Evaluation of a spatial relationship by the concept of intrinsic spatial distance

Yee Leung[1], Deyu Meng[2], Zongben Xu[2]

[1]Department of Geography and Resource Management, Institute of Environment, Energy and Sustainability, The Chinese University of Hong Kong, Shatin, Hong Kong, P.R. China, [2]Institute for Information and System Sciences, Faculty of Science and Ministry of Education Key Lab for Intelligent Networks and Network Security, Xi'an Jiaotong University, Xiąŕan 710049, P.R. China.

Correspondence: Yee Leung, Department of Geography and Resource Management, The Chinese University of Hong Kong, Shatin, Hong Kong, P.R. China

e-mail: yeeleung@cuhk.edu.hk

We propose the concept of intrinsic spatial distance (ISD) for the study of a spatial relationship between any two points in space. The ISD is a distance measure that takes into account the separation of two points with respect to their physical and attribute closeness. We construct an algorithm to implement this concept as an ISD measurement. Based on the ISD concept, two points in space are related through a transitional path linking one to the other. As an ISD measurement decreases, the spatial relationship between two points becomes increasingly stronger. We argue theoretically and demonstrate empirically that the ISD concept is not predisposed in favor of the first law of geography, but directly considers variance of nearness in physical distance and attribute distance to derive the extent to which two points are spatially associated. Specifically, in single attribute cases, the information uncovered by an ISD measurement is more elaborate than that revealed by Moran's I, local Moran's I, and a semivariogram, giving a meticulous account of relatedness in both local and global contexts. The ISD concept is also sufficiently general to be used to study multiple attributes of relationships.

Introduction

"Everything is related to everything else, but near things are more related than distant things." This is the so-called first law of geography invoked by Tobler in 1970. This statement essentially posits that things near each other in space are more similar in profile for a set of attributes than are things distant from each other in space. That is, the degree of relatedness possesses a distance decay structure whereby near phenomena are more closely related than distant phenomena. This conceptualization has aroused much interest and debate over the years (see, for example, Sui 2004; Barnes 2004; Miller 2004; Phillips 2004; Smith 2004; Goodchild 2004; Tobler 2004 in a special issue of the Annals of the AAG). Whether or not it can be regarded as a law, Tobler's invocation did bring to the fore the essential premises that have long served as a basis for the study of spatial relationships. In brief, spatial relationships should be evaluated in terms of their similarity in attributes and proximity in space (Gatrell 1983; Elhorst 2001).

To quantitatively describe the covariation of properties of proximal observations/locations /points (to avoid confusion, the terms observation, location and point are used interchangeably in our discussion), measures of relationships relative to distance have been formulated in spatial statistics. In this paper, we mainly focus on spatial relationship with respect to a single attribute variable. All our analysis can be easily generalized to spatial relatedness with respect to a profile of attributes.

In single attribute cases, a spatial relationship among proximal observations is commonly characterized as spatial autocorrelation, which is the correlation among values of a single variable strictly attributable to the proximity of such values in a two-dimensional (2-D) geographic space, introducing a deviation from the independent observations assumption of classical statistics (Griffith 2003). The functions most often used to describe spatial autocorrelation are related to variance, covariance, and correlation. Area-based techniques are aimed at constructing quantitative criteria for measuring global or local spatial autocorrelation. Moran's I and Geary's c are widely adopted spatial correlation statistics. Moran's I takes the form of the classical Pearson product moment correlation coefficient with values approximately ranging from -1 to $+1$ (Moran 1950). Asymptotically, the value 0 indicates randomness (no spatial autocorrelation). A positive value indicates a positive spatial autocorrelation among proximal locations, whereas a negative value indicates a negative spatial autocorrelation. Geary's c is always positive with a semi-variance type of numerator (Geary 1954). Its values tend to range from 0 to $+2$, where positive spatial autocorrelation is indicated by a value

less than 1 and negative spatial autocorrelation is indicated by a value greater than 1. Both the classic Moran's I and Geary's c require a binary weight/link matrix to reflect distances/linkages between neighbors, and give only a global measure of spatial autocorrelation embedded in a dataset.

