CHIN. PHYS. LETT. Vol. 26, No. 3 (2009) 034212

Dynamics of Incoherent Photovoltaic Spatial Solitons *

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(Received 6 November 2008)

Propagation properties of bright and dark incoherent beams are numerically studied in photovoltaic-photorefractive crystal by using coherent density approach for the first time. Numerical simulations not only exhibit that bright incoherent photovoltaic quasi-soliton, grey-like incoherent photovoltaic soliton, incoherent soliton doublet and triplet can be established under proper conditions, but also display that the spatial coherence properties of these incoherent beams can be significantly affected during propagation by the photovoltaic field.

PACS: 42. 65. Tg, 42. 25. Kb, 73. 50. Pz, 42. 65. Jx, 42. 25. Bs, 42. 81. Dp

The discovery of photovoltaic solitons[1,2] and incoherent spatial solitons[3−10] which have been attracted considerable attention from all over the world in the past decades not only opens up a new research field for soliton science and nonlinear optics but also broaden the potential applications of optical solitons. To be frank, both of the two types of spatial solitons have been relatively thoroughly studied both in theory and experiment,[1−10] and incoherent photovoltaic solitons are also reported now and again over the past several years,[11,12] but the incoherent photovoltaic soliton are still not studied at length. The case to the point is that the propagation properties as well as the spatial coherence properties of multiple incoherent photovoltaic spatial soliton in photovoltaic-photorefractive crystal are not investigated at all. In this Letter, we firstly set up the dynamic equations of incoherent photovoltaic soliton and then discuss the propagation properties and spatial coherence properties in detail through numerical simulations for the first time. In the theoretical research, as is well known, there are three traditionally used research approaches, i.e., coherent density approach,[6,7] self-consistent multimode approach[8] and geometric optics approach (in the diffractionless limit).[9] It is worth mentioning the three approaches have been demonstrated to be equivalent to each other in inertial nonlinear media.[10] In this study, coherent density approach is adopted, and by using this approach we observe the forming of incoherent soliton triplet and doublet under odd and even initial conditions respectively as well as bright incoherent photovoltaic quasi-soliton and grey-like incoherent photovoltaic soliton. The results are similar to the pioneering studies,[7] however the significance is quite different. It is important that incoherent photovoltaic soliton is closely linked with photovoltaic field.

The space-charge field E_{sc} in terms of the optical irradiance is depicted by[1]

$$E_{sc} = -E_p \frac{I/I_d}{1+I/I_d}, \tag{1}$$

where E_p is the photovoltaic field, I is the light intensity, I_d is the dark irradiance. According to the method offered by Ref. [6], we can obtain the following infinite set of coupled nonlinear Schrödinger-like equations,

$$\begin{aligned} i\left(\frac{\partial \Psi_j}{\partial z} + (j\Delta\theta)\frac{\partial \Psi_j}{\partial x}\right) + \frac{1}{2k}\frac{\partial^2 \Psi_j}{\partial x^2} \\ + \frac{k_0}{2} n_b^3 r_{\rm eff} E_p \frac{I(x,z)}{1+I(x,z)} \Psi_j = 0, \end{aligned} \tag{2}$$

where j is the discrete index ($j = 0, 1, 2 \ldots$), $j\Delta\theta$ is the angle with respect to the z axis, Ψ_j is the field of each coherent component called coherent density function, $k = k_0 n_b$, $k_0 = 2\pi/\lambda$, n_b is the background refractive index, λ is the wavelength, $r_{\rm eff}$ is the effective electro-optic coefficient, $I(x, z)$ is the intensity profile expressed in units of the dark irradiance of the incoherent beam, given by

$$I(x,z) = \sum_{j=-\infty}^{\infty} \left|\Psi_j(x,z)\right|^2, \tag{3}$$

We set the coherent density functions at $z = 0$,[6,7,13]

$$\Psi_j(x, z=0) = (r,\rho)^{1/2} G_N^{1/2}(j\Delta\theta)\varphi_0(x), \tag{4}$$

where r is the maximum intensity of the bright incoherent beam [i.e., $r = \max(I)$], ρ is the normalized intensity of the dark beam at $x \to \pm\infty$ [i.e., $I(x \to \pm\infty, z)$], $G_N(j\Delta\theta)$ is the normalized angular power spectrum of the incoherent source, $\varphi_0(x)$ is the

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*Supported by the Key Project of the National Natural Science Foundation of China under Grant No 10674176.

