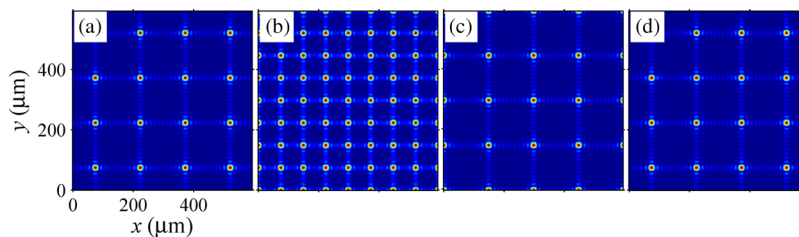


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Abstract: We theoretically study the Talbot effects resulted from the four-wave mixing and six-wave mixing signals, which are periodically modulated due to the coherence control effect. Corresponding to different dressing states, the enhancement and suppression conditions that will affect the properties of the multiwave mixing signals are also discussed in detail. Such proposal can be useful in all-optical-controlled pattern formation and propagation of light.

Index Terms: Talbot effects, multiwave mixing (MWM), nonlinear optics, atomic coherence.

1. Introduction

Talbot effect, which was observed by H. F. Talbot in 1836 [1] and first analytically explained by Lord Rayleigh in 1881 [2], is a near-field diffraction phenomenon that the light field spatially imprinted with periodic structure can have self-imaging at certain periodic imaging planes (the so-called Talbot planes). Such self-imaging effect holds a range of applications from image preprocessing and synthesis, photolithography, optical testing, optical metrology, spectrometry, to optical computing [3].

To date, studies on Talbot effect have been reported associated with atomic waves [4], [5], surface waves [6], [7], nonclassical light [8], or waveguide arrays [9]. Among these research works, the nonlinear Talbot effect with second harmonic (SH) were demonstrated both experimentally [10] and theoretically [11] in periodically poled LiTaO₃ (PPLT) crystal for the first time. The Talbot effect with beam imprinted with an electromagnetically induced grating (EIG) has been also proposed [12]. However, no progress in higher nonlinear optical process was reported before to our knowledge.

The crucial point and the guiding ideology to observe the Talbot effect is how to produce a spatial periodical incidence in transverse dimension. From this point of view, it is natural to realize that researchers have created spatially periodic four-wave mixing (FWM) and six-wave mixing (SWM) signals due to the periodic atomic coherence induced by standing waves (SWs) [13]–[16], which can play an important role in lasing without inversion [17], slow light generation [18], photon controlling and information storage [19]–[21], fiber lasers [22], [23], etc. The periodic pattern of the multiwave mixing (MWM) signals can be flexibly controllable by adjusting the atomic coherence via changing the beam detunings.

In this paper, we investigate the MWM Talbot effect for the first time. And as the coexisting FWM and SWM processes have been observed in the Cd (S, Se) semiconductor-doped glasses [24], the idea can be also executed in some solid crystals such as Pr-doped YSO crystals [24]–[27] besides

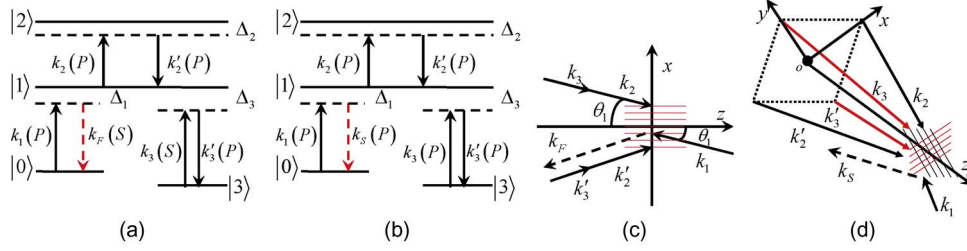


Fig. 1. Schematic of the reverse Y-type atomic system to produce MWM signals. (a) \mathbf{k}_F represents the singly or doubly dressed FWM corresponds to a weak or strong \mathbf{k}_3 , respectively. (b) Emission of the doubly dressed SWM signal \mathbf{k}_S . The letters S and P in the brackets mean the polarization of the wave. (c) and (d) The Cartesian geometric configurations to generate FWM and SWM signals, respectively; The SWs along x and y are constructed by \mathbf{k}_2 , \mathbf{k}'_2 and \mathbf{k}_3 , \mathbf{k}'_3 , respectively.

atomic vapors. This paper is organized as follows: in Section 2, we introduce the basic theory that includes the energy level systems, some conceptions, and how to prepare the periodic FWM and MWM signals; in Section 3, we discuss the enhancement and suppression conditions, due to which the energy levels split; in Section 4, we investigate the Talbot effects of the periodic MWM signals; and in the last section, we conclude this paper.