Because of the presence of spatial heterogeneity, the degree of spatial autocorrelation may vary significantly over space. Because Moran's I and Geary's c measure only the global pattern of spatial association, they do not identify local areas exhibiting spatial heterogeneities with significant local departures from randomness. Local indicators of spatial association (LISA statistics) ameliorate this disadvantage of such global measures to a certain extent (Anselin 1995). A LISA decomposes a global indicator, such as Moran's I, into the contributions of its individual locations. In essence, LISA statistics measure the degree of spatial autocorrelation within the neighborhood of each location; i.e., they describe the spatial autocorrelation of certain local areas. Local Moran's I and local Geary's c (Anselin 1995), as well as Ord and Getis G statistics (Getis and Ord 1992; Ord and Getis 1995, 2001) are widely used local measures of spatial autocorrelation. Each can be expressed as the ratio of quadratic forms in observations (Leung et al. 2003). These statistics have been employed to study the spatial autocorrelation of various geographical phenomena (see for example, Griffith 2003, 2009; Lee and Rogerson 2007; Yamada et al. 2009). Again, each of these measures requires the specification of a weight/link matrix stipulating the contiguity of locations over space, with the distance effect being the same in all specified directions (Aldstadt and Getis 2006; Cressie and Chan 1989).

Although the preceding area-based global or local spatial measures incorporate distance in geographic space into measures of spatial autocorrelation, they still fall short in describing the degree of spatial relatedness between all pairs of locations. The weight matrix is too crude a representation of the intricate local variations between all pairs of locations in the presence of sizeable spatial heterogeneity. Furthermore, its specification is predisposed to obey Tobler's first law that "near things are more related than distant things."

In addition to area-based techniques, distance-based methods also have been formulated to measure spatial autocorrelation. The semivariogram, commonly referred to as the variogram (a component of kriging in geostatistics), is perhaps the method most widely used to describe spatial autocorrelation (Cressie 1993; Wackernagel 2003). The variogram is a plot of semivariance structured as a function of distance between points. Instead of using covariance or correlation, both of which are measures of the similarity of

a variable in a geographic space, semivariance depicts the squared differences of a variable as a function of the distances between all known points, and measures the dissimilarity of subjects in terms of that variable (Cressie 1993). Recently, the semivariogram has been further extended to non-stationary and directional biases (Goovaerts 1997; Deutsch and Journel 1998). However, similar to area-based methods, the semivariogram does not capture detailed inter-point variations for all pairs of observations essential to the description of global and local spatial autocorrelation, and does not locate the paths along which spatial data are intrinsically related.

The purpose of this paper is to present a newly constructed measure called intrinsic spatial distance (ISD) to quantitatively describe spatial association between all pairs of locations over space. The main idea underlying this concept is to integrate proximities in space and attribute values into a unified measure to depict the intrinsic relatedness of observations. This method involves finding a chain between two locations by linking a long series of interconnected relatedness observations. Along such a chain, spatial relatedness within the context of a particular attribute variable can be gradually and optimally differentiated over space. As such, the spatial relationship between a pair of locations can be calculated as the sum of the variable variances in that chain.

The measure we propose here has four main advantages over existing measures. First, the technique can measure in detail spatial relatedness between all pairs of observations, and can be utilized to study spatial relationships in the presence of non-stationarity and anisotropy. The method is particularly important to the study of interaction proximity, where the relative proximity of any location to each designated location needs to be evaluated (Brown and Horton 1970; Gatrell 1983). Second, the spatial relationship measure we construct forms a metric on the observation space, and thus can be treated as a distance measure in the conventional sense. Third, the interconnected chains between observation pairs meticulously reflect the location-to-location interrelationships essential to spatial analysis in global and local contexts. The transitional path so constructed is tractable, and can reflect complexity/heterogeneity in the continuous surface between any two locations. It also is in line with the convention of measuring distance by a path (see, for example, Lösch 1954; Warntz 1966; Leung 1984). Moreover, the corresponding matrix of distance represents a certain spatial relation. Fourth, the proposed spatial relationship measure not only provides a quantitative realization of "everything is related to everything else" in Tobler's first law by giving the value of the ISD, but also can measure relatedness when observations are: (1) near in both geographical

