcal{F}_0 -measurable E -valued random variable. They assumed that F satisfies the Lipschitz condition on bounded sets of E . This property of F together with some other conditions ensure that (5) has a unique non-exploding solution. When $\sigma(u) = \sigma$ (positive constant), W is a 2-dimensional Brownian sheet, f is a nonnegative, convex function such that $\int_0^\infty 1/f < \infty$, Bonder and Groisman [3]

proved that the solution to (3) blows up in finite time with probability one for every nonnegative initial datum $u(x, 0) \geq 0$. Dozzi and López-Mimbela [11] considered the equation (3) with $\sigma(u) = \kappa u$, where κ are given positive numbers, W_t is a standard one-dimensional Brownian motion. They proved that the solution of (3) blows up in finite time with positive probability if $f(u) \geq Cu^{1+\beta}$ ($u > 0$, $C, \beta > 0$).

Chow [8, 5] considered the initial-boundary value problem for the parabolic Itô equation

$$\begin{cases} \frac{\partial u}{\partial t} = Au + f(u, x, t) + \sigma(u, \nabla u, x, t) \partial_t W(x, t) \\ u(x, 0) = g(x), \quad x \in D, \\ u(x, t) = 0, \quad t \in (0, T), x \in \partial D, \end{cases} \quad (6)$$

where $D \subset \mathbb{R}^d$, $A = \sum_{i,j=1}^d \frac{\partial}{\partial x_i} [a_{ij}(x) \frac{\partial}{\partial x_j}]$ is a symmetric, uniformly elliptic operator with smooth coefficients, f and σ are given functions. For $x \in \mathbb{R}^d$, $t \geq 0$, $W(x, t)$ is a continuous Wiener random field defined in a complete probability space (Ω, \mathcal{F}, P) with a filtration \mathcal{F}_t . $W(x, t)$ has mean $\mathbb{E}W(x, t) = 0$ and covariance function $q(x, y)$ defined by

$$\mathbb{E}\{W(x, t)W(y, s)\} = (t \wedge s)q(x, y), \quad x, y \in \mathbb{R}^d,$$

where $t \wedge s = \min\{t, s\}$ for $0 \leq t, s \leq T$. Let $H = L^2(D)$, $H^1 = H^1(D)$ be the L^2 -Sobolev space of first order and H_0^1 the closure in H^1 of the space of C^1 -functions with compact support in D . Under the usual conditions, such as coercivity conditions, Lipschitz continuity and boundedness conditions, Eq.(6) has a unique global strong solution $u \in C([0, T]; H) \cap L^2([0, T]; H_0^1)$ for any $T > 0$ (see [7, Theorem 3-7.2]). To consider positive (nonnegative) solutions, the author assume that the following conditions hold:

(P1) There exists a constant $\delta \geq 0$ such that

$$\frac{1}{2}q(x, x)\sigma^2(r, \xi, x, t) - \sum_{i,j=1}^d a_{ij}(x)\xi_i\xi_j \leq \delta r^2,$$

for all $r \in \mathbb{R}$, $x \in \overline{D}$, $\xi \in \mathbb{R}^d$ and $t \in [0, T]$.

(P2) The function $f(r, x, t)$ is continuous on $\mathbb{R} \times \overline{D} \times [0, T]$ such that $f(r, x, t) \geq 0$ for $r \leq 0$ and $x \in \overline{D}$, $t \in [0, T]$.

(P3) The initial datum $g(x)$ on \overline{D} is positive and continuous.

Chow [8] proved that the solution of Eq.(6) is positive. Under some suitable conditions, Chow [8, 5] showed that the positive solutions of a class of stochastic reaction-diffusion equations will blow up in the L^p -norm sense, $p \geq 1$. Chow and Liu [6] considered the problem of explosive solutions in mean L^p -norm sense of semilinear stochastic functional parabolic differential equations of retarded type.

Lv and Duan [19] considered the Eq.(6) with $A = \Delta$, the Laplacian operator, the nonlinear term f is assumed to be satisfied by an inequality, which is weaker than the condition (P2), the noise intensity σ allows to be higher nonlinear than the square nonlinear (see [19, formula (3.2)]). They proved that the noise could induce finite time blow up of solutions.

Recently, great attention has been devoted to the study of stochastic partial differential equations driven by Lévy noise (see, for example, [4, 25, 27, 18, 22, 10, 26, 23, 24]). But there are few results about the existence of explosive solutions to stochastic partial differential equations with Lévy noise in the literature. Bao and

Yuan [2] considered the stochastic reaction-diffusion equations with jumps

$$\begin{cases} \frac{\partial u}{\partial t} = Au + b(t, u, x) + \sigma(t, u, \nabla u, x) \partial_t W(x, t) \\ \quad + \int_{\mathbb{Y}} \Upsilon(t, u, x, y) \partial_t \tilde{N}(t, dy), \quad t > 0, \\ u(x, 0) = \phi(x), \quad x \in \mathcal{O}, \quad u(t, x)|_{\partial \mathcal{O}} = 0, \quad t > 0, \end{cases} \quad (7)$$

where $\mathcal{O} \subset \mathbb{R}^n$ is a bounded domain with C^∞ boundary, $A = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} (a_{ij}(x) \frac{\partial}{\partial x_j})$ is a symmetric, uniformly elliptic operator with smooth coefficients, $W(x, t)$ is a Wiener random defined on the completed probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$, $\tilde{N}(dt, du)$ is the compensated Poisson measure. Under some conditions, they showed that the solution of (7) blows up in finite time. It should be pointed out that the nonlinear term $b : [0, \infty) \times \mathbb{R} \times \overline{\mathcal{O}} \mapsto \mathbb{R}$ is assumed to be locally Lip-continuous w.r.t the second variable such that $b(t, r, x) \geq 0$ for any $r \leq 0$, however, there are many functions don't satisfy this condition, for example, $b(r) = r(1 - r^2)$. And $\mathcal{O} \subset \mathbb{R}^n$ is assumed to be bounded, the proof of Theorem 2.1 in [2] depends on the boundedness of volume of \mathcal{O} . The results of [2] can't be generalized to the case for unbounded domain, such as $\mathcal{O} = \mathbb{R}^n$.

In this paper, we study the problem of explosive solutions to a class of semilinear stochastic parabolic differential equations driven by Lévy noise. The paper is organized as follows. In Section 2, we recall some basic results for semilinear stochastic parabolic equations with Lévy noise. In Section 3, under some assumptions, we prove that the existence of positive solutions of a semilinear stochastic reaction-diffusion equation. In Section 4, under some suitable conditions on the drift or diffusion term, we prove that the solutions of stochastic parabolic differential equations will blow up in a finite time in mean L^p -norm sense, $p \geq 1$. Some examples are presented to illustrate the theory. In Section 5, we establish a global existence theorem based on a Lyapunov functional. We show that the existence of global solution to stochastic Allen-Cahn equation driven by Lévy noise.

2. Preliminaries. Let D be a domain in \mathbb{R}^d , which has a smooth boundary if it is bounded. Denote $L^2(D)$ by H , the usual L^2 real Hilbert space with the inner product (\cdot, \cdot) and norm $\|\cdot\|$, respectively. Let $H^1 = H^1(D)$ be the L^2 -Sobolev space of first order. Denote H_0^1 the closure in H^1 of the space of C^1 -functions with compact support in D . Denote by $D([0, T], H)$ the space of all càdlàg paths from $[0, T]$ into H . Let $W(x, t)$ be a continuous Wiener random field defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a filtration \mathcal{F}_t . $W(x, t)$ has mean zero and covariance function $q(x, y)$ such that

$$\mathbb{E}W(x, t) = 0, \quad \mathbb{E}\{W(x, t)W(y, s)\} = (t \wedge s)q(x, y), \quad s, t \in [0, T], x, y \in \mathbb{R}^d.$$

The associated covariance operator Q in H with kernel $q(x, y)$ is defined by

$$(Q\phi)(x) = \int_D q(x, y)\phi(y)dy, \quad x \in D, \quad \phi \in H.$$

In this paper, we assume that the covariance function $q(x, y)$ is bounded, continuous and there is $q_0 > 0$ such that

$$\sup_{x, y \in D} |q(x, y)| \leq q_0 \quad \text{and} \quad \text{Tr } Q = \int_D q(x, x)dx < \infty.$$

Let $(Z, \mathcal{B}(Z))$ be a measurable space. Denote by $N(dt, dz)$ the Poisson random measure with intensity measure $dt\nu(dz)$ on $\mathbb{R}_+ \times Z$, here $\mathbb{R}_+ = [0, \infty)$, dt is the Lebesgue measure on \mathbb{R}_+ , $\nu(dz)$ is a σ -finite measure on $(Z, \mathcal{B}(Z))$. Denote by $\tilde{N}(dt, dz) = N(dt, dz) - dt\nu(dz)$ the compensated Poisson measure. Assume that W and N are independent.