2. Theoretical Model and Analysis

We consider the FWM and SWM processes in the reverse Y-type atomic levels system derived by five beams, as shown in Fig. 1(a) and (b). A candidate for the systems are rubidium atomic vapors. Also, in Pr:YSO crystal, the energy-level system with $|0\rangle = |^3H_4(\pm 3/2)\rangle$, $|1\rangle = |^1D_2(\pm 3/2)\rangle$, and $|3\rangle = |^3H_4(\pm 1/2)\rangle$ can be used as a subsystem of our proposed system. Here, we note that, to guarantee the applicability of our proposal to all the possible candidate systems, we choose parameters in simulation arbitrarily. Fig. 1(c) and (d) is the corresponding beam geometric configurations. The transition $|0\rangle \rightarrow |1\rangle$ with resonant frequency Ω_{10} is probed by beam \mathbf{E}_1 (with frequency ω_1 , frequency detuning Δ_1 , and wave vector \mathbf{k}_1). $|1\rangle \rightarrow |2\rangle$ with resonant frequency Ω_{21} is pumped by beams \mathbf{E}_2 (with ω_2 , Δ_2 , and \mathbf{k}_2) and \mathbf{E}'_2 (with ω_2 , Δ_2 , and \mathbf{k}'_2); $|3\rangle \rightarrow |1\rangle$ with resonant frequency Ω_{31} is pumped by beams \mathbf{E}_3 (with ω_3 , Δ_3 , and \mathbf{k}_3) and \mathbf{E}'_3 (with ω_3 , Δ_3 , and \mathbf{k}'_3). The frequency detunings are defined as $\Delta_1 = \Omega_{10} - \omega_1$, $\Delta_2 = \Omega_{21} - \omega_2$, and $\Delta_3 = \Omega_{13} - \omega_3$. In the Cartesian coordinate introduced in Fig. 1(c) and (d), the wave vectors are elaborated designed that \mathbf{E}_1 propagates z-negative with an angle θ_1 ; \mathbf{E}_2 and \mathbf{E}_3 propagate along the opposite direction of \mathbf{k}_1 , deviating from the z-positive direction with θ_1 ; and \mathbf{E}'_2 and \mathbf{E}'_3 propagate symmetrically to \mathbf{E}_2 and \mathbf{E}_3 with respect to z-negative direction.

As shown in Fig. 1(a), if the fields \mathbf{E}_2 , \mathbf{E}'_2 , \mathbf{E}_3 , and \mathbf{E}'_3 are not strong sufficiently, we consider the undressed FWM signal \mathbf{E}_F with $\omega_F = \omega_1$ and $\mathbf{k}_F = \mathbf{k}_1 + \mathbf{k}_3 - \mathbf{k}'_3$; if \mathbf{E}_3 and \mathbf{E}'_3 are weak but \mathbf{E}_2 and \mathbf{E}'_2 are strong sufficiently to induce dressing effect (act as the dressing fields), we consider the singly dressed FWM signal \mathbf{E}_{F1} with $\omega_{F1} = \omega_1$ and $\mathbf{k}_{F1} = \mathbf{k}_F$. While, if \mathbf{E}_3 and \mathbf{E}'_3 are also strong sufficiently to induce the dressing effect, we consider the doubly dressed FWM signal \mathbf{E}_{F2} with $\omega_{F2} = \omega_1$ and $\mathbf{k}_{F2} = \mathbf{k}_F$. Although \mathbf{E}_2 and \mathbf{E}'_2 will induce another FWM signal (P polarization), we can still get the expected FWM signal via a polaroid for its polarization selectivity. By using five beams with polarizations, as shown in Fig. 1(b), a SWM signal \mathbf{E}_S with $\omega_S = \omega_1$ and $\mathbf{k}_S = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}'_2 + \mathbf{k}_3 - \mathbf{k}'_3$ can be obtained.

To obtain the MWM signal, we solve the density matrix equations and the Liouville pathways for $\rho_{10}^{(3)}$ and $\rho_{10}^{(5)}$, the amplitude of which is proportional to that of the MWM signal. First, for the undressed FWM signal \mathbf{E}_F , we can obtain $\rho_{10}^{(3)}$ via the Liouville pathway [28] as

$$\rho_{00}^{(0)} \xrightarrow{\mathbf{E}_1} \rho_{10}^{(1)} \xrightarrow{(\mathbf{E}_3)^*} \rho_{30}^{(2)} \xrightarrow{\mathbf{E}'_3} \rho_{10}^{(3)} \quad (1)$$

