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Robust Stability of Singularly Perturbed Systems with State Delays

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***Abstract*—In this paper, robust stability of uncertain singularly perturbed systems with state delays is studied. Based on Lyapunov-Krasovskii stability theorem and linear matrix inequality (LMI) technique, sufficient conditions are given to guarantee that systems are asymptotically stable, and explicit stability bounds are attained by solving convex optimization problem. Numerical example is presented to show the application of the proposed techniques.**

***Index Terms*—singularly perturbed systems; delay; robust stability**

I. Introduction

Singularly perturbed systems have been studied extensively due not only to theoretical interest but also to its wide application in control engineering [1-7]. In most of physical systems, the singular perturbations are characterized by some small parasitic parameters. For example, in power systems the singularity often results from the small machine reactance or transients in voltage regulators. In nuclear reactor models the singularity is due to fast neutrons, which can be separated from the slow neutrons through introducing small time scales [2, 3, 4, 5]. For such parameterized singular perturbation systems, a commonly used approach to system analysis and control design is the system reduction technique, a key of which lies in the construction of the slow and fast subsystems [1-3]. However, when there exist time delays in both slow and fast states, the reduction technique fails to be usable as the slow and fast state can not be separated completely [4,5]. Here, the LMI technique is used to study the stability of singularly perturbed system with delay on both slow and fast states.

In the past decade, LMI technique has been extensively exploited to solve control problems. The LMIs that arise in systems and control theory can be formulated as convex optimization problems that are amenable to compute solution and can be solved effectively [8]. Some significant advances have been achieved for developing LMI based approaches to the control synthesis for singularly perturbed systems [9, 10, 11]. To our knowledge, there are few LMI-based formulation for analysis and synthesis of uncertain singularly perturbed system with delay, which is the motivation of this study.

In this paper, the robust stability of uncertain singularly perturbed system with delay on both slow and fast states are considered by using LMI technique and Lyapunov-Krasovskii stability theorem. Sufficient conditions are given to guarantee that systems are asymptotically stable, and explicit stability bounds are obtained by solving convex optimization problem. An example is illustrated to show the applications of the results derived.

II. Main Result

Consider the following linear uncertain singularly perturbed system with state delay

$$I_\varepsilon \dot{x}(t) = (A+\Delta A)x(t) + (D+\Delta D)x(t-d) \qquad (\Sigma_0)$$

Where， $x(t)=col\{x_1(t),x_2(t)\}$, $x_1(t)\in\mathbb{R}^{n_1}$ is the slow state vector, $x_2(t)\in\mathbb{R}^{n_2}$ is the fast state vector. A, D are known real constant matrices, $\Delta A, \Delta D$ are the time-varying norm bounded uncertainties, and are assumed to be of the form

$$\begin{bmatrix} \Delta A & \Delta D \end{bmatrix} = E\Delta(t)\begin{bmatrix} F & F_d \end{bmatrix}, \quad \Delta^T(t)\Delta(t)\le I$$

The matrix I_ε is given by $I_\varepsilon = \begin{bmatrix} I_{n1} & 0 \\ 0 & \varepsilon I_{n2} \end{bmatrix}$, where $\varepsilon>0$ is a small parameter. d is the state time delay.

The matrices in (1) have the following structure: $A=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22}\end{bmatrix}$, $D=\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22}\end{bmatrix}$, $E=\begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$, $F^T=\begin{bmatrix} F_1^T \\ F_2^T \end{bmatrix}$, $F_d^T=\begin{bmatrix} F_{d1}^T \\ F_{d2}^T \end{bmatrix}$.

The stability of singularly perturbed systems with delay were considered in [12] by Nyquist criterion, and by small gain theorem in [13], where only delay on slow states were discussed. In [14， 15], delay-dependent criterion were derived, but uncertainty were not considered. In [16], both system matrices and time delays may be uncertain, but $D_{22}=0$. Here, stability criterion of uncertain singularly perturbed system with state delay is derived based on Lyapunov-Krasovskii stability theorem.

Lemma 1[17] (Lyapunov-Krasovskii stability theorem)

A time-delay system is asymptotically stable if there exists a bounded quadratic Lyapunov-Krasovskii

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functional $V(\phi)$ such that for some $\varepsilon > 0$, it satisfies $V(\phi) \geq \varepsilon\|\phi(0)\|^2$, and its derivative along the system trajectory satisfies $\dot{V}(\phi) \leq -\varepsilon\|\phi(0)\|^2$. Where $\phi = x(t+\theta),\ -d \leq \theta \leq 0$.

Before presenting the main results, two matrix inequalities are given which is extensively used in uncertain system research.

