

Fig. 7. The propagation of a hexapole azimuthon with $A = 0.4$, $B = 0.5$ in the circular waveguide. Left panel shows the original hexapole azimuthon and the corresponding phase; right panel shows the iso-surface plot of the propagation.

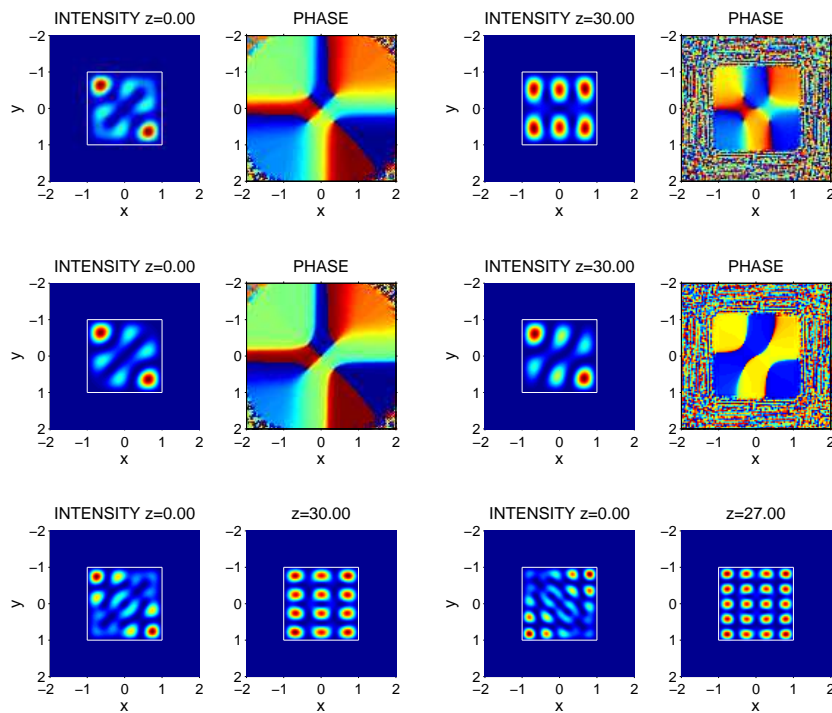


Fig. 8. The propagation of higher order azimuthons in the square waveguide. The first row shows a rotating hexapole with $A = 0.4$, $B = 0.5 > B_{cr}$ (Media 2), and the second row shows a twisting hexapole with $A = 0.4$, $B = 0.2 < B_{cr}$ (Media 1). The superposed dodecapoles (left two panels), and superposed icosapoles (right two panels) as well as their deformations are shown in the third row; in both cases we choose $A = 0.4$ and $B = 0.5$.

5. Conclusion

In conclusion, we demonstrated numerically stable propagation of azimuthons, i.e. localized rotating nonlinear waves in weakly nonlinear waveguides. Depending on the waveguide profile, different nonlinearity induced propagation dynamics can be observed. We showed that azimuthons in circular waveguides rotate continuously. The analytically predicted dependence of rotation frequency ω as a function of soliton parameters was found to be in excellent agreement with numerical simulations. Further on, we discussed propagation of azimuthon-like structures in square waveguides. We showed that their dynamics critically depends on the initial conditions. In particular, we found a threshold-like behavior in the propagation dynamics, separating rotating and wobbling solutions. We showed that this effect is associated with different values of the Hamiltonian for different azimuthon orientations. As our analysis relies on physical parameters of actual multi-mode waveguides, our findings may open a relatively easy route to experimental observations of stable azimuthons.

Acknowledgement

Numerical simulations were partly performed on the SGI XE Cluster and the Sun Constellation VAYU Cluster of the Australian Partnership for Advanced Computing (APAC). This research was supported by the Australian Research Council.