$$\rho_{10}^{(3)} = \frac{-iG_{\text{FWM}}}{d_1^2 d_3} \quad (2)$$

where $G_{\text{FWM}} = G_{10}G_{30}(G'_{30})^* \exp(i\mathbf{k}_F \cdot \mathbf{r}) \exp(-i\omega_1 t)$, $d_1 = \Gamma_{10} + i\Delta_1$, and $d_3 = \Gamma_{30} + i(\Delta_1 - \Delta_3)$. The terms $G_{i0} = \mu_{mn}E_{i0}/\hbar$ and $G'_{i0} = \mu_{mn}E'_{i0}/\hbar$, with $i = 1, 2$, are the amplitudes of the Rabi frequencies of \mathbf{E}_i and \mathbf{E}'_i , respectively, where μ_{mn} is the dipole moment of transition between $|m\rangle$ and $|n\rangle$, which is driven by \mathbf{E}_i ; E_{i0} and E'_{i0} are the amplitudes of \mathbf{E}_i and \mathbf{E}'_i , respectively. Γ_{pq} represents the transverse relaxation rate between $|p\rangle$ and $|q\rangle$ for $p \neq q$, and the longitudinal relaxation rate of $|p\rangle$ for $p = q$. It is obvious to see that the amplitude of $\rho_{10}^{(3)}$ in (2) is spatial independent, so \mathbf{E}_F is uniform.

Next, we consider the singly dressed FWM process from \mathbf{E}_2 and \mathbf{E}'_2 , which are strong sufficiently. In the spatial interaction region, \mathbf{E}_2 and \mathbf{E}'_2 will interfere with each other and create a SW, which leads to periodic Rabi frequency amplitude $|G_{2t}(x)|^2 = G_{20}^2 + G_{20}'^2 + 2G_{20}G_{20}'\cos[2(k_2\sin\theta_1)x]$. Therefore, the Liouville pathway for \mathbf{E}_{F1} will be modified into

$$\rho_{00}^{(0)} \xrightarrow{\mathbf{E}_1} \rho_{|G_{2t}(x)\pm 0}^{(1)} \xrightarrow{(\mathbf{E}_3)^*} \rho_{30}^{(2)} \xrightarrow{\mathbf{E}'_3} \rho_{|G_{2t}(x)\pm 0}^{(3)} \quad (3)$$

where $|G_{2t}(x)\pm\rangle$ represents two dressed states produced by the spatial periodic dressing effect. According to (3), we can obtain the spatial-dependent dressed density matrix element as

$$\rho_{F1}^{(3)}(x) = \frac{-iG_{\text{FWM}}}{[d_1 + |G_{2t}(x)|^2/d_2]^2 d_3} \quad (4)$$

where $d_2 = \Gamma_{20} + i(\Delta_1 + \Delta_2)$. The amplitude of $\rho_{F1}^{(3)}$ shows obvious periodic variation along x direction with a period of $d_x = \lambda_2/(2\sin\theta_1)$.

When \mathbf{E}_3 and \mathbf{E}'_3 get strong significantly, we will get the doubly dressed FWM signal \mathbf{E}_{F2} . The Liouville pathway and the doubly dressed density matrix element for this case are

$$\rho_{00}^{(0)} \xrightarrow{\mathbf{E}_1} \rho_{|G_{3t}\pm G_{2t}(x)\pm 0}^{(1)} \xrightarrow{(\mathbf{E}_3)^*} \rho_{30}^{(2)} \xrightarrow{\mathbf{E}'_3} \rho_{|G_{3t}\pm G_{2t}(x)\pm 0}^{(3)} \quad (5)$$

$$\rho_{F2}^{(3)}(x) = \frac{-iG_{\text{FWM}}}{[d_1 + |G_{30}|^2/d_3 + |G_{2t}(x)|^2/d_2]^2 d_3} \quad (6)$$

In Fig. 1(b), the spatially periodic dressing effect from \mathbf{E}_2 and \mathbf{E}'_2 remains, and the dressing fields \mathbf{E}_3 and \mathbf{E}'_3 with same polarization (compared with the case of (6) with \mathbf{E}_3 and \mathbf{E}'_3 having orthogonal polarizations) can also interfere with each other and create periodic Rabi frequency amplitude $|G_{3t}(y)|^2 = G_{30}^2 + G_{30}'^2 + 2G_{30}G_{30}'\cos[2(k_3\sin\theta_2)y]$. Therefore, this SWM will suffer from two dressing effects, which are both spatial periodic with different periods d_x and $d_y = \lambda_3/(2\sin\theta_2)$, respectively. The corresponding Liouville pathway and doubly dressed density matrix element are

$$\rho_{00}^{(0)} \xrightarrow{\mathbf{E}_1} \rho_{|G_{3t}(y)\pm G_{2t}(x)\pm 0}^{(1)} \xrightarrow{(\mathbf{E}_3)^*} \rho_{30}^{(2)} \xrightarrow{\mathbf{E}'_3} \rho_{|G_{3t}(y)\pm G_{2t}(x)\pm 0}^{(3)} \xrightarrow{(\mathbf{E}_2)^*} \rho_{2\pm 0}^{(4)} \xrightarrow{\mathbf{E}'_2} \rho_{|G_{3t}(y)\pm G_{2t}(x)\pm 0}^{(5)} \quad (7)$$

$$\rho_{10}^{(5)} = \frac{iG_{\text{SWM}}}{[d_1 + |G_{2t}(x)|^2/d_2 + |G_{3t}(y)|^2/d_3]^3 d_2 d_3} \quad (8)$$

where $G_{\text{SWM}} = G_{10}G_{20}(G'_{20})^*G_{30}(G'_{30})^* \exp(i\mathbf{k}_S \cdot \mathbf{r}) \exp(-i\omega_1 t)$.