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spatial modulation function. The spatial coherence properties of these incoherent beams can be depicted by coherence length l_c,[7,8] which is given by

$$l_c(x) = \int_{-\infty}^{+\infty} dx' \frac{1}{I(x,z)I(x',z)} \Big| \sum_{j=-\infty}^{+\infty} \Psi_j(x,z,j\Delta\theta) \cdot \Psi_j^*(x',z,j\Delta\theta) \exp[ik(j\Delta\theta)(x-x')]\Delta\theta \Big|^2 . \qquad (5)$$

We assume that the normalized angular power spectrum of the incoherent source is Gaussian, i.e., $G_N(j\Delta\theta) = (\sqrt{\pi}\theta_0)^{-1}\exp[-(j\Delta\theta)^2/\theta_0^2]$, where θ_0 is the width of the angular power spectrum.[13] We also assume that the spatial modulation function at the input is Gaussian when bright incoherent photovoltaic soliton is considered, i.e., $\varphi_0(x) = \exp[-x^2/(2x_0^2)]$, where x_0 is associated with the full width at half maximum (FWHM) of the beam intensity and the relation between them is $x_0 = \mathrm{FWHM}/2\sqrt{\ln 2}$; as to dark incoherent photovoltaic soliton $\varphi_0(x) = \tanh(x/x_0)$ under odd initial conditions and $\varphi_0(x) = \left[1 - \varepsilon^2 \mathrm{sech}^2(x/x_0)\right]^{1/2}$ under even initial conditions, where $x_0 = \mathrm{FWHM}/\ln(3+2\sqrt{2})$, $\varepsilon^2 \approx 1$. According to the equations mentioned above, we can obtain $l_c = \sqrt{2\pi}/(k\theta_0)$ at $z = 0$, which is independent of the spatial modulation function $\phi_0(x)$, but relies on k and θ_0.

In our numerically simulations, we choose LiNbO$_3$ as the photovoltaic-photorefractive crystal with $n_b = 2.2$, $r_{\mathrm{eff}} = 30\,\mathrm{pm/V}$,[1] $\lambda = 500\,\mathrm{nm}$, $r(\text{or } \rho) = 3$, $\theta_0 = 7.9\,\mathrm{mrad}$, so the coherence length $l_c \approx 11.5\,\mu\mathrm{m}$ at $z = 0$. We use 201 components ($-100 \le j \le 100$) equidistantly spanning the range $\pm 2.5\theta_0$ (i.e., $\Delta\theta = 5\theta_0/200$), similar to that used in Ref. [6].

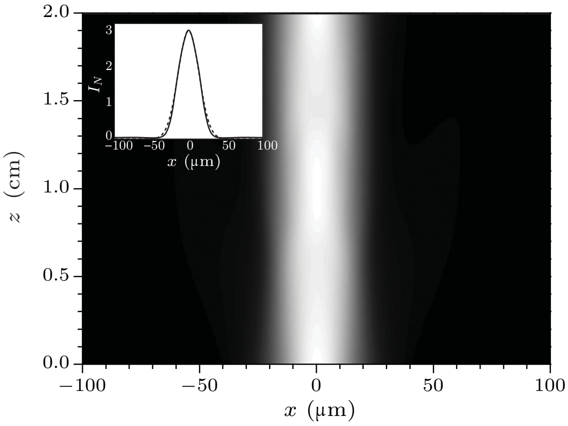

Fig. 1. Formation of a bright incoherent photovoltaic quasi-soliton with $x_0 = 20\,\mu\mathrm{m}$ and $E_p = 20\,\mathrm{kV/cm}$. The inset shows the input beam (dashed curve) and output beam at $z = 2\,\mathrm{cm}$ (solid curve).