Consider the initial-boundary problem of a semilinear stochastic reaction-diffusion equation in domain $D \subset \mathbb{R}^d$:

$$\begin{cases} \frac{\partial u}{\partial t} = Au + f(u, x, t) + \sigma(u, \nabla u, x, t)\partial_t W(x, t) \\ \quad + \int_Z \varphi(u, x, z, t)\partial_t \tilde{N}(t, dz), \\ u(x, 0) = g(x), \quad x \in D, \\ u(x, t) = 0, \quad t \in (0, T), x \in \partial D, \end{cases} \quad (8)$$

where $A = \sum_{i,j=1}^d \frac{\partial}{\partial x_i} [a_{ij}(x) \frac{\partial}{\partial x_j}]$ is a symmetric, uniformly elliptic operator with smooth coefficients, that is, there exists a constant $c > 0$ such that $b(x, \xi) := \sum_{i,j=1}^d a_{ij}(x) \xi_i \xi_j \geq c|\xi|^2$ for all $x \in \bar{D}$ and $\xi = (\xi_1, \dots, \xi_d) \in \mathbb{R}^d$.

Let $u_t = u(\cdot, t)$, $F_t(u) = f(u, \cdot, t)$, $\Sigma_t(u) = \sigma(u, \nabla u, \cdot, t)$, $\Gamma_t(u, z) = \varphi(u, \cdot, z, t)$ and $W_t = W(\cdot, t)$, then we can rewrite the equation (8) as

$$\begin{cases} du_t = [Au_t + F_t(u_t)]dt + \Sigma_t(u_t)dW_t + \int_Z \Gamma_t(u_t, z)\tilde{N}(dt, dz), \\ u_0 = g, \end{cases} \quad (9)$$

where A is regarded as a linear operator from H^1 into H^{-1} with domain $D(A) = H_0^1 \cap H^2$, $F_t : H \rightarrow H$ is continuous. If A satisfies the coercivity condition, f and σ satisfy the Lipschitz continuity and boundedness conditions, the equation (9) has a unique global strong solution $u \in L^2([0, T]; H_0^1) \cap D([0, T]; H)$ for any $T > 0$ (see Theorem 3.2, [23]).

To consider the positive solutions, we assume that (8) has a unique (strong) solution. In addition, we assume that

(A1)

$$f(u, x, t) \geq a_1 u^\beta + a_2 u,$$

where $a_1, a_2 \in \mathbb{R}$, $\beta > 1$, $(-1)^\beta \in \mathbb{R}$ and

$$a_1 \begin{cases} > 0, & \text{if } (-1)^\beta = 1, \\ < 0, & \text{if } (-1)^\beta = -1. \end{cases} \quad (10)$$

(A2) There exist constants $b_1, b_2 \geq 0$ such that

$$\frac{1}{2}q(x, x)\sigma^2(u, \xi, x, t) - \sum_{i,j=1}^d a_{ij}(x)\xi_i \xi_j \leq b_1 |u|^m + b_2 u^2,$$

for all $u \in \mathbb{R}$, $x \in \bar{D}$, $\xi \in \mathbb{R}^d$ and $t \in [0, T]$, where $2 < m < \beta + 1$.

(A3) There exist a constant $\mu \in [2, \beta + 1)$ and mappings $\psi : \bar{D} \rightarrow \mathbb{R}_+$ with $\int_Z \psi(z)\nu(dz) < +\infty$, such that

$$\varphi^2(u, x, z, s) \leq \psi(z)|u(x, s)|^\mu. \quad (11)$$

(A4) the initial datum $g(x)$ on \bar{D} is positive and continuous.

As in [8], let $\eta(r) = r^-$ denote the negative part of r for $r \in \mathbb{R}$, or $\eta(r) = 0$, if $r \geq 0$ and $\eta(r) = -r$ if $r < 0$. Set $k(r) = \eta^2(r)$ so that $k(r) = 0$ for $r \geq 0$ and $k(r) = r^2$ for $r < 0$. For $\varepsilon > 0$, let $k_\varepsilon(r)$ be a C^2 -regularization of $k(r)$ defined by

$$k_\varepsilon(r) = \begin{cases} r^2 - \frac{\varepsilon^2}{6}, & r < -\varepsilon, \\ -\frac{r^3}{\varepsilon}(\frac{r}{2\varepsilon} + \frac{4}{3}), & -\varepsilon \leq r < 0, \\ 0, & r \geq 0. \end{cases} \quad (12)$$

It is easy to see that $k_\varepsilon(r)$ has the following properties.

Lemma 2.1. (see [8]). The first two derivatives $k'_\varepsilon, k''_\varepsilon$ of k_ε are continuous and satisfy the conditions: $k'_\varepsilon(r) = 0$ for $r \geq 0$; $k'_\varepsilon(r) \leq 0$ and $k''_\varepsilon(r) \geq 0$ for any $r \in \mathbb{R}$. Moreover, as $\varepsilon \rightarrow 0$, we have

$$k_\varepsilon(r) \rightarrow k(r), \quad k'_\varepsilon(r) \rightarrow -2\eta(r) \quad \text{and} \quad k''_\varepsilon(r) \rightarrow 2\theta(r), \quad (13)$$

where $\theta(r) = 0$ for $r \geq 0$, $\theta(r) = 1$ for $r < 0$, and the convergence is uniform for $r \in \mathbb{R}$.

3. Positive solutions. In this section, we will consider the existence of positive solution of Eq. (8).

Theorem 3.1. Suppose that the conditionss (A1)-(A4) hold. Then the solution of initial-boundary value problem (8) with nonnegative and continuous data remains positive so that $u(x, t) \geq 0$, a.s. for almost every $x \in D$ and for all $t \in [0, T]$.

Proof. Let $u_t = u(\cdot, t)$ and

$$\Phi_\varepsilon(u_t) = (1, k_\varepsilon(u_t)) = \int_D k_\varepsilon(u(x, t)) dx.$$

From Itô's formula, it follows that

$$\begin{aligned} \Phi_\varepsilon(u_t) &= \Phi_\varepsilon(g) + \int_0^t \int_D k'_\varepsilon(u(x, s)) Au(x, s) dx ds \\ &\quad + \int_0^t \int_D k'_\varepsilon(u(x, s)) f(u(x, s), x, s) dx ds \\ &\quad + \int_0^t \int_D k'_\varepsilon(u(x, s)) \sigma(u(x, s), \nabla u(x, s), x, s) dW(x, s) dx \\ &\quad + \frac{1}{2} \int_0^t \int_D k''_\varepsilon(u(x, s)) q(x, x) \sigma^2(u(x, s), \nabla u(x, s), x, s) dx ds \\ &\quad + \int_0^t \int_Z \int_D (k_\varepsilon(u(x, s) + \varphi(u, x, z, s)) - k_\varepsilon(u(x, s))) dx \tilde{N}(ds dz) \\ &\quad + \int_0^t \int_Z \int_D (k_\varepsilon(u(x, s) + \varphi(u, x, z, s)) - k_\varepsilon(u(x, s))) \\ &\quad \quad - \varphi(u, x, z, s) k'_\varepsilon(u(x, s))) dx \nu(dz) ds \\ &= \Phi_\varepsilon(g) + \int_0^t \int_D k''_\varepsilon(u(x, s)) \left(\frac{1}{2} q(x, x) \sigma^2(u(x, s), \nabla u(x, s), x, s) \right. \\ &\quad \quad \left. - b(x, \nabla u(x, s)) \right) dx ds \end{aligned}$$

$$\begin{aligned}
& + \int_0^t \int_D k'_\varepsilon(u(x, s)) f(u(x, s), x, s) dx ds \\
& + \int_0^t \int_D k'_\varepsilon(u(x, s)) \sigma(u(x, s), \nabla u(x, s), x, s) dW(x, s) dx \\
& + \int_0^t \int_Z \int_D (k_\varepsilon(u(x, s) + \varphi(u, x, z, s)) - k_\varepsilon(u(x, s))) dx \tilde{N}(ds dz) \\
& + \int_0^t \int_Z \int_D (k_\varepsilon(u(x, s) + \varphi(u, x, z, s)) \\
& - k_\varepsilon(u(x, s)) - \varphi(u, x, z, s) k'_\varepsilon(u(x, s))) dx \nu(dz) ds.
\end{aligned}$$