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and attribute distances; (2) far in both geographical and attribute distances; (3) near in geographical distance but far in attribute distance; and, (4) far in geographical distance but near in attribute distance. These four situations simply represent a crude classification of relatedness for the convenience of discussion. The ISD concept and its related algorithm cover the entire range of both distance and attribute values. In brief, our proposed concept of ISD faithfully reflects relatedness through proximity in distance and attribute values.

Measuring a spatial relationship by the concept of intrinsic spatial distance

In this section, we propose the concept of intrinsic spatial distance (ISD) and the ISD-estimation algorithm for the characterization and measurement of the spatial relationship between two locations.

Three basic assumptions underlying a spatial relationship

The building block of a spatial relationship is the association of any two locations in geographic space. Two points are related through a path that provides a smooth transition of relatedness among all relevant in-between neighboring points. A point is related to a distant point through this smooth propagation of relatedness. That is, attribute values should vary gradually along a path connecting two designated points in a geographical space. The fundamental problem then becomes the search for an underlying path along which two points are related, with its inter-point spatial relationship being determined by integrating local variations in the physical and attribute spaces. Consequently, the basic question asks how to find such a path that relates two points by both geography and attributes.

   The path along which spatial objects are intrinsically related should possess three basic properties:

 A(i) Attribute values should vary along the path as smoothly as possible;

 A(ii) The overall attribute-value variation of the path should be as small as possible; and,

A(iii) The physical length of the path should be as short as possible.

These three properties are both intuitive and natural. Property A(i) captures the gradual local variation of attribute values along a path, while properties A(ii) and A(iii) mean that a spatial relationship between

objects should reflect the most direct physical and attribute relationships between them. These two properties also ensure the uniqueness of the spatial association path between spatial objects obtained by our proposed method.

As an illustration, Figure 1 depicts spatial association paths between two locations that satisfy the preceding three properties. Such paths capture inter-location spatial relatedness in an intuitive and natural way. Each path provides a smooth gradation of attribute values (sandstorm intensity in this case) with the shortest physical length.

Our strategy to find an intrinsic path for the measurement of a spatial relationship is to construct a quantitative measure of the path in the geographical space that appropriately assesses the extent to which the path deviates from the three basic spatial relationship properties. A path between any two spatial points with the smallest deviation from these properties can be employed to measure an inter-point spatial relationship. The starting point of this task is to construct a measure for the direct connection between neighboring spatial points. This becomes the local measure of a spatial relationship. Once the local measure is obtained, we can establish a global measure that integrates all relevant local measures. Then the intrinsic distance between two locations can be obtained along the path that gives the smallest global measure. In what follows, we give a detailed description of these related tasks.

The local measure of a spatial relationship

We construct the local measure of a spatial relationship based on the following observation. If two points are a very short physical distance apart, and their attribute values are significantly different (points A and B in Figure 2(b)), then the indicated path connecting them seriously violates property A(i). This violation implies that the spatial relationship of the two points should be weak despite their nearness in physical distance. In other words, the local measure in terms of attribute distance should give a remarkably high value. In contrast, if the two points have very similar attribute values (points A and B in Figure 2(a)), then the direct path connecting them reflects all three properties of a spatial relationship. This means that the two points are strongly associated, and the local measure in terms of attribute distance should give a very low value. Thus, the local measure to be constructed should severely penalize large differences in attribute values along a path.