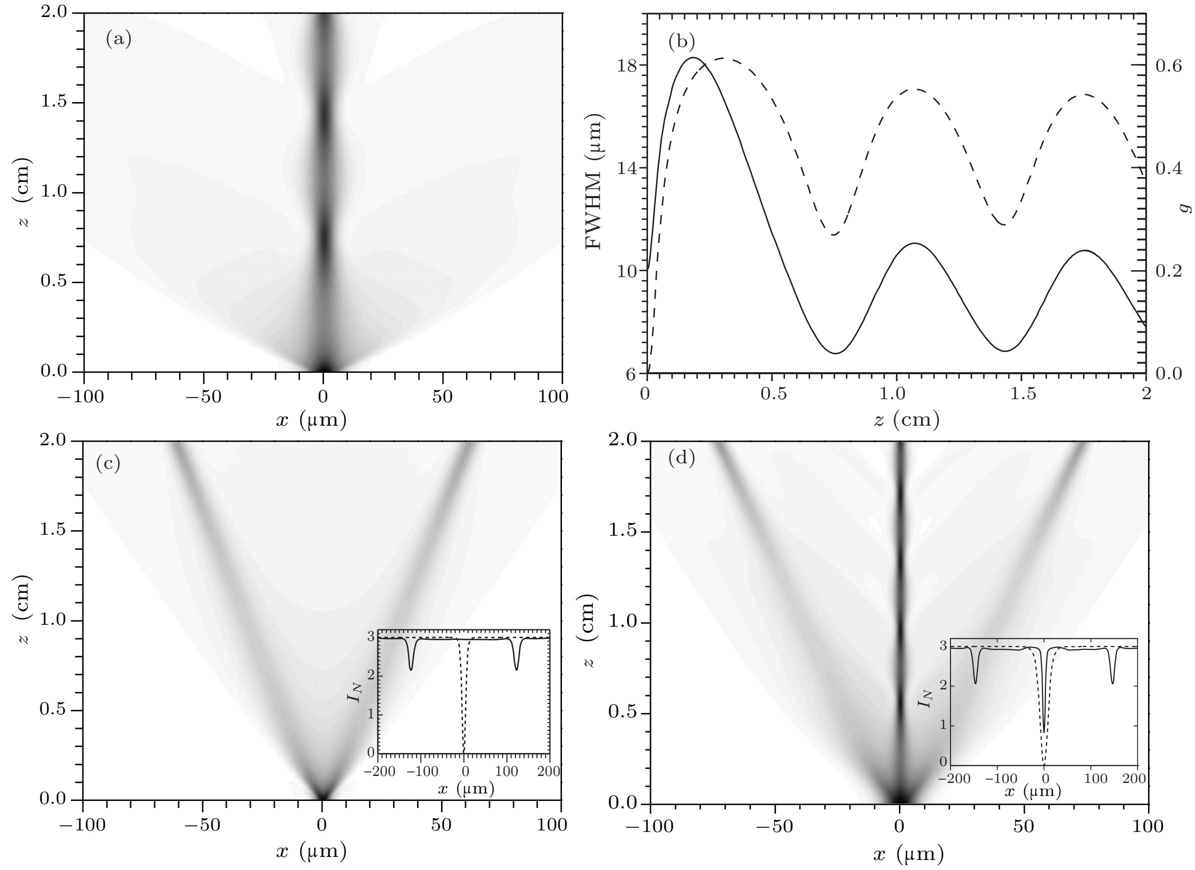

Fig. 2. (a) Formation of a grey-like incoherent photovoltaic soliton under odd initial conditions, (b) FWHM (solid curve, left y-axis) and the grayness (dashed curve, right y-axis) of the grey-like soliton as a function of distance, (c) formation of a incoherent soliton doublet under even initial conditions, (d) formation of a incoherent soliton triplet under odd initial conditions. The insets show the input beam (dashed curve) and output beam at $z = 2\,\mathrm{cm}$ (solid curve).

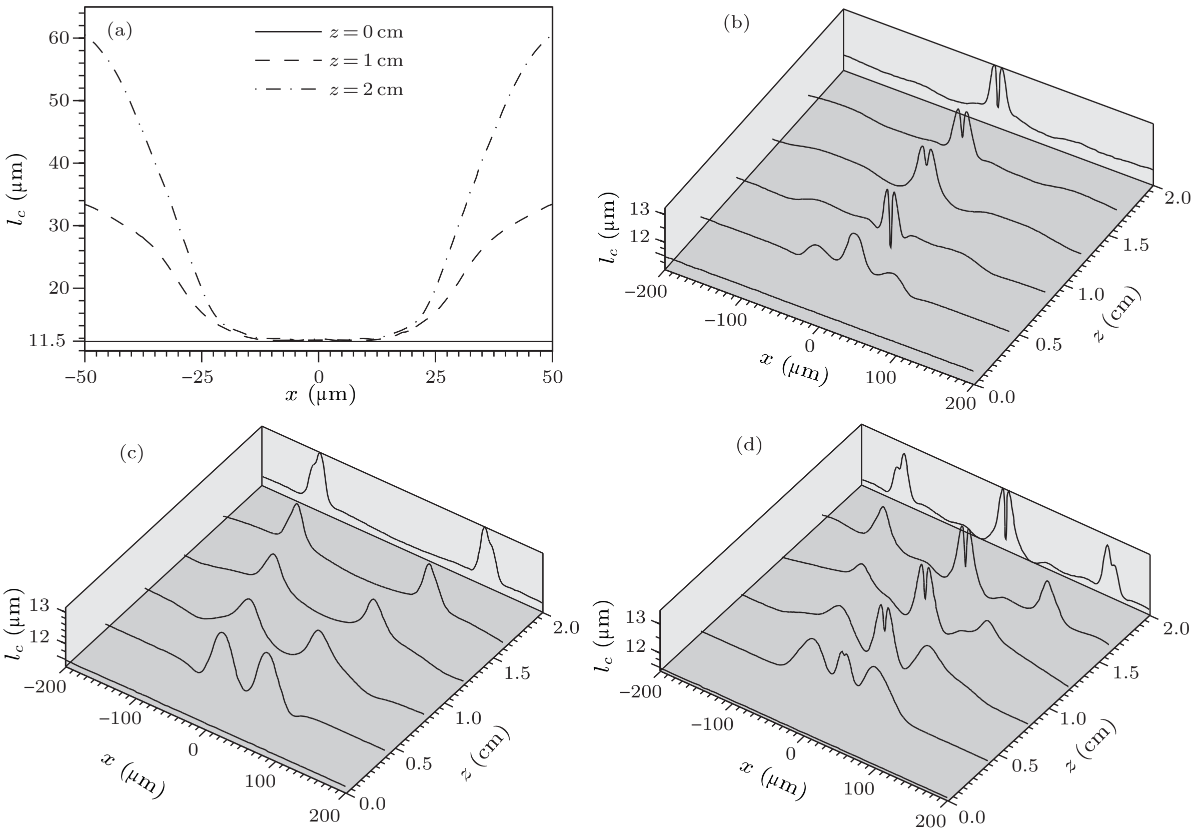

Fig. 3. Coherence length of (a) bright incoherent photovoltaic quasi-soliton, (b) grey-like incoherent photovoltaic soliton, (c) incoherent soliton doublet and (d) incoherent soliton triplet as a function of distance.