With these expressions, the spatial periodic pattern formation of MWM signals can be investigated. It is clear that the periodical variation of singly dressed FWM, doubly dressed FWM, and doubly dressed SWM signals with respect to x -axis at $z = 0$, are all derived from the periodical dressing effects. With different probe detunings, these MWM signals will show profiles with significant differences. So, in order to analyze such dependence, we consider the so-called enhancement and suppression conditions due to dressing effect before investigating the MWM Talbot effects.

3. Suppression and Enhancement Conditions

The characteristics of FWM are affected by the dressing effect, which depends on Δ_1 , Δ_2 , and $|G_{2t}(x)|^2$. As shown in Fig. 2, because of $|G_{2t}(x)|^2$, $|1\rangle$ is split into two dressing states $|G_{2t}(x)\pm\rangle$

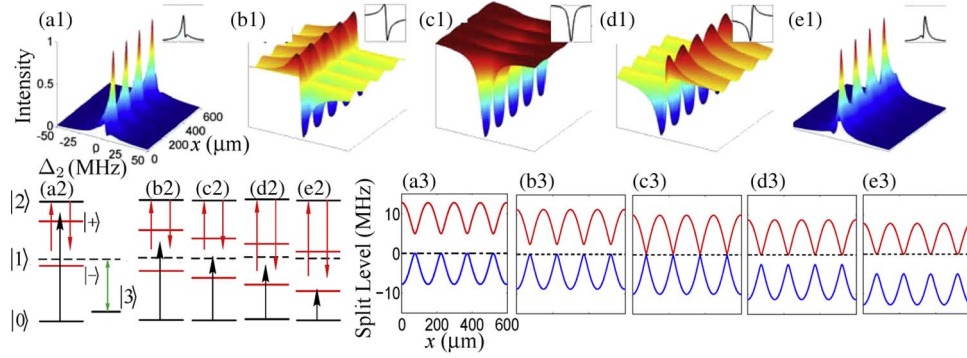


Fig. 2. Normalized intensities versus Δ_2 and x of singly dressed FWM with (a1) enhancement effect, (b1) enhancement-suppression effect, (c1) suppression effect, (d1) suppression-enhancement effect, and (e1) enhancement effect, corresponding to $\Delta_1 = -5$ MHz, -4 MHz, 0 , 4 MHz, and 5 MHz, respectively. The insets are the incident intensity versus Δ_2 . (a2)–(e2) The corresponding split energy level (red lines marked $|+\rangle$ and $|-\rangle$ on the right are split energy levels, the dotted line is the initial energy level) diagrams with dressing fields at $x = 0$. (a3)–(e3) The corresponding periodic split energy levels versus x . The other parameters are $G_2 = 15$ MHz, $\Gamma_{10} = 5$ MHz, and $\Gamma_{20} = 1$ kHz.

with eigenfrequencies $\lambda_{|G_{2t}(x)\pm\rangle} = \Delta_2/2 \pm \sqrt{\Delta_2^2/4 + |G_{2t}(x)|^2}$. Therefore, $|G_{2t}(x)\pm\rangle$ values are periodic along x .

Absorption will be enhanced when the probe resonates with the dressing states, i.e., $\Delta_1 = -\lambda_{|G_{2t}(x)\pm\rangle}$, which corresponds to the electromagnetically induced absorption (EIA) condition. Accordingly, the FWM signal will get enhanced resonantly. Thus, we define $\Delta_1 = -\lambda_{|G_{2t}(x)\pm\rangle}$ as the enhancement condition. While when the probe reaches two-photon resonance ($\Delta_1 + \Delta_2 = 0$), absorption will be suppressed (the EIT condition), and the FWM signal in such case will be suppressed correspondingly. Thus, we define $\Delta_1 + \Delta_2 = 0$ as the suppression condition. So, if Δ_1 is set at discrete values orderly from negative to positive and Δ_2 is scanned, we can obtain the periodic enhancement/suppression condition and FWM signals along x . In Fig. 2(a1)–(e1), we display the periodic FWM signals along x corresponding to different Δ_1 , and the insets are the dependence of the FWM signal intensity on Δ_2 at $z = 0$ and $x = 0$. In Fig. 2(a2)–(e2), we exhibit the corresponding split energy level diagrams, and in Fig. 2(a3)–(e3), the corresponding periodic properties of the split energy levels along x . In Fig. 2(a1), Δ_1 is relatively large and positive, and we only consider the enhancement because of the weakness of the suppression. When Δ_1 decreases but is still positive, Δ_2 will meet the enhancement condition ($\Delta_1 = -\lambda_{|G_{2t}(x)+\rangle}$) first and then the suppression condition as shown in Fig. 2(b1). Because of $\Delta_1 = 0$ in Fig. 2(c1), so that Δ_2 can never meet the enhancement condition. The case in Fig. 2(d1) is opposite to that in Fig. 2(b1). And in Fig. 2(e1), the enhancement is dominant again, the same as that in Fig. 2(a1), for Δ_1 is relatively large and negative.

For the doubly dressed MWM signals \mathbf{E}_{F2} and \mathbf{E}_S , we take \mathbf{E}_{F2} to execute our discussions. First, the interference between \mathbf{E}_3 and \mathbf{E}'_3 leads to periodic dressing effect and therefore splits the naked state into two dressing states $|G_{3t}(y)\pm\rangle$ with eigenfrequencies $\lambda_{|G_{3t}(y)\pm\rangle} = -\Delta_3/2 \pm \sqrt{\Delta_3^2/4 + |G_{3t}(y)|^2}$. \mathbf{E}_2 and \mathbf{E}'_2 also act as dressing fields, so the first-order dressing state $|G_{3t}(y)\pm\rangle$ will split into second-order dressing states $|G_{3t}(y) \pm G_{2t}(x)\pm\rangle$ with frequencies

$$\lambda_{|G_{3t}(y)-G_{2t}(x)\pm\rangle} = \frac{-\Delta_3 - \sqrt{\Delta_3^2 + 4|G_{3t}(y)|^2}}{2} + \frac{\Delta'_2 \pm \sqrt{\Delta_2'^2 + 4|G_{2t}(x)|^2}}{2} \quad (9)$$

$$\lambda_{|G_{3t}(y)+G_{2t}(x)\pm\rangle} = \frac{-\Delta_3 + \sqrt{\Delta_3^2 + 4|G_{3t}(y)|^2}}{2} + \frac{\Delta''_2 \pm \sqrt{\Delta_2''^2 + 4|G_{2t}(x)|^2}}{2} \quad (10)$$

where $\Delta'_2 = \Delta_2 - \{-\Delta_3 - \sqrt{\Delta_3^2 + 4|G_{3t}(y)|^2}\}/2$, and $\Delta''_2 = \Delta_2 - \{-\Delta_3 + \sqrt{\Delta_3^2 + 4|G_{3t}(y)|^2}\}/2$, respectively.

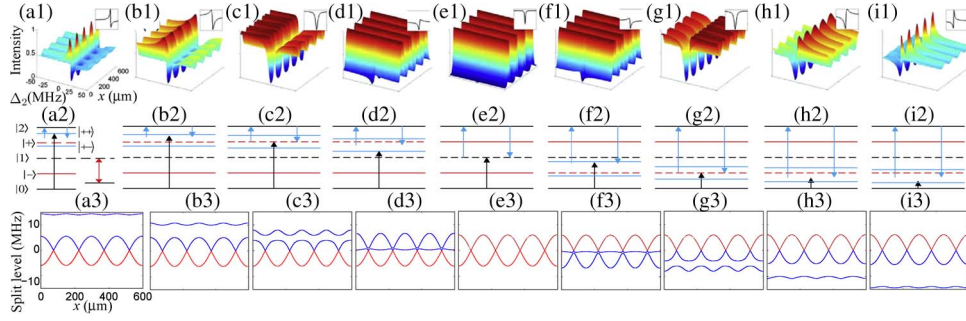


Fig. 3. Normalized intensities versus Δ_2 and x of (a1) doubly dressed FWM with enhancement effect, (b1) enhancement-suppression effect, (c1) suppression effect, (d1) suppression-enhancement effect, (e1) suppression effect, (f1) enhancement-suppression effect, (g1) suppression effect, (h1) suppression-enhancement effect, and (i1) enhancement effect, corresponding to $\Delta_1 = -18$ MHz, -8 MHz, -5 MHz, -2 MHz, 0 , 2 MHz, 5 MHz, 8 MHz, and 18 MHz, respectively. The insets are the incident intensity versus Δ_2 . (a2)-(i2) The corresponding split energy level diagrams with dressing fields at $x = 0$. (a3)-(i3) The corresponding periodic split energy levels versus x . The other parameters are $\Delta_3 = 0$, $G_2 = G_3 = 15$ MHz, $\Gamma_{10} = 5$ MHz, and $\Gamma_{20} = 1$ kHz.