Before formulating the local measure, we first undertake a formal analysis of the spatial relationship between two georeferenced locations. Let Ω be the spatial data space. For any $x \in \Omega$, a spatial relationship is composed of two features: the geographical feature (coordinates in space) $x^g \in R^2$ and the attribute feature $x^a \in R$, where $x = (x^g, x^a)^1$. For any pair of data points $x, y \in \Omega$, our aim is to measure the spatial relationship between them.

First, we construct the local measure of a spatial relationship in terms of distance. For any data pair $x, y \in \Omega$, we denote their physical distance as $d^g(x,y) = \|x^g - y^g\|$, and their attribute distance as $d^a(x,y) = |x^a - y^a|$. In this context, physical distance also can be defined on the basis of travel time, cost of separation, social relations, and cognitive distance between the spatial objects, as investigated by Gatrell (1983). The Minkowski metric (Bertazzon and Olson 2009) or L_p-norms (Josselin and Ladiray 2002) also can be adopted to measure this distance.

Integrating these two distances, we define the ε-*spatial-distance* between x and y as

$$d^\varepsilon(x,y) = \begin{cases} e^{cd^a(x,y)} - 1 + cd^g(x,y) & \text{, if } d^g(x,y) < \varepsilon, \\ \infty & \text{, otherwise,} \end{cases} \tag{1}$$

where c is a positive constant for penalizing significant variations in attribute values between x and y, and ε specifies the neighborhood.

The ε-spatial-distance $d^\varepsilon(x,y)$ constructs a local measure of a spatial relationship between neighboring points along a path by simultaneously accounting for their physical and attribute nearness. This measure penalizes sudden changes in attribute values. Moreover, it fully adheres to the preceding three properties of a rational measure of a local spatial relationship.

Specifically, for situations in which the attribute distance $d^a(x,y)$ between the ε-neighboring x and y is short, $d^\varepsilon(x,y)$ can be approximated by $c(d^a(x,y) + d^g(x,y))$ because $e^{cd^a(x,y)} - 1 \approx cd^a(x,y)$. In this case, a local measure $d^\varepsilon(x,y)$ corresponds to the simple summation of the geographical and attribute distances between x and y with a scale parameter c, and yields a relatively small value.

For situations in which the attribute distance $d^a(x,y)$ is large while $d^g(x,y)$ is small, $d^\varepsilon(x,y)$ imposes an exponential penalty upon abrupt changes in attribute values between x and y. Due to the well-known property of the exponential function $e^{cd^a(x,y)}$, a larger difference in attribute values $d^a(x,y)$ leads to a much greater ε-spatial-distance $d^\varepsilon(x,y)$; i.e., the penalty is more severe. In this case, the local measure $d^\varepsilon(x,y)$

imposes a substantial penalty on attribute variation between x and y, and thus gives a much larger value.

According to the preceding analysis, $d^\varepsilon(x,y)$ is a reasonable measure of a local spatial relationship along the connection path between points x and y. The smaller the value of $d^\varepsilon(x,y)$, the stronger the spatial relatedness between two locally neighboring points, and vice versa.

Based on the local measure given by equation (1), we can construct a measure of a global spatial relationship between any two locations along a path in space.

The global measure of a spatial relationship

A natural way of evaluating the global measure along a path (a smooth curve) connecting two locations is to partition the curve into local connections between neighboring points residing on the path before integrating all local measures between such neighboring points to form a spatial relationship measure of the entire path. Because each local measure reflects the extent of deviation from basic spatial relationship properties along the corresponding local path, as explained in the previous section, the global measure so obtained naturally represents the extent to which the entire path deviates from the basic spatial relationship properties.

To facilitate construction of a global measure of a spatial relationship, we first define a specific path. A sequence $(x_0, \cdots, x_n)$ is called an ε-*spatial-path* between x and y in Ω if $x_0 = x$, $x_n = y$, $x_i \in \Omega (i = 1, \cdots, n-1)$ and $d^g(x_i, x_{i-1}) < \varepsilon (i = 1, \cdots, n)$ are satisfied. All ε-spatial-paths between x and y in Ω constitute the ε-*spatial-path-set* $\Gamma_\varepsilon(x,y)$.