Figure 1 presents the evolution of the bright incoherent beam in the photovoltaic-photorefractive crystal with $x_0 = 20\,\mu\text{m}$ and $E_p = 20\,\text{kV/cm}$. We can see that the input incoherent beam propagates almost undistorted and it behaves rather stable. The inset also exhibits that the input beam (dashed line, FWHM ≈ 33.3 μm) and the output beam (solid line, FWHM ≈ 30 μm) match each other very well. Here the incoherent beam can be called a bright incoherent photovoltaic quasi-soliton. Figure 2(a) shows the formation of a grey-like incoherent photovoltaic soliton with $x_0 = 5.7\,\mu\text{m}$ and $E_p = 55\,\text{kV/cm}$ under odd initial conditions. Even though the incoherent beam exhibits somehow oscillatory behaviour, it can also be called a grey-like incoherent photovoltaic soliton. Furthermore, we introduce the g parameter ($0 < g < 1$) to describe the soliton grayness, and define it as the ratio of the lowest intensity of the beam to the intensity at infinity, i.e., $g = \min[I(x, z)]/I(x \to \pm\infty, z)$.[14] Figure 2(b) depicts the grayness (dashed curve, right y-axis) of the grey-like soliton and the intensity FWHM (solid curve, left y-axis) as a function of distance. We can see clearly that both the grayness (changing between about 0.25 and 0.6) and the intensity FWHM (changing between about 7 μm and 11 μm) have a sine-like shape, and what is more, they are also changing synchronism. Figure 2(c) presents the formation of an incoherent photovoltaic soliton doublet (2-splitting) with $x_0 = 5.7\,\mu\text{m}$ and $E_p = 55\,\text{kV/cm}$ under even initial conditions. From this phenomenon, we predict that if the parameters are properly chosen, 4-splitting, 6-splitting and even higher splitting can also emerge in a photovoltaic-photorefractive crystal as in a photorefractive crystal.[15] As known to all, researchers have demonstrated that incoherent soliton triplet (3-splitting) can form in photorefractive crystal under odd initial conditions with proper values of the parameters used,[7,15] so can the phenomenon also emerge in photovoltaic-photorefractive crystal? Figure 2(d) not only positively answers the question with $x_0 = 11.4\,\mu\text{m}$ and $E_p = 70\,\text{kV/cm}$ but also connotes 5-splitting and even higher splitting can also form.

The coherence length of bright incoherent photovoltaic quasi-soliton is depicted in Fig. 3(a) within a spatial window of ±50 μm, which reveals that within the range of ±20 μm, the coherence length is almost at the same level whereas increases sharply beyond the range, and this behaviour is in good agreement with the results of Refs. [7,8]. Figure 3(b) displays the coherence length of grey-like incoherent photovoltaic soliton corresponding to Fig. 2(a). From the figure, we can see clearly that the coherence length increases around the notch and sharply falls at the centre with a depression. Figure 3(c) shows the coherence length of the incoherent photovoltaic soliton doublet corresponding to Fig. 2(b). Different from Fig. 3(b), the coherence length increases around the notch without falling and depression at the centre. However, according to the coherence length at output, we predict that the situation will change if the incoherent soliton doublet propagates in an enough long distance. Figure 3(d) exhibits the coherence length of the incoherent photovoltaic soliton triplet corresponding to Fig. 2(d), the case around the central notch is similar to that of the grey-like incoherent photovoltaic soliton and the other two cases are similar to these of the incoherent photovoltaic soliton doublet. This phenomenon shows that the coherence length is significantly affected during propagation by the photovoltaic field.

In conclusion, we have studied the propagation properties of bright and dark incoherent photovoltaic solitons in a photovoltaic-photorefractive crystal by coherent density approach. For properly given parameters, we obtain bright incoherent photovoltaic quasi-soliton, incoherent photovoltaic soliton doublet under even initial conditions, and grey-like incoherent photovoltaic soliton and incoherent photovoltaic soliton triplet under odd initial conditions. We have also predicted that 4-splitting, 5-splitting and even higher splitting can emerge in photovoltaic-photorefractive crystal. We find that the coherence properties of these incoherent beams can be significantly affected during propagation of the photovoltaic field.

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