By taking expectations of both sides of the above equality, we have

$$\begin{aligned}
\mathbb{E}\Phi_\varepsilon(u_t) & = \Phi_\varepsilon(g) + \mathbb{E} \int_0^t \int_D k''_\varepsilon(u(x, s)) \left(\frac{1}{2} q(x, x) \sigma^2(u(x, s), \nabla u(x, s), x, s) \right. \\
& \quad \left. - b(x, \nabla u(x, s)) \right) dx ds \\
& + \mathbb{E} \int_0^t \int_D k'_\varepsilon(u(x, s)) f(u(x, s), x, s) dx ds \\
& + \mathbb{E} \int_0^t \int_Z \int_D (k_\varepsilon(u(x, s) + \varphi(u, x, z, s)) - k_\varepsilon(u(x, s)) \\
& \quad - \varphi(u, x, z, s) k'_\varepsilon(u(x, s))) dx \nu(dz) ds.
\end{aligned}$$

From (A1) and Lemma 2.1, it follows that

$$\begin{aligned}
\mathbb{E}\Phi_\varepsilon(u_t) & \leq \Phi_\varepsilon(g) + \mathbb{E} \int_0^t \int_D k''_\varepsilon(u(x, s)) (b_1 |u(x, s)|^m + b_2 |u(x, s)|^2) dx ds \\
& + \mathbb{E} \int_0^t \int_D k'_\varepsilon(u(x, s)) (a_1 u^\beta(x, s) + a_2 u(x, s)) dx ds \\
& + \mathbb{E} \int_0^t \int_Z \int_D (k_\varepsilon(u(x, s) + \varphi(u, x, z, s)) - k_\varepsilon(u(x, s)) \\
& \quad - \varphi(u, x, z, s) k'_\varepsilon(u(x, s))) dx \nu(dz) ds.
\end{aligned} \tag{14}$$

By Taylor's theorem, in view of the integral form of the remainder, we have

$$\begin{aligned}
& k_\varepsilon(u(x, s) + \varphi(u, x, z, s)) - k_\varepsilon(u(x, s)) - k'_\varepsilon(u(x, s)) \varphi(u, x, z, s) \\
& = \int_0^1 (1 - \tau) k''_\varepsilon(\varphi(u, x, z, s) \tau + u(x, s)) \varphi^2(u, x, z, s) d\tau
\end{aligned} \tag{15}$$

Substitute (15) into (14), we get

$$\begin{aligned}
\mathbb{E}\Phi_\varepsilon(u_t) & \leq \Phi_\varepsilon(g) + \mathbb{E} \int_0^t \int_D k''_\varepsilon(u(x, s)) (b_1 |u(x, s)|^m + b_2 |u(x, s)|^2) dx ds \\
& + \mathbb{E} \int_0^t \int_D k'_\varepsilon(u(x, s)) (a_1 u^\beta(x, s) + a_2 u(x, s)) dx ds \\
& + \mathbb{E} \int_0^t \int_Z \int_D \int_0^1 (1 - \tau) k''_\varepsilon(\varphi(u, x, z, s) \tau + u(x, s)) \varphi^2(u, x, z, s) d\tau \\
& \quad dx \nu(dz) ds.
\end{aligned} \tag{16}$$

Since $\lim_{\varepsilon \rightarrow 0} \mathbb{E}\Phi_\varepsilon(u_t) = \mathbb{E}\|\eta(u_t)\|^2$, taking the limits on both sides of (16) as $\varepsilon \rightarrow 0$, by (13) we obtain

$$\begin{aligned}
\mathbb{E}\|\eta(u_t)\|^2 &\leq \int_D |\eta(g(x))|^2 dx + 2\mathbb{E} \int_0^t \int_D \theta(u(x, s)) (b_1 |u(x, s)|^m + b_2 |u(x, s)|^2) dx ds \\
&\quad - 2\mathbb{E} \int_0^t \int_D \eta(u(x, s)) (a_1 u^\beta(x, s) + a_2 u(x, s)) dx ds \\
&\quad + 2\mathbb{E} \int_0^t \int_Z \int_D \int_0^1 (1 - \tau) \theta(\varphi(u, x, z, s) \tau \\
&\quad + u(x, s)) \varphi^2(u, x, z, s) d\tau dx \nu(dz) ds.
\end{aligned} \tag{17}$$

By the definition of η , it follows that $\eta(g) = 0$. This together with (A4), Lemma 2.1 and $(-1)^\beta a_1 = |a_1|$ yield

$$\begin{aligned}
\mathbb{E}\|\eta(u_t)\|^2 &\leq 2\mathbb{E} \int_0^t \int_D [b_1 (u^-)^m(x, s) + b_2 (u^-)^2(x, s)] dx ds \\
&\quad - 2\mathbb{E} \int_0^t \int_D [|a_1| (u^-)^{\beta+1}(x, s) - a_2 (u^-)^2(x, s)] dx ds \\
&\quad + \int_Z \psi(z) \nu(dz) \mathbb{E} \int_0^t \int_D (u^-)^\mu(x, s) dx ds.
\end{aligned} \tag{18}$$

It is known that the following L^p interpolation inequality and Young inequality hold (see [16])

$$\|u\|_{L^r} \leq \|u\|_{L^p}^\alpha \|u\|_{L^q}^{1-\alpha}, \tag{19}$$

$$ab \leq \varepsilon a^\delta + \varepsilon^{-\frac{\omega}{\delta}} b^\omega, \tag{20}$$

where $\alpha \in (0, 1)$, $\varepsilon > 0$, $\delta > 0$, $\omega > 0$, $a > 0$, $b > 0$,

$$\begin{aligned}
\frac{1}{r} &= \frac{\alpha}{p} + \frac{1-\alpha}{q}, \quad 1 \leq p \leq r \leq q \leq \infty, \\
\frac{1}{\delta} + \frac{1}{\omega} &= 1.
\end{aligned}$$

Since $2 < m < \beta + 1$, it follows that from (19) and (20)

$$\begin{aligned}
2b_1 \int_D (u^-)^m(x, t) dx &= 2b_1 \|u^-\|_{L^m}^m \\
&\leq C \|u^-\|_{L^2}^{m\alpha} \|u^-\|_{L^{\beta+1}}^{m(1-\alpha)} \\
&\leq \varepsilon \|u^-\|_{L^{\beta+1}}^{m(1-\alpha)\frac{2}{2-m\alpha}} + C(\varepsilon, m, \beta) \|u^-\|_{L^2}^2 \\
&= \varepsilon \|u^-\|_{L^{\beta+1}}^{\beta+1} + C(\varepsilon, m, \beta) \|u^-\|_{L^2}^2,
\end{aligned} \tag{21}$$

where $\alpha = \frac{2(\beta+1-m)}{m(\beta-1)}$.

Similarly, for $\mu \in [2, \beta + 1)$, we obtain

$$\begin{aligned}
\int_Z \psi(z) \nu(dz) \int_D (u^-)^\mu(x, s) dx ds &\leq C \|u^-\|_{L^\mu}^\mu \\
&\leq C \|u^-\|_{L^2}^{\mu\alpha'} \|u^-\|_{L^{\beta+1}}^{\mu(1-\alpha')} \\
&\leq \varepsilon \|u^-\|_{L^{\beta+1}}^{\mu(1-\alpha')\frac{2}{2-\mu\alpha'}} + C(\varepsilon, \mu, \beta) \|u^-\|_{L^2}^2 \\
&= \varepsilon \|u^-\|_{L^{\beta+1}}^{\beta+1} + C(\varepsilon, \mu, \beta) \|u^-\|_{L^2}^2,
\end{aligned} \tag{22}$$

where $\alpha' = \frac{2(\beta+1-\mu)}{\mu(\beta-1)}$.

Putting (21) and (22) into (18), we obtain

$$\begin{aligned} \mathbb{E}\|\eta(u_t)\|^2 &\leq \int_0^t (2\varepsilon - 2|a_1|)\mathbb{E}\|u_s^-\|_{L^{\beta+1}}^{\beta+1} ds + (2b_2 + 2a_2 \\ &\quad + C(\varepsilon, m, \beta) + C(\varepsilon, \mu, \beta)) \int_0^t \mathbb{E}\|u_s^-\|_{L^2}^2 ds \end{aligned}$$

Let $\varepsilon \in (0, |a_1|)$. Then

$$\mathbb{E}\|\eta(u_t)\|^2 \leq C \int_0^t \mathbb{E}\|\eta(u_s)\|^2 ds,$$

From Gronwall's inequality, it follows that $\mathbb{E}\|\eta(u_t)\|^2 = 0$. This implies that $\eta(u_t) = u^-(x, t) = 0$ a.s. for a.e. $x \in D$ and $t \in [0, T]$. The proof is complete. \square

Remark 1. The assumption (A1) is weaker than the assumption (H1) in [2]. For example, if we consider the Allen-Cahn type equation, $f(u) = u - u^3$ doesn't satisfy (H1), but f satisfies (A1).

Remark 2. Since $A = \sum_{i,j=1}^d \frac{\partial}{\partial x_i} [a_{ij}(x) \frac{\partial}{\partial x_j}]$ is more general than the Laplacian operator Δ , Theorem 3.1 is the generalization of Theorem 3.1 in [19].

Remark 3. If it is assumed that $\beta \in (0, 1)$, for the case $1 + \beta \leq m < 2$ and $1 + \beta \leq q < 2$, by the L^p interpolation inequality and Young inequality, we can get the corresponding results.

4. Explosive solutions. In this section, we consider the unbounded solutions of the equation (8).

For the elliptic equation:

$$\begin{cases} A\vartheta = -\lambda\vartheta, & \text{in } D, \\ \vartheta = 0, & \text{on } \partial D, \end{cases} \quad (23)$$

it is well known that all the eigenvalues of $-A$ are strictly positive, increasing and the eigenfunction ϕ corresponding to the smallest eigenvalue λ_1 does not change sign in the domain D (see p. 355, [12]). We can normalize it in such a way that

$$\phi(x) \geq 0, \quad \int_D \phi(x) dx = 1. \quad (24)$$

Theorem 4.1. Suppose the initial-boundary value problem (8) has a unique local solution and the conditions (A1)-(A4) hold. In addition, we assume that $\lambda_1 > a_2$, $a_1 > 0$, and

$$\int_D g(x)\phi(x)dx > \left(\frac{\lambda_1 - a_2}{a_1}\right)^{\frac{1}{\beta-1}},$$

and if $\lambda_1 < a_2$, we assume that $\int_D g(x)\phi(x)dx > 0$, where λ_1 is the smallest eigenvalue of $-A$ and ϕ is the corresponding eigenfunction. Then, for any $p \geq 1$, there exists a constant $T_p > 0$ such that

$$\lim_{t \rightarrow T_p^-} \mathbb{E}\|u_t\|_{L^p} = \lim_{t \rightarrow T_p^-} \mathbb{E}\left\{\int_D |u(x, t)|^p dx\right\}^{1/p} = \infty. \quad (25)$$

That is, the solution explodes in mean L^p -norm sense.

Proof. By Theorem 3.1, Eq. (8) has a unique positive solution. We will prove the theorem by contradiction. We suppose (25) is false. Then there exists a global positive solution u such that

$$\sup_{0 \leq t \leq T} \mathbb{E} \left\{ \int_D |u(x, t)|^p dx \right\}^{1/p} < \infty,$$

for any $T > 0$. Let ϕ be the the eigenfunction defined in (23). Define

$$\hat{u}(t) := \int_D u(x, t) \phi(x) dx \geq 0. \quad (26)$$

By (24), ϕ can be regarded as the probability density function of a random variable ξ in D , independent of W_t . The equality (26) can be interpreted as an expectation $\hat{u}(t) = \mathbb{E}_\xi \{u(\xi, t)\}$. From (8), (26) and the self-adjointness of A , it follows that

$$\begin{aligned} \hat{u}(t) &= (g, \phi) + \int_0^t \int_D [Au(x, s)] \phi(x) dx ds + \int_0^t \int_D f(u, x, s) \phi(x) dx ds \\ &\quad + \int_0^t \int_D \sigma(u, \nabla u, x, s) \phi(x) dx dW(x, s) \\ &\quad + \int_0^t \int_Z \int_D \varphi(u, x, z, s) \phi(x) dx \tilde{N}(ds, dz) \\ &= (g, \phi) - \lambda_1 \int_0^t \int_D u(x, s) \phi(x) dx ds + \int_0^t \int_D f(u, x, s) \phi(x) dx ds \\ &\quad + \int_0^t \int_D \sigma(u, \nabla u, x, s) \phi(x) dx dW(x, s) \\ &\quad + \int_0^t \int_Z \int_D \varphi(u, x, z, s) \phi(x) dx \tilde{N}(ds, dz). \end{aligned} \quad (27)$$

Taking the expectation to both sides of (27) and by Fubini's theorem, we have

$$\mathbb{E} \hat{u}(t) = (g, \phi) - \lambda_1 \int_0^t \mathbb{E} \hat{u}(s) ds + \int_0^t \mathbb{E} \int_D f(u, x, s) \phi(x) dx ds,$$

or, in the differential form,

$$\begin{cases} \frac{d\xi(t)}{dt} = -\lambda_1 \xi(t) + \mathbb{E} \int_D f(u, x, s) dx \\ \xi(0) = \xi_0, \end{cases} \quad (28)$$

where $\xi(t) = \mathbb{E} \hat{u}(t)$, $\xi_0 = (g, \phi)$. By (A1) and Jensen's inequality, we obtain

$$\begin{cases} \frac{d\xi(t)}{dt} \geq -\lambda_1 \xi(t) + a_1 \xi^\beta(t) + a_2 \xi(t), \\ \xi(0) = \xi_0, \end{cases} \quad (29)$$

If $\lambda_1 \geq a_2$, for $\xi_0 > \left(\frac{\lambda_1 - a_2}{a_1}\right)^{\frac{1}{\beta-1}}$, we can show that $\xi(\cdot)$ is strictly increasing. It follows from (29) that

$$T \leq \int_{\xi_0}^{\xi(T)} \frac{ds}{a_1 s^\beta - (\lambda_1 - a_2)s} \leq \int_{\left(\frac{\lambda_1 - a_2}{a_1}\right)^{\frac{1}{\beta-1}}}^{\infty} \frac{ds}{a_1 s^\beta - (\lambda_1 - a_2)s} < \infty. \quad (30)$$

If $\lambda_1 < a_2$, for $\xi_0 > 0$, we can show that $\xi(\cdot)$ is strictly increasing. We have

$$T = \int_0^T dt \leq \int_0^T \frac{d\xi(t)}{a_1 \xi^\beta(t)} = \int_{\xi_0}^{\xi(T)} \frac{ds}{a_1 s^\beta} < \infty. \quad (31)$$

Since T is arbitrary, either (30) or (31) results in a contradiction. Therefore, for $\lambda_1 \geq a_2$, $\xi_0 > \left(\frac{\lambda_1 - a_2}{a_1}\right)^{\frac{1}{\beta-1}}$, $\xi(t)$ must blow up at a time $T_p \leq \int_{\xi_0}^{\xi(T)} \frac{ds}{a_1 s^\beta - (\lambda_1 - a_2)s}$. For $\lambda_1 < a_2$, $\xi(t)$ must blow up at a time $T_p \leq \int_{\xi_0}^{\xi(T)} \frac{ds}{a_1 s^\beta}$.