A global spatial relationship measure for a path $(x_0, \cdots, x_n) \in \Gamma_\varepsilon(x,y)$ can be constructed as

$$\sum_{i=1}^{n} d^\varepsilon(x_i, x_{i-1}).$$

With respect to the two cases referred to in the discussion of the ε-spatial-distance, we justify this global measure as follows. When variations in attribute values along the ε-spatial-path $(x_0, \cdots, x_n)$ are gradual, we have

$$\sum_{i=1}^{n} d^\varepsilon(x_i, x_{i-1}) \approx \sum_{i=1}^{n} c(d^a(x_i, x_{i-1}) + d^g(x_i, x_{i-1})). \tag{2}$$

In this case, Equation (2) shows that $\sum_{i=1}^{n} d^\varepsilon(x_i, x_{i-1})$ is the sum (with a scale transformation) of all variations in both the geographical and attribute space along the ε-spatial-path. If the global measure of a path is a

large value, then either the degree of attribute variation is large or the path is physically long. This means the path deviates from property A(ii) or A(iii), and the global spatial relationship should be weak. In contrast, if the global measure is a small value, then both the degree of attribute variation and the physical length of the path are small. That is, the path satisfies properties A(i), A(ii) and A(iii), and the global spatial relationship should be strong.

For some spatial points $\{x_i\}_{i \in I}$ along the ε-spatial-path $(x_0, \cdots, x_n)$ with attribute values that differ greatly in their ε neighborhoods, if we denote the variation in attribute values at x_i $(i \in I)$ as $\Delta_i = d^a(x_i, x_{i-1})$, then we have

$$\sum_{i=1}^{n} d^{\varepsilon}(x_i, x_{i-1}) \approx \sum_{i=1}^{n} cd^g(x_i, x_{i-1}) + \sum_{\substack{i=1 \\ (i \notin I)}}^{n} cd^a(x_i, x_{i-1}) + \sum_{i \in I} (e^{c\Delta_i} - 1)). \tag{3}$$

Equation (3) shows that $\sum_{i=1}^{n} d^{\varepsilon}(x_i, x_{i-1})$ corresponds to the sum of the curve length of the spatial path, the variations of the attribute values along the path, and the specific exponential penalty for each abrupt change in attribute values along the path. Specifically, as the magnitude of this change increases, the severity of the penalty increases. This case seriously violates the spatial relationship property A(i). Because there is an exponential penalty term $\sum_{i \in I} (e^{c\Delta_i} - 1))$ in the global measure, it always has a large value. Gradual attribute variation (i.e., $\sum_{\substack{i=1 \\ (i \notin I)}}^{n} cd^a(x_i, x_{i-1})$) and curve length (i.e., $\sum_{i=1}^{n} cd^g(x_i, x_{i-1})$) still play a smaller but non-negligible role in this global measure. That is, this measure also reflects the extent to which the path deviates from properties A(ii) and A(iii). Therefore, the presented global measure still faithfully reflects the extent to which a path deviates from the three spatial relationship properties.

Based on the aforementioned properties of $\sum_{i=1}^{n} d^{\varepsilon}(x_i, x_{i-1})$, Equation (3) gives a reasonable spatial relationship measure of the ε-spatial-path $(x_0, \cdots, x_n) \in \Gamma_{\varepsilon}(x, y)$ between x and y. Given that the ε-spatial-path is not unique, our final task is to choose a path that reflects the maximum spatial relatedness between x and y. We propose the concept of intrinsic spatial distance to achieve this goal.

Intrinsic spatial distance

Based on the preceding formulation of a global spatial relationship measure of a path in geographical space, the path along which a pair of spatial points is intrinsically associated naturