

# FOREWORD

This book is the outcome of years of collaboration among the three coauthors. The collaboration resulted in a number of papers, whose most significant results compose the major portion of this book. The principal coauthor is Dr. Yiqi Zhang. He joined the postdoctoral research station of Electronic Science and Technology at Xi'an Jiaotong University in 2012, under the supervision of Prof. Yanpeng Zhang, who is in charge of the “Quantum Control of Multi-Wave Mixing-Key Scientific and Technological Innovation Team” of the Shaanxi Province. In the past few decades, Prof. Y.P. Zhang and his research team generated a lot of notable scientific results, owing to a solid foundation and inspiring academic atmosphere, which provided abundant nourishment for quick development of younger researchers. Prof. Milivoj Belić is team’s international collaborator, who started collaborating with Dr. Y.Q. Zhang during his stay in Germany. Prof. Belić is professor in physics at the Texas A&M University at Qatar and the team leader of the Qatar Nonlinear Science Initiative. Thanks to the strong support for research by the Qatar National Research Fund, Dr. Y.Q. Zhang was able to visit Doha for extended periods in the past few years.

The book is a summary of coauthors research during the past five years, and the results are obtained and published jointly. The results involve not only analytical analysis but also extensive numerical simulations. The book covers a series of research topics in photonics of high current interest, including photonic topological insulators, optical rogue waves, Airy beams, Talbot effect, optical vortices, and other. The contents of the book are as follows.

In Chapter 1, the theory of physical models that will be expounded in this book is briefly introduced, which includes the derivation of the paraxial wave equation and the development of susceptibilities in atomic vapors.

In Chapter 2, the spatial periodic modulation of light is considered. By using the three-beam interference method and nonlinear phase shift modulation, we first investigate the photonic topological insulators in atomic vapors. Secondly, we investigate the Talbot effect resulting from periodically modulated multi-wave mixing. Thirdly, we discuss the nonlinear Talbot effect of rogue waves, which is a real nonlinear optical effect. The effects mentioned are generated in atomic or bulk dielectric media. In the last section of the Chapter, we discuss spatial light modulation in

discrete systems, resulting in the proposal of a beam combiner and splitter.

In Chapter 3, the role of nonlinearities in light modulation is discussed. We first demonstrate that optical vortices (as well as vortex pairs) appear in atomic vapors during propagation, when the third- and fifth-order nonlinearities (the so-called cubic-quintic competing nonlinearities) are considered. Secondly, the interaction of incoherent solitons in a photorefractive medium is investigated, in which the nonlinearity is saturable. The last topic discussed in this Chapter is that of azimuthons, which connect necklace solitons and optical vortices. In this part, we consider a weak Kerr nonlinearity, but with deep potentials of different symmetries.

In Chapter 4, the propagation dynamics of some novel optical beams is investigated, including Airy, Bessel-Gauss, and Laguerre-Gauss beams, as well as Fresnel diffraction patterns. In addition, Mathieu and Weber beams are discussed from the same point of view. The media in which these beams propagate include linear media, Kerr and saturable nonlinear media, and media with harmonic potential. We find that spatial solitons can be formed during interaction of Airy beams in nonlinear media, but the solitons do not exhibit the self-accelerating property. We also show how Airy wave functions, Airy breathers and (dual) Airy-Talbot effect can be considered from a unified viewpoint. Based on the harmonic potential model, we discover a new class of self-Fourier beams – the beams whose Fourier transform are the beams themselves. In addition, if the harmonic potential is inserted into the fractional Schrödinger equation, we show that a Gaussian beam propagates along a zigzag and a funnel-like path in one and two dimensions.

In Chapter 5, a summary of the book is presented, with an outlook on future investigations.

Such an arrangement of the book not only provides for a relative independence of topics discussed in different chapters, but also allows for imminent connections among the topics. We believe this book may become a useful reference for researchers in photonics. Despite our careful exposition, mistakes cannot be avoided in a book addressing very recent research advances. Therefore, comments and criticisms are welcome.

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