Lemma 2 [18] For appropriate dimension matrix E, F, symmetric matrix Y, all the $\Delta(t)$ satisfied $\Delta^T(t)\Delta(t) \leq I$,

$$Y + E\Delta(t)F + F^T\Delta^T(t)E^T < 0$$

if and only if there exists a constant $\eta > 0$ such that

$$Y + \eta EE^T + \eta^{-1}F^TF < 0.$$

In the sequel, we shall present our main results on parameter-independent robust stability analysis based on the LMI method. A sufficient condition is given such that system (2.1) is asymptotically stable and stability bound ε^* is also derived .

Theorem 1 If there exist positive matrices P_1, P_3, S_1, S_3 , matrices P_2, S_2 and scalar $\eta > 0$, such that $\Phi < 0$ is feasible, then system (Σ) is asymptotically stable for $0 < \varepsilon \leq \varepsilon^*$, where ε^* is obtained by solving convex optimization problem

$$\max_{P,S}\ \varepsilon \qquad (1)$$

$$s.t \qquad \Phi + \varepsilon\Psi < 0, \qquad (2)$$

$$P > 0, \qquad (3)$$

$$S > 0. \qquad (4)$$

Where $\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} \\ * & * & \Phi_{33} & \Phi_{34} & 0 \\ * & * & * & \Phi_{44} & 0 \\ * & * & * & * & -\eta I \end{bmatrix}$,

$$\Psi = \begin{bmatrix} 0 & \Psi_{12} & 0 & 0 & 0 \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix},$$

$$P = \begin{bmatrix} P_1 & \varepsilon P_2 \\ \varepsilon P_2^T & \varepsilon P_3 \end{bmatrix}, S = \begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix},$$

$\Phi_{11} = A_{11}^TP_1 + A_{21}^TP_2^T + P_1A_{11} + P_2A_{21} + S_1 + \eta F_1^TF_1,$

$\Phi_{12} = A_{21}^TP_3 + P_1A_{12} + P_2A_{22} + S_2 + \eta F_1^TF_2,$

$\Phi_{13} = P_1D_{11} + P_2D_{21} + \eta F_1^TF_{d1},$

$\Phi_{14} = P_1D_{12} + P_2D_{22} + \eta F_1^TF_{d2},$

$\Phi_{15} = P_1E_1 + P_2E_2, \Phi_{25} = P_3E_2, \Psi_{25} = P_2^TE_1,$

$\Phi_{22} = A_{22}^TP_3 + P_3A_{22} + S_3 + \eta F_2^TF_2,$

$\Phi_{23} = P_3D_{21} + \eta F_2^TF_{d1}, \Psi_{23} = P_2^TD_{11},$

$\Phi_{24} = P_3D_{22} + \eta F_2^TF_{d2}, \Psi_{24} = P_2^TD_{12},$

$\Phi_{33} = -S_1 + \eta F_{d1}^TF_{d1}, \Psi_{12} = A_{11}^TP_2,$

$\Phi_{34} = -S_2 + \eta F_{d1}^TF_{d2}, \Psi_{22} = A_{12}^TP_2 + P_2A_{12},$

$\Phi_{44} = -S_3 + \eta F_{d2}^TF_{d2}.$

PROOF: Denote

$$A_\varepsilon = \begin{bmatrix} A_{11} & A_{12} \\ \frac{1}{\varepsilon}A_{21} & \frac{1}{\varepsilon}A_{22} \end{bmatrix}, \quad E_\varepsilon = \begin{bmatrix} E_1 \\ \frac{1}{\varepsilon}E_2 \end{bmatrix},$$

$$D_\varepsilon = \begin{bmatrix} D_{11} & D_{12} \\ \frac{1}{\varepsilon}D_{21} & \frac{1}{\varepsilon}D_{22} \end{bmatrix},$$

Then system (Σ_0) can be rewritten as follows (Σ_1)

$$\dot{x}(t) = (A_\varepsilon + E_\varepsilon\Delta(t)F)x(t) + (D_\varepsilon + E_\varepsilon\Delta(t)F_d)x(t-d),$$

Introduce the following Lyapunov-Krasovskii functional candidate for the system :

$$V(t, x_t) = x^T(t)Px(t) + \int_{t-d}^{t} x^T(s)Sx(s)ds,$$

The time-derivation of $V(t, x_t)$ along with system is given by

$$\begin{aligned}\dot{V}(t) &= x^T(t)[(A_\varepsilon + E_\varepsilon\Delta(t)F)^TP + P(A_\varepsilon + E_\varepsilon\Delta(t)F)]x(t) \\ &\quad + x^T(t)Sx(t) + 2x^T(t)P(D_\varepsilon + E_\varepsilon\Delta(t)F_d)x(t-d) \\ &\quad - x^T(t-d)Sx(t-d) \\ &= \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T M_\varepsilon \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix},\end{aligned}$$

where

$$M_\varepsilon = \begin{bmatrix} (A_\varepsilon + E_\varepsilon\Delta(t)F)^TP + P(A_\varepsilon + E_\varepsilon\Delta(t)F) + S & P(D_\varepsilon + E_\varepsilon\Delta(t)F_d) \\ (D_\varepsilon + E_\varepsilon\Delta(t)F_d)^TP & -S \end{bmatrix}.$$