Since ϕ is bounded and continuous on \bar{D} , by Hölder's inequality, we have

$$\xi(t) \leq \left(\int_D |\phi(x)|^q dx \right)^{1/q} \left(\mathbb{E} \int_D |u(x, t)|^p dx \right)^{1/p}, \quad (32)$$

where $q = p/(p-1)$, $p \geq 1$. So the positive solution explodes at some time $T' \leq T_e$ in the mean L^p -norm for each $p \geq 1$. The proof is complete. \square

Example 4.1. Consider the following problem in a spherical domain $D = B(R)$ in \mathbb{R}^3 :

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta u + u^4 - u + \gamma_0(u^3 + |\nabla u|^2)^{1/2} \partial_t W(x, t) \\ \quad + c_0 \int_0^\infty z u^3 \partial_t \tilde{N}(t, dz), \quad t > 0, x \in D, \\ u(x, 0) = a_0 e^{-\alpha|x|}, \quad x \in D, \\ u(x, t)|_{|x|=R} = 0, \quad t > 0, \end{cases} \quad (33)$$

where $\tilde{N}(dt, dz) = N(dt, dz) - dt\nu(dz)$ is a compensated Poisson measure corresponding to the Poisson random measure $N(dt, dz)$, $W(x, t)$ is a continuous Wiener random field with the covariance function

$$q(x, y) = b_0 \exp\{-\rho(x \cdot y)\}, \quad x, y \in \mathbb{R}^3.$$

The constants $\sigma_0, c_0, a_0, \alpha$ are strictly positive and $x \cdot y = \sum_{i=1}^3 x_i y_i$. Here $A = \Delta$, $f = u^4 - u$, $\sigma = \gamma_0(u^3 + |\nabla u|^2)^{1/2}$, $\varphi = c_0 z u^3$, $Z = (0, \infty)$. It is obvious that conditions (A1) and (A4) are satisfied. If $\frac{1}{2}b_0\gamma_0^2 < 1$, since

$$\frac{1}{2}b_0\gamma_0^2 \exp\{-\rho|x|^2\}(u^3 + |\xi|^2) - |\xi|^2 \leq \left(\frac{1}{2}b_0\gamma_0^2 - 1\right)|\xi|^2 + \frac{1}{2}b^2\gamma_0^2 u^3.$$

then condition (A2) is satisfied. If $\int_0^\infty z^2 \nu(dz) < \infty$, take $\mu = 6$ and $\psi(z) = c_0^2 z^2$, then the condition (A3) is satisfied. From Theorem 3.1, it follows that the solution of Eq. (33) is positive. The smallest eigenvalue of the elliptic equation (23) is $\lambda_1 = \left(\frac{\pi}{R}\right)^2$ and the corresponding normalized eigenfunction is $\phi(x) = \frac{1}{4R^2|x|} \sin \frac{\pi|x|}{R}$, $0 < |x| < R$. If a_0 is sufficiently large, then we have

$$\int_D g(x)\phi(x)dx = \int_0^R \frac{a_0 e^{-\alpha r}}{4R^2 r} \sin \frac{\pi r}{R} dr > \frac{a_0}{4R^3} \int_0^R e^{-\alpha r} \sin \frac{\pi r}{R} dr > \left(\frac{\pi^2}{R^2} + 1\right)^{\frac{3}{5}}, \quad (34)$$

Therefore, by Theorem 4.1, the solutions to the Eq. (33) will blow up in finite time in mean L^p -norm for any $p \geq 1$. Note that Theorem 3.1 in [2] is not suitable for Eq. (33).

To discuss the noise-induced explosion, we consider the following stochastic reaction-diffusion equation:

$$\begin{cases} \frac{\partial u}{\partial t} = Au + f(u, x, t) + \sigma(u, x, t)\partial_t W(x, t) \\ \quad + \int_Z \varphi(u, x, z, t)\partial_t \tilde{N}(t, dz), \quad t > 0, x \in D \\ u(x, 0) = g(x), \quad x \in D, \\ u(x, t) = 0, \quad t > 0, x \in \partial D, \end{cases} \quad (35)$$

which is a special case of Eq. (8), where σ is independent of ∇u . We assume that the noise terms satisfy the following conditions:

(A1') The correlation function $q(x, y)$ is continuous and positive for $x, y \in \overline{D}$ such that

$$\int_D \int_D q(x, y)v(x)v(y)dx dy \geq \kappa \left(\int_D v(x)dx \right)^2$$

for any non-negative $v \in H$ and some constant $\kappa > 0$.

(A2') The function $f(u, x, t)$ is continuous on $\mathbb{R} \times \overline{D} \times [0, \infty)$ such that $f(u, x, t) \geq 0$ for $u \geq 0$ and $x \in \overline{D}$, $t \in [0, \infty)$.

(A3') There exist continuous functions σ_0, G such that they are both positive, convex and satisfy

$$\sigma(u, x, t) \geq \sigma_0(u), \quad \sigma_0^2(u) \geq G(u^2),$$

for $x \in \overline{D}$, $u \geq 0$, $t \in [0, \infty)$.

(A4') There exist continuous functions φ_0, K such that they are both positive, convex and satisfy

$$\begin{aligned} \int_Z \left(\int_D \varphi(u, x, z, t)\phi(x)dx \right)^2 \nu(dz) &\geq \int_Z \left(\int_D \varphi_0(u, z)\phi(x)dx \right)^2 \nu(dz), \\ \int_Z \varphi_0^2(u, z)\nu(dz) &\geq K(u^2), \end{aligned}$$

for $x \in \overline{D}$, $z \in Z$, $u \geq 0$, $t \in [0, \infty)$.

(A5') There exists a constant $M > 0$ such that $\kappa G(u) + K(u) > 2\lambda_1 u$ for $u > M$, and

$$\int_M^\infty \frac{du}{\kappa G(u) + K(u) - 2\lambda_1 u} < \infty.$$

(A6') The initial datum satisfies the following

$$(g, \phi) = \int_D g(x)\phi(x)dx > M.$$

Theorem 4.2. Assume that the initial-boundary value problem (8) has a unique positive local solution and the conditions (A1') – (A6') hold. Then for each $p \geq 2$, there exists a constant T_p such that

$$\lim_{t \rightarrow T_p^-} \mathbb{E} \|u_t\|_{L^p} = \lim_{t \rightarrow T_p^-} \mathbb{E} \left\{ \int_D |u(x, t)|^p dx \right\}^{1/p} = \infty. \quad (36)$$

That is, the solution explodes in mean L^p -norm sense.

Proof. We assume the conclusion is false. Then there exists the solution u and for some $p \geq 2$, $\mathbb{E} \|u_t\|^p < \infty$, $t \in [0, T]$, for any $T > 0$. Let $\hat{u}(t) = (\phi, u_t)$ be defined as

before. By (35),

$$\begin{aligned}
\hat{u}(t) &= (g, \phi) + \int_0^t \int_D [Au(x, s)] \phi(x) dx ds + \int_0^t \int_D f(u, x, s) \phi(x) dx ds \\
&\quad + \int_0^t \int_D \sigma(u, x, s) \phi(x) dW(x, s) dx \\
&\quad + \int_0^t \int_Z \int_D \varphi(u, x, z, s) \phi(x) dx \tilde{N}(ds, dz) \\
&= (g, \phi) - \lambda_1 \int_0^t \int_D u(x, s) \phi(x) dx ds + \int_0^t \int_D f(u, x, s) \phi(x) dx ds \\
&\quad + \int_0^t \int_D \sigma(u, x, s) \phi(x) dW(x, s) dx \\
&\quad + \int_0^t \int_Z \int_D \varphi(u, x, z, s) \phi(x) dx \tilde{N}(ds, dz). \tag{37}
\end{aligned}$$

By (37), we apply the Itô's formula to $\hat{u}^2(t)$ to get

$$\begin{aligned}
\hat{u}^2(t) &= (g, \phi)^2 - 2\lambda_1 \int_0^t \hat{u}^2(s) ds + 2 \int_0^t \int_D \hat{u}(s) f(u, x, s) \phi(x) dx ds \\
&\quad + 2 \int_0^t \int_D \hat{u}(s) \sigma(u, x, s) \phi(x) dx dW(x, s) \\
&\quad + \int_0^t \int_D \int_D q(x, y) \phi(x) \phi(y) \sigma(u, x, s) \sigma(u, y, s) dx dy ds \\
&\quad + \int_0^t \int_Z \left[\left(\hat{u}(s) + \int_D \varphi(u, x, z, s) \phi(x) dx \right)^2 - \hat{u}^2(s) \right] \tilde{N}(ds, dz) \\
&\quad + \int_0^t \int_Z \left[\left(\hat{u}(s) + \int_D \varphi(u, x, z, s) \phi(x) dx \right)^2 - \hat{u}^2(s) \right. \\
&\quad \left. - 2\hat{u}(s) \int_D \varphi(u, x, z, s) \phi(x) dx \right] \nu(dz) ds. \tag{38}
\end{aligned}$$