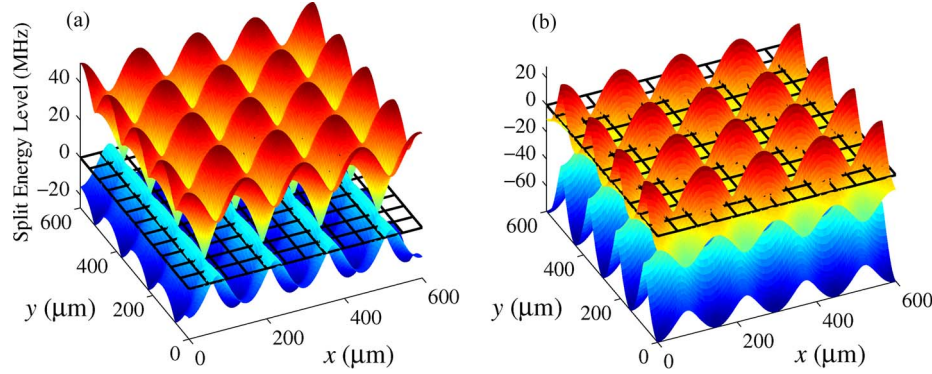


Fig. 4. (a) The surfaces represent $|G_{3t}(y) + G_{2t}(x)+\rangle$ (top), $|1\rangle$ (grid), and $|G_{3t}(y) + G_{2t}(x)-\rangle$ (bottom), respectively. (b) The surfaces are $|G_{3t}(y) - G_{2t}(x)+\rangle$ (top), $|1\rangle$ (grid), and $|G_{3t}(y) - G_{2t}(x)-\rangle$ (bottom), respectively. $\Delta_2 = 2$ MHz for (a) and -2 MHz for (b). The values of the other parameters are $\Delta_3 = -3$ MHz, $G_{20} = G'_{20} = G_{30} = G'_{30} = 15$ MHz, $\Gamma_{10} = 5$ MHz, $\Gamma_{20} = \Gamma_{30} = 1$ kHz, $\lambda_1 = 776$ nm, $\lambda_2 = \lambda_3 = 780$ nm, $\theta_1 = \theta_2 = 0.3^\circ$.

Similar to the case for \mathbf{E}_{F1} , we can also investigate the enhancement and suppression for \mathbf{E}_{F2} and \mathbf{E}_S . And their enhancement and suppression correspond to $\Delta_1 = -\lambda|G_{3t}(y) \pm G_{2t}(x) \pm\rangle$ and $\Delta_1 + \Delta_2 = 0$, respectively. The periodic doubly dressed FWM signals along x corresponding to different conditions are shown by the mesh plots in Fig. 3(a1)–(i1) with insets being the dependence of the signal intensity on Δ_2 at $z = 0$ and $x = 0$. The corresponding split energy level schematics are shown in Fig. 3(a2)–(i2), and the corresponding spatial periodic energy levels are shown in Fig. 3(a3)–(i3). Doubly dressed SWM enhancement and suppression effects are similar to the results shown in Fig. 3. In light of the SWM here being a two-dimension (2-D) case, we exhibit two cases of the second-order splitting energy levels, as shown in Fig. 4(a) and (b).

4. Talbot Effect of MWM Signals

In the perspective of Fourier optics, the transfer function of a Fresnel diffraction system with z as the propagation axis can be expressed as $H_F(\xi) = \exp(ik_z z) \exp(-i\pi\lambda z \xi^2)$ in frequency domain [29], where ξ is the spatial frequency and k_z is the projection of \mathbf{k} along z . The field of the MWM signal

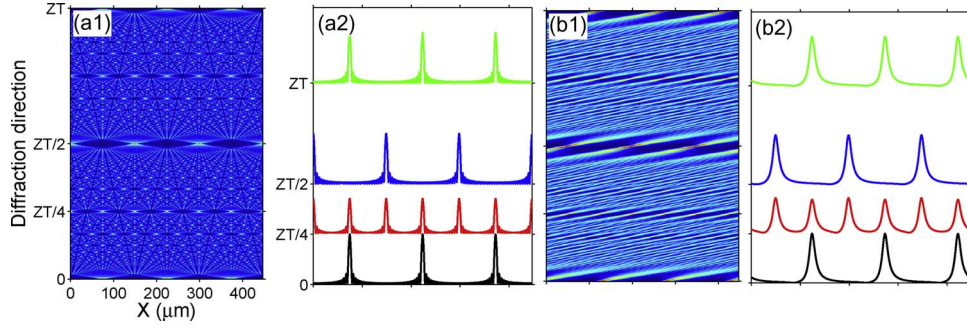


Fig. 5. (a1) The Talbot effect carpets for \mathbf{E}_{F1} under suppression conditions with $\Delta_1 = 3$ MHz and $\Delta_2 = -3$ MHz. (b1) The Talbot effect carpets for \mathbf{E}_{F1} under enhancement conditions with $\Delta_2 = -3$ MHz and $-\Delta_1 = \Delta_2/2 + \sqrt{\Delta_2^2/4 + |G_{2T}(x)|^2}$. (a2) and (b2) The intensity profiles at $z = 0$ (black curves), $z = z_T/4$ (red curves), $z = z_T/2$ (blue curves), and $z = z_T$ (green curves), respectively. The other parameters are $G_{20} = G_{20} = 15$ MHz, $\Gamma_{10} = 5$ MHz, and $\Gamma_{20} = \Gamma_{30} = 1$ kHz.