Note that by lemma 1, $M_\varepsilon < 0$ if and only if there exists $\eta > 0$ such that $\overline{M_\varepsilon} < 0$, where

$$\overline{M_\varepsilon} = \begin{bmatrix} A_\varepsilon^TP + PA_\varepsilon + S & PD_\varepsilon \\ * & -S \end{bmatrix} + \eta\begin{bmatrix} F^T \\ F_d^T \end{bmatrix}\begin{bmatrix} F^T \\ F_d^T \end{bmatrix}^T + \eta^{-1}\begin{bmatrix} PE_\varepsilon \\ 0 \end{bmatrix}\begin{bmatrix} PE_\varepsilon \\ 0 \end{bmatrix}^T$$

Applying the Schur complement to (2.3) , it follows that $\overline{M_\varepsilon} < 0$, then $M_\varepsilon < 0$. By Lyapunov-Krasovskii stability theorem, the result follows immediately.

Remark 1: Though the proposed LMI-based approach is of full dimensions, it provides a convex alternative method. In general, the LMI problems with reasonable large dimensions can also be effectively solved by using LMI control toolbox.

As special case, when there are no uncertainties in systems (2.1). i.e. $\Delta A = 0, \Delta D = 0$. We have the following result.

Corollary 1 If there exist positive matrix $P_1 > 0, P_3 > 0, S_1 > 0, S_3 > 0$ and matrix P_2, S_2 such that $\hat{\Phi} < 0$ is feasible, then system without uncertainty is asymptotically stable for $0 < \varepsilon \leq \varepsilon_1^*$, where ε_1^* is obtained by solving convex optimization problem

$$\max_{P,S}\ \varepsilon \qquad (1)$$

$$s.t \qquad \hat{\Phi} + \hat{\Psi} < 0, \qquad (2)$$

$$P > 0. \qquad (3)$$

Where

$$\hat{\Phi} = \begin{bmatrix} \hat{\Phi}_{11} & \hat{\Phi}_{12} & \hat{\Phi}_{13} & \hat{\Phi}_{14} \\ * & \hat{\Phi}_{22} & \hat{\Phi}_{23} & \hat{\Phi}_{24} \\ * & * & -S_1 & -S_2 \\ * & * & * & -S_3 \end{bmatrix},$$

$$\hat{\Psi} = \begin{bmatrix} 0 & \Psi_{12} & 0 & 0 \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}.$$

$$\hat{\Phi}_{11} = A_{11}^T P_1 + A_{21}^T P_2^T + P_1 A_{11} + P_2 A_{21} + S_1,$$
$$\hat{\Phi}_{12} = A_{21}^T P_3 + P_1 A_{12} + P_2 A_{22} + S_2,$$
$$\hat{\Phi}_{13} = P_1 D_{11} + P_2 D_{21}, \hat{\Phi}_{14} = P_1 D_{12} + P_2 D_{22},$$
$$\hat{\Phi}_{22} = A_{22}^T P_3 + P_3 A_{22} + S_3,$$
$$\hat{\Phi}_{23} = P_3 D_{21}, \hat{\Phi}_{24} = P_3 D_{22}.$$

Remark 2: This case was considered in [15], where stability criterion were given for small enough $\varepsilon > 0$, but there was no explicit bound of ε. By corollary 3, explicit stability bound can be obtained by solving convex optimization problem.

We illustrate our results by considering the following singularly perturbed system with time-invariant delay in [15].

$$\begin{cases} \dot{x_1}(t) = x_2(t) + x_1(t-d) \\ \varepsilon \dot{x_2}(t) = -2x_1(t) - x_2(t) + 0.5x_2(t-d) \end{cases}$$

In corollary 3, $\hat{\Phi} < 0$ has feasible solve $P_1 = 1.0804$, $P_2 = 0.8642$, $P_3 = 0.6074$, $S_1 = 1.5978$, $S_2 = 0.5378$, $S_3 = 0.6472$.The solution to (1), (2), (3) are obtained: $\varepsilon_1^* = 0.4638$. System is asymptotically stable for $0 < d < \infty$, $0 < \varepsilon \leq 0.4638$.

III. CONCLUSIONS

The main contribution of this work is the derivation of stability criterion for uncertain singularly perturbed systems with state delays by LMI technique. As corollary, the stability of certain singularly perturbed systems is also considered. The stability condition and explicit stability bound are given by solving LMIs which can be dealt by MATLAB. Finally, a numerical example is given to illustrate the applications of the proposed results.

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