Let $\eta(t) = \mathbb{E}\hat{u}^2(t)$. Taking expectations of both sides of (38), we obtain

$$\begin{aligned}
\eta(t) &= (g, \phi)^2 - 2\lambda_1 \int_0^t \eta(s) ds + 2\mathbb{E} \int_0^t \int_D \hat{u}(s) f(u, x, s) \phi(x) dx ds \\
&\quad + \mathbb{E} \int_0^t \int_D \int_D q(x, y) \phi(x) \phi(y) \sigma(u, x, s) \sigma(u, y, s) dx dy ds \\
&\quad + \mathbb{E} \int_0^t \int_Z \left(\int_D \varphi(u, x, z, s) \phi(x) dx \right)^2 \nu(dz) ds,
\end{aligned}$$

or, in the differential form,

$$\left\{ \begin{aligned} \frac{d\eta(t)}{dt} &= -2\lambda_1 \eta(t) + 2\mathbb{E} \hat{u}(t) \int_D f(u, x, t) \phi(x) dx \\ &\quad + \mathbb{E} \int_D \int_D q(x, y) \phi(x) \phi(y) \sigma(u, x, t) \sigma(u, y, t) dx dy \\ &\quad + \mathbb{E} \int_Z \left(\int_D \varphi(u, x, z, t) \phi(x) dx \right)^2 \nu(dz), \\ \eta(0) &= \eta_0 = (g, \phi)^2. \end{aligned} \right. \tag{39}$$

By (A2'), we have $\mathbb{E}\hat{u}(t) \int_D f(u, x, t) \phi(x) dx \geq 0$. By (A1'), (A3'), Jensen's inequality, we have

$$\begin{aligned} & \int_D \int_D q(x, y) \phi(x) \phi(y) \sigma(u, x, t) \sigma(u, y, t) dx dy \\ & \geq \kappa \left(\int_D \phi(x) \sigma(u, x, t) dx \right)^2 \\ & \geq \kappa \left(\int_D \phi(x) \sigma_0(u) dx \right)^2 \\ & \geq \kappa \sigma_0^2(\hat{u}(t)) \\ & \geq \kappa G(\hat{u}^2(t)). \end{aligned} \quad (40)$$

By (A4'), Jensen's inequality, we get

$$\begin{aligned} & \int_Z \left(\int_D \varphi(u, x, z, t) \phi(x) dx \right)^2 \nu(dz) \\ & \geq \int_Z \left(\int_D \varphi_0(u, z) \phi(x) dx \right)^2 \nu(dz) \\ & \geq \int_Z \varphi_0^2(\hat{u}(t), z) \nu(dz) \\ & \geq K(\hat{u}^2(t)). \end{aligned} \quad (41)$$

From (40), (41), (39) and Jensen's inequality, it follows that

$$\begin{aligned} \frac{d\eta(t)}{dt} & \geq -2\lambda_1 \eta(t) + \kappa \mathbb{E}G(\hat{u}^2(t)) + \mathbb{E}K(\hat{u}^2(t)) \\ & \geq -2\lambda_1 \eta(t) + \kappa G(\eta(t)) + K(\eta(t)). \end{aligned} \quad (42)$$

Similar to (30), for $\eta_0 > M$, we obtain

$$T \leq \int_{\eta_0}^{\eta(T)} \frac{du}{\kappa G(u) + K(u) - 2\lambda_1 u} \leq \int_M^\infty \frac{du}{\kappa G(u) + K(u) - 2\lambda_1 u} < \infty.$$

Since T is arbitrary, this results in a contradiction. Therefore, the mean square $\eta(t) = \mathbb{E}\hat{u}^2(t)$ must blow up at some finite time $T_* > 0$. Applying the Hölder inequality, we see that (36) holds for each $p \geq 2$. \square

Remark 4. In [5], [6], the correlation function $q(x, y)$ is assumed to satisfy the inequality

$$\int_D \int_D q(x, y) v(x) v(y) dx dy \geq q_1 \int_D v^2(x) dx \quad (43)$$

for any positive $v \in H$ and for some $q_1 > 0$.

In fact, this assumption is not suitable. If the domain D is bounded and $q \in (0, 1]$, by the Cauchy-Schwarz inequality, we have

$$\int_D \int_D q(x, y) v(x) v(y) dx dy \leq \left(\int_D v(x) dx \right)^2 \leq \mu(D) \int_D v^2(x) dx, \quad (44)$$

where $\mu(D)$ is the volume of D . By (43), (44), we have $\mu(D) \geq q_1$. If the bounded domain D is small enough, then we get a contradiction.

Remark 5. We consider the problem (35) with a Levy-type noise, and the coefficient operator A is more general than the Laplacian, it is easy to see that Theorem 4.2 is the generalization of Theorem 4.3 in [19].

Example 4.2. Consider the following problem in a spherical domain $D = B(R)$ in \mathbb{R}^3 :

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta u + u^{1+\alpha} + \mu u^4 \partial_t W(x, t) + c_0 \int_0^\infty zu^6 \partial_t \tilde{N}(t, dz), & t > 0, x \in D, \\ u(x, 0) = a_0 e^{-\beta|x|}, & x \in D, \\ u(x, t)|_{|x|=R} = 0, & t > 0, \end{cases} \quad (45)$$

where $\tilde{N}(dt, dz) = N(dt, dz) - dt\nu(dz)$ is a compensated Poisson measure corresponding to the Poisson random measure $N(dt, dz)$, $W(x, t)$ is a continuous Wiener random field with the covariance function

$$q(x, y) = b_0 \exp\{-\rho(x \cdot y)\}, \quad x, y \in \mathbb{R}^3.$$

The constants $\mu, c_0, a_0, \alpha, \beta$ are strictly positive and $x \cdot y = \sum_{i=1}^3 x_i y_i$. Here $A = \Delta, f = u^{1+\alpha}, \sigma = \mu u^4, \varphi = c_0 z u^6, Z = (0, \infty)$.

For $x, y \in B(R)$, we have

$$q(x, y) \geq \kappa = b_0 \exp\{-\rho R^2\}.$$

Then for any non-negative $v \in H$,

$$\int_D \int_D q(x, y) v(x) v(y) dx dy \geq \kappa \left(\int_D v(x) dx \right)^2.$$

The condition (A1') holds. It is obvious that $f = u^{1+\alpha} > 0$ for $u > 0$, the condition (A2') holds. Let $G(u) = \mu^2 u^4, \sigma_0(u) = \mu u^4$. Then $\sigma(u, x, t) = \mu u^4 = \sigma_0(u)$ and $\sigma_0^2(u) = G(u^2)$. σ_0 and G are both continuous, positive and convex. The condition (A3') is satisfied. Let $\varphi_0(u, z) = c_0 z u^6$. Assume $\int_0^\infty z^2 \nu(dz) < \infty$, let $K(u) = (c_0 \int_0^\infty z^2 \nu(dz)) u^6$. Then $\varphi(u, x, z, t) = c_0 z u^6 = \varphi_0(u, z)$. φ_0 and K are both positive and convex. The condition (A4') holds. The smallest eigenvalue of the elliptic equation (23) is $\lambda_1 = (\frac{\pi}{R})^2$. If b_0 or c_0 is large enough, it is easy to see that $b_0 \exp\{-\rho R^2\} \mu^2 u^4 + (c_0 \int_0^\infty z^2 \nu(dz)) u^6 > 2(\frac{\pi}{R})^2 u$ for $u > M$, and

$$\int_M^\infty \frac{du}{b_0 \exp\{-\rho R^2\} \mu^2 u^4 + (c_0 \int_0^\infty z^2 \nu(dz)) u^6 - 2(\frac{\pi}{R})^2 u} < \infty.$$

The condition (A5') is satisfied. If a_0 is sufficiently large, similar to (34), we have $\int_D g(x) \phi(x) dx > M$. By Theorem 4.2, the solution of (45) will blow up in finite time in L^p -norm for any $p \geq 2$.

Now we consider the explosive solution problem for (8), when the domain D is unbounded, for example, $D = \mathbb{R}^d$. Let $B(R) = \{x \in \mathbb{R}^d : |x| < R\}$.

Theorem 4.3. Assume that the initial-boundary value problem (8) has a unique local solution and the conditions (A1)-(A3) hold, where $D = \mathbb{R}^d$. Then for any $R > 0$, there exists a constant $T_p(R) > 0$ such that

$$\lim_{t \rightarrow T_p(R)-} \mathbb{E} \left\{ \int_{B(R)} |u(t, x)|^p \right\}^{1/p} = \infty,$$

provided that the conditions of Theorem 4.1 holds for $p \geq 1$ or the conditions of Theorem 4.2 holds for $p \geq 2$, where $D = \mathbb{R}^d$.

Proof. The proof follows the spirit of the one for Theorem 3.3 in [6]. For the sake of completeness, we present it. We only consider the case under the conditions of Theorem 4.1, since the proof under the conditions of Theorem 4.2 is similar.

By restricting the solution u to $\overline{B(R)}$, let $\hat{u}(t) = \int_{B(R)} u(x, t) \phi(x) dx \geq 0$ as defined by (26). Since $u \geq 0$ on the boundary $\partial B(R)$, by Green's identity,

$$(Au_t, \phi) = -\lambda_1 \phi + \int_{\partial B(R)} u(x, t) \left(-\frac{\partial \phi(x)}{\partial \nu} \right) dS. \quad (46)$$

Denote $n = (n_1, n_2, \dots, n_d)$ as the unit outward normal vector to the boundary $\partial B(R)$. Since there exists a constant $c > 0$ such that $\sum_{i,j=1}^d a_{ij}(x) \xi_i \xi_j \geq c|\xi|^2$ for all $x \in \overline{D}$ and $\xi = (\xi_1, \dots, \xi_d) \in \mathbb{R}^d$. We have $\nu \cdot n = \sum_{i,j}^d a_{ij} n_i n_j \geq 0$. This implies that the conormal $\nu(x)$ is an exterior direction field. Since $\phi > 0$ in $B(R)$ and $\phi = 0$ on $\partial B(R)$, we have

$$\frac{\partial \phi(x)}{\partial \nu} \leq 0. \quad (47)$$

Putting (46) into (27), by (47), we obtain the inequality (29). The rest of proof can be completed as in Theorem 4.1. \square

Remark 6. In [2], when the domain $D = \mathbb{R}^d$, it seems impossible to consider the existence of the position solution of initial-boundary value problem (8) and the explosion of the position solution. The reason is that the proof of Theorem 2.1 in [2] relies on the fact that the volume $V(D)$ of domain D is bounded. In [2], the proofs of Theorem 3.1 and Theorem 4.1 are both rely on Theorem 2.1. So for $D = \mathbb{R}^d$, they are not valid.

5. Global solutions for a stochastic Allen-Cahn equation driven by a Lévy type noise. In this section, we consider the following stochastic Allen-Cahn equation driven by a Lévy type noise,

$$\begin{cases} du = (\Delta u + u(1 - u^2))dt + bu^m dW_t + cu^n \int_Z z \tilde{N}(dt, dz), & t > 0, x \in D, \\ u(x, 0) = h(x), & x \in D, \\ u(x, t) = 0, & t > 0, x \in \partial D, \end{cases} \quad (48)$$

where $1 < m < 2$, $1 < n < 2$, $b, c \in \mathbb{R}$, $D \subset \mathbb{R}^3$, $Z = (0, \infty)$.

Let V be a real separable Hilbert space. We first consider the more general equation

$$\begin{cases} du_t = (Au_t + F_t(u_t)dt + \Sigma_t(u_t)dW_t + \int_Z \Gamma_t(u_t, z)\tilde{N}(dt, dz), & t \geq 0, \\ u_0 = h(x), \end{cases} \quad (49)$$

where the coefficients A , F_t , Σ_t and Γ_t are assumed to be non-random or deterministic. $W(x, t)$ is a Wiener random field, $(Z, \mathcal{B}(Z))$ is a measurable space. $\tilde{N}(dt, dz)$ is the compensated Poisson measure. Here we say that an \mathcal{F}_t -adapted V -valued process u_t is a *strong solution*, or a *variational solution*, of the equation (49) if $u \in L^2([0, T]; V)$, and for any $\varphi \in V$, the following equation

$$\begin{aligned} (u_t, \varphi) &= (h, \varphi) + \int_0^t (Au_s, \varphi) ds + \int_0^t (F_s(u_s), \varphi) ds + \int_0^t (\varphi, \Sigma_s(u_s) dW_s) \\ &\quad + \int_0^t \left(\int_Z \Gamma_s(u_s, z) \tilde{N}(ds, dz), \varphi \right) \end{aligned}$$

holds for each $t \in [0, T]$ a.s.

Denote $L_1(V)$ the space of nuclear (trace class) operators on V . Let $U \subset V$ be an open set and let $U \times [0, T] = U_T$. Here a functional $\Phi : U_T \rightarrow \mathbb{R}$ is said to be a

strong Itô functional if it satisfies the following (see [7, pp. 226]):

- (1) $\Phi : U_T \rightarrow \mathbb{R}$ is locally bounded and continuous such that its first two partial derivatives $\partial_t \Phi(v, t)$, $\Phi'(v, t)$ and $\Phi''(v, t)$ exist for each $(v, t) \in U_T$.
- (2) The derivatives $\partial_t \Phi$ and $\Phi' \in V$ are locally bounded and continuous in U_T .
- (3) For any $\Gamma \in \mathcal{L}_1(V)$, the map: $(v, t) \rightarrow \text{Tr}[\Phi''(v, t)\Gamma]$ is locally bounded and continuous in $(v, t) \in U_T$.
- (4) $\Phi'(\cdot, t) : U \rightarrow V$ is such that $(\Phi'(\cdot, t), v)$ is continuous in $t \in [0, T]$ for any $v \in V$ and

$$\|\Phi'(v, t)\| \leq \kappa(1 + \|v\|), \quad (v, t) \in U \times [0, T],$$

for some $\kappa > 0$.

In the following, we will present the definition of *Lyapunov functional*. Let $U \subset V$ be a neighborhood of the origin. Define the operator \mathcal{L}_t as follows:

$$\begin{aligned} \mathcal{L}_t \Phi(v, t) &= \frac{\partial}{\partial t} \Phi(v, t) + \frac{1}{2} \text{Tr}[\Phi''(v, t) \Sigma_t(v) Q \Sigma_t^*(v)] + (Av, \Phi'(v, t)) \\ &\quad + (F_t(v), \Phi'(v, t)) + \int_Z [\Phi(v + \Gamma_t(v, z), t) - \Phi(v, t) \\ &\quad - (\Gamma_t(v, z), \Phi'(v, t))] \nu(dz), \end{aligned}$$

where Q is the covariance operator.

A strong Itô functional $\Phi : U \times \mathbb{R}^+ \rightarrow \mathbb{R}$ is said to be a *Lyapunov functional* for the equation (49), if

- (1) $\Phi(0, t) = 0$ for all $t \geq 0$, and, for any $\varepsilon > 0$, there is a $\delta > 0$ such that

$$\inf_{t \geq 0, \|h\| \geq \varepsilon} \Phi(h, t) \geq \delta, \quad \text{and}$$

- (2) for any $t \geq 0$ and $v \in U$,

$$\mathcal{L}_t \Phi(v, t) \leq 0.$$

Let u_t^h be a strong solution of the equation (48) with $u_0^h = h$.

Definition 5.1. The solution u_t^h is said to be nonexplosive if

$$\lim_{r \rightarrow \infty} P\left\{ \sup_{0 \leq t \leq T} \|u_t^h\| \geq r \right\} = 0,$$

for any $T > 0$. If the above holds for $T = \infty$, the solution is said to be ultimately bounded.