$g_0(x, y) \propto \rho(x, y)$, so it can be expanded into 2-D Fourier series as $g_0(x, y) = \sum_{m,n=-\infty}^{\infty} c_{m,n} \times \exp[i2\pi(nx/d_x + my/d_y)]$, and in frequency domain, the equation above can be written as

$$G_0(\xi, \eta) = \sum_{m,n=-\infty}^{\infty} c_{m,n} \delta\left(\xi - \frac{n}{d_x}\right) \cdot \delta\left(\eta - \frac{m}{d_y}\right) \quad (11)$$

where $c_{m,n}$ is the Fourier coefficient. So, considering the Fresnel diffraction, the MWM signal at a z distance is

$$G(\xi, \eta) = G_0(\xi, \eta) \exp(ik_z z) \exp[-i\pi\lambda_1 z(\xi^2 + \eta^2)]. \quad (12)$$

Plugging (11) into (12), we end up with

$$G(\xi, \eta) = \sum_{m,n=-\infty}^{\infty} c_{m,n} \delta\left(\xi - \frac{n}{d_x}\right) \cdot \delta\left(\eta - \frac{m}{d_y}\right) \exp(ik_z z) \exp\left\{-i\pi\lambda_1 z \left[\left(\frac{n}{d_x}\right)^2 + \left(\frac{m}{d_y}\right)^2\right]\right\}. \quad (13)$$

For simplicity, we let $\lambda_2 = \lambda_3$ and $\theta_1 = \theta_2$; thus, the periods along x and y are the same, i.e., $d_x = d_y$. Singly and doubly dressed FWM signals only concern one SW formed by \mathbf{k}_2 and \mathbf{k}_2^* , and therefore, we do not consider y component. In such a case, $\exp[-i\pi\lambda_1 z(n/d_x)^2] = 1$ if $z = 2qd_x^2/\lambda_1$ ($q = 1, 2, 3, \dots$), so after inverse Fourier transformation, we get

$$g(x) = g_0(x) \exp(ik_z z). \quad (14)$$

Because of $|g(x)|^2 = |g_0(x)|^2$, we can see the self-imaging of the MWM signals at $z = 2qd_x^2/\lambda_1$, and $z_T = z|_{q=1}$ is the Talbot length. It is worth mentioning that the images on the z_T/N planes are fractional Talbot images [30], where N is an integer bigger than 1.

We first choose \mathbf{E}_{F1} as the incidence to execute the simulation. As shown in Fig. 2, the spatial profile of the incident signal varies along Δ_2 with different Δ_1 , and this variation leads to different diffraction process of FWM signal under different conditions. Fig. 5(a1) and (b1) are the corresponding Talbot effect carpets under suppression conditions and enhancement conditions, respectively. Fig. 5(a2) and (b2) are the intensity profiles of the repeated images cut at certain fractional Talbot lengths. From Fig. 5, we cannot only clearly see the periodic singly dressed FWM can reappear along z but also see that the carpet stripes are oblique and such obliquity is more obvious under enhancement conditions.

For \mathbf{E}_{F2} , the results are shown in Fig. 6. Clearly, we obtain the self-imaging of the incident \mathbf{E}_{F2} at the Talbot plane again, whether it is under the suppression or the enhancement conditions. In contrary to the case in Fig. 5(a1), which is obtained under suppression conditions, the Talbot effect shown in Fig. 6(a1) seems oblique. Under enhancement conditions, the carpet stripes are more

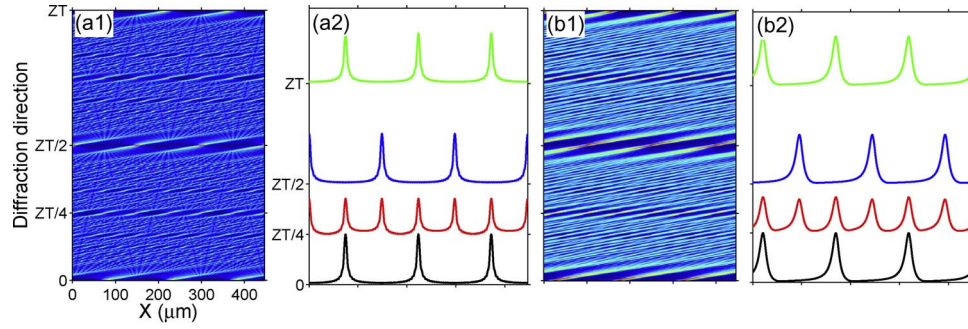


Fig. 6. (a1) The Talbot effect carpets for \mathbf{E}_{F2} under suppression conditions with $\Delta_1 = 18$ MHz, $\Delta_2 = -18$ MHz, and $\Delta_3 = 18$ MHz. (b1) The Talbot effect carpets for \mathbf{E}_{F2} under enhancement conditions with $\Delta_2 = -18$ MHz, $\Delta_3 = 18$ MHz, and Δ_1 based on (9). (a2) and (b2) The setup is as Fig. 5(a2) and (b2). The other parameters are $G_{20} = G'_{20} = 15$ MHz, $G_{30} = 2$ MHz, $\Gamma_{10} = 5$ MHz, and $\Gamma_{20} = \Gamma_{30} = 1$ kHz.

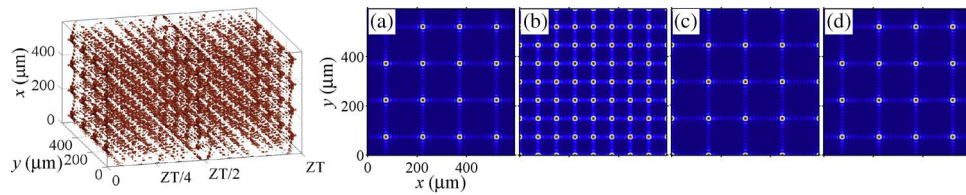


Fig. 7. Iso-surface plot is the Talbot effect for \mathbf{E}_S under the suppression condition. The four panels are the contour plots of the Talbot effect at (a) $z = 0$, (b) $z = z_T/4$, (c) $z = z_T/2$, and (d) $z = z_T$, respectively. The parameters are $\Delta_1 = 18$ MHz, $\Delta_2 = -18$ MHz, $\Delta_3 = 18$ MHz, $G_{20} = G'_{20} = G_{30} = G'_{30} = 15$ MHz, $\Gamma_{10} = 5$ MHz, $\Gamma_{20} = \Gamma_{30} = 1$ kHz.

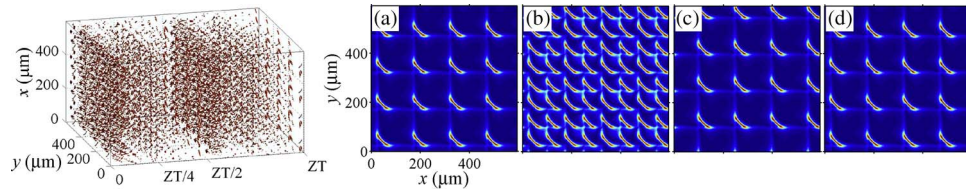


Fig. 8. Talbot effect for \mathbf{E}_S under the enhancement condition. The setup is as the Fig. 7, but with Δ_1 according to (10).

obvious, as shown in Fig. 6(b1), but the obliquity is almost the same with that shown in Fig. 6(a1). By comparing the Talbot carpets shown in Figs. 6(b1) and 5(b1), we find the obliquity and the width of the stripes are almost unchanged.

Last but not the least, we discuss the Talbot effect from \mathbf{E}_S . We consider two orthogonal SWs from two couple of dressing fields \mathbf{E}_2 , \mathbf{E}'_2 and \mathbf{E}_3 , \mathbf{E}'_3 simultaneously, to form a 2-D lattice, which is periodic both along x and y , as shown in Fig. 1(d); thus, a 2-D SWM signal will be excited. In Figs. 7 and 8, we first give the iso-surface plots of the Talbot effect of the 2-D SWM signal under suppression conditions and enhancement conditions, respectively. And then, we choose four intensity plots at certain places during propagation to show the details more clearly. We can see that, at the Talbot length as shown in Figs. 7(d) and 8(d), the 2-D SWM signals are reproduced. At half the Talbot length, as shown in Figs. 7(c) and 8(c), the self-images shifted half period both along x and y . At one quarter of the Talbot length, fractional self-images can be seen, as shown in Figs. 7(b) and 8(b), in which the images are twice as many as those in Figs. 7(a) and 8(a). Under enhancement conditions shown in Fig. 8, the images are clearer than those under suppression conditions, as shown in Fig. 7. Here, we want to point out the reason that the periods of fractional Talbot effects, shown in Figs. 5(b2), 6(a2) and (b2), and 8(b) seemingly not the half period at $z = 0$, is due to the

inertial weakness of the numerical simulations. The Talbot effect changes dramatically during propagation, especially around small fractional Talbot lengths. Even a tiny deviation from the exact $z_T/4$ will bring seemingly imperfect quarter Talbot effect. Calculations with very high resolution will help us approach perfect fractional Talbot effects but beyond our computer's memory.

5. Conclusion

In conclusion, we have studied the Talbot effects with completely controllable MWM signals. We have obtained the spatially periodic FWM and SWM signals by the interference between two dressing fields in a reverse Y-type atomic level system. The intensities of the MWM signals can be effectively controlled via enhancement conditions and suppression conditions derived from dressing effect. Talbot effect from singly and doubly dressed FWM has been observed in the numerical experiment. Different from the case of FWM, for SWM, we have constructed a 2-D lattice, from which the SWM is 2-D modulated, to investigate the 2-D Talbot effect. We have found that the numerical simulations agree with the theoretical predictions very well. Our scheme is more advantageous in the controllability, compared with the previous studies, mainly attributed to the modulation of the MWM signal by the dressing effect. Our findings not only enrich the understanding of the MWM theory but also offer a different method to investigate the Talbot effect.

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