Lemma 5.2. Let $\Phi : U \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ be a Lyapunov functional and let u_t^h denote the strong solution of (49). For $r > 0$, let $B_r = \{h \in V : \|h\| < r\}$ such that $B_r \subset U$. Define

$$\tau = \inf\{t > 0 : u_t^h \in B_r^c, h \in B_r\},$$

with $B_r^c = V \setminus B_r$. We put $\tau = T$ if the set is empty. Then the process $\phi_t = \Phi(u_{t \wedge \tau}^h, t \wedge \tau)$ is a local \mathcal{F}_t -supermartingale and the following Chebyshev inequality holds

$$P\left\{ \sup_{0 \leq t \leq T} \|u_t^h\| \geq r \right\} \leq \frac{\Phi(h, 0)}{\Phi_r},$$

where

$$\Phi_r = \inf_{0 \leq t \leq T, h \in U \cap B_r^c} \Phi(h, t).$$

Proof. From Itô's formula, it follows that

$$\begin{aligned}\Phi(u_{t \wedge \tau}^h) &= \Phi(h, 0) + \int_0^{t \wedge \tau} \mathcal{L}_s \Phi(u_s^h, s) ds + \int_0^{t \wedge \tau} (\Phi'(u_s^h, s), \Sigma_s(u_s^h)) dW_s \\ &\quad + \int_0^{t \wedge \tau} \int_Z (\Phi(v + \Gamma_s(v, z), s) - \Phi(v, s)) \tilde{N}(ds, dz) \\ &\leq \Phi(h, 0) + \int_0^{t \wedge \tau} (\Phi'(u_s^h, s), \Sigma_s(u_s^h)) dW_s \\ &\quad + \int_0^{t \wedge \tau} \int_Z (\Phi(v + \Gamma_s(v, z), s) - \Phi(v, s)) \tilde{N}(ds, dz),\end{aligned}$$

therefore, $\phi_t = \Phi(u_{t \wedge \tau}^h, t \wedge \tau)$ is a local supermartingale and

$$\mathbb{E}\phi_t \leq \mathbb{E}\phi_0 = \Phi(h, 0).$$

By definition,

$$\begin{aligned}\mathbb{E}\phi_T &= \mathbb{E}\Phi(u_{T \wedge \tau}^h, T \wedge \tau) \\ &\geq \mathbb{E}\{\Phi(u_\tau^h; \tau \leq T)\} \\ &\geq \inf_{0 \leq t \leq T, \|h\|=r} \Phi(h, t) P\{\tau \leq T\} \\ &\geq \Phi_r P\left\{\sup_{0 \leq t \leq T} \|u_t^h\| \geq r\right\},\end{aligned}$$

the proof is complete. \square

Theorem 5.3. *If there exists an Itô functional $\Psi : V \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ and a constant $\alpha > 0$ such that*

$$\mathcal{L}_t \Psi \leq \alpha \Psi(v, t) \quad \text{for any } v \in V,$$

and the infimum $\inf_{t \geq 0, \|h\| \geq r} \Psi(h, t) = \Psi_r$ exists such that $\lim_{r \rightarrow \infty} \Psi_r = \infty$, then the solution u_t^h does not explode in finite time.

Proof. Let $\Phi(v, t) = e^{-\alpha t} \Psi(v, t)$. We have

$$\begin{aligned}\mathcal{L}_t \Phi(v, t) &= \frac{\partial}{\partial t} \Phi(v, t) + \frac{1}{2} \text{Tr}[\Phi''(v, t) \Sigma_t(v) Q \Sigma_t^*(v)] + (Av, \Phi'(v, t)) \\ &\quad + (F_t(v), \Phi'(v, t)) + \int_Z [\Phi(v + \Gamma_t(v, z), t) - \Phi(v, t) \\ &\quad - \Phi'(v, t) \Gamma_t(v, z)] \nu(dz) \\ &= -\alpha e^{-\alpha t} \Psi(v, t) + e^{-\alpha t} \frac{\partial}{\partial t} \Psi(v, t) + e^{-\alpha t} \left(\frac{1}{2} \text{Tr}[\Psi''(v, t) \Sigma_t(v) Q \Sigma_t^*(v)] \right. \\ &\quad \left. + (Av, \Psi'(v, t)) \right. \\ &\quad \left. + (F_t(v), \Psi'(v, t)) + \int_Z [\Psi(v + \Gamma_t(v, z), t) - \Psi(v, t) \right. \\ &\quad \left. - \Psi'(v, t) \Gamma_t(v, z)] \nu(dz) \right) \\ &= -\alpha e^{-\alpha t} \Psi(v, t) + e^{-\alpha t} \mathcal{L}_t \Psi \leq 0.\end{aligned}$$

Therefore Φ is a Lyapunov functional. By Lemma 5.2, we have

$$P\left\{\sup_{0 \leq t \leq T} \|u_t^h\| > r\right\} \leq \frac{\Phi(h, 0)}{\Phi_r} = \frac{\Psi(h, 0)}{\Psi_r} \rightarrow 0$$

as $r \rightarrow \infty$, for any $T > 0$. \square

Theorem 5.4. *Let $1 < m < 2$, $1 < n < 2$ and the initial datum $h(x)$ on \overline{D} is positive and continuous. Suppose that $\int_0^\infty z^2 \nu(dz) < \infty$ and there is $q_0 > 0$ such that $\sup_{x,y \in D} |q(x,y)| \leq q_0$. Then the equation (48) has a global strong solution.*

Proof. In view of the proof of Theorem 3-6.5 in [7, pp. 86] and the proof of Theorem 3.2 in [23], we can show that the equation (48) has a local strong solution. By Theorem 3.1, the solution is positive. Define $\Phi(v, t) = \|v\|_{L^2}^2$. The infimum $\inf_{t \geq 0, \|h\| \geq r} \Phi(h, t) \rightarrow \infty$ as $r \rightarrow \infty$. We have

$$\begin{aligned} \mathcal{L}_t \Phi(v, t) &= \frac{\partial}{\partial t} \Phi(v, t) + \frac{b^2}{2} \text{Tr}[\Phi''(v, t) v^m Q v^m] + (\Delta v, \Phi'(v, t)) \\ &\quad + (v - v^3, \Phi'(v, t)) + \int_Z [\Phi(v + cv^n z, t) - \Phi(v, t) \\ &\quad - (cv^n z, \Phi'(v, t))] \nu(dz) \\ &\leq b^2 \int_D q(x, x) v^{2m}(x) dx - 2 \int_D |\nabla v|^2 dx + 2 \int_D (v^2 - v^4) dx \\ &\quad + \int_0^\infty (cv^n z, cv^n z) \nu(dz) \\ &\leq b^2 q_0 \|v\|_{L^{2m}}^{2m} + 2 \|v\|_{L^2}^2 - 2 \|v\|_{L^4}^4 + c^2 \|v\|_{L^{2n}}^{2n} \int_0^\infty z^2 \nu(dz). \end{aligned} \quad (50)$$

By (19) and (20), we have

$$\begin{aligned} \|v\|_{L^{2m}}^{2m} &\leq \|v\|_{L^2}^{2m\alpha} \|v\|_{L^4}^{2m(1-\alpha)} \\ &\leq \varepsilon \|v\|_{L^4}^{\frac{2m(1-\alpha)}{1-m\alpha}} + \varepsilon^{-\frac{1-m\alpha}{m\alpha}} \|v\|_{L^2}^2 \\ &\leq \varepsilon \|v\|_{L^4}^4 + \varepsilon^{-\frac{1-m\alpha}{m\alpha}} \|v\|_{L^2}^2, \end{aligned} \quad (51)$$

where $\alpha = \frac{2-m}{m}$. Similarly,

$$\|v\|_{L^{2n}}^{2n} \leq \varepsilon \|v\|_{L^4}^4 + \varepsilon^{-\frac{1-n\beta}{n\beta}} \|v\|_{L^2}^2, \quad (52)$$

where $\beta = \frac{2-n}{n}$. By (50), (51) and (52),

$$\begin{aligned} \mathcal{L}_t \Phi(v, t) &= (b^2 q_0 \varepsilon + c^2 \varepsilon \int_0^\infty z^2 \nu(dz) - 2) \|v\|_{L^4}^4 \\ &\quad + (b^2 q_0 \varepsilon^{-\frac{1-m\alpha}{m\alpha}} + c^2 \varepsilon^{-\frac{1-n\beta}{n\beta}} \int_0^\infty z^2 \nu(dz) + 2) \|v\|_{L^2}^2. \end{aligned}$$

Choose ε small such that $b^2 q_0 \varepsilon + c^2 \varepsilon \int_0^\infty z^2 \nu(dz) - 2 \leq 0$, we have

$$\mathcal{L}_t \Phi(v, t) \leq C \Phi(v, t).$$

Therefore, by Theorem 5.3, the equation (48) has a global strong solution. \square

Acknowledgments. Kexue Li is supported by National Natural Science Foundation of China under the contract No.11571269, China Postdoctoral Science Foundation Funded Project under grants 2015M572539, 2016T90899. Junxiong Jia is supported by National Natural Science Foundation of China under the contract No.11501439. Jigen Peng is supported partially by National Natural Science Foundation of China under the contract No.11131006, and by the National Basic Research Program of China under the grant No. 2013CB329404.

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Received August 2016; revised May 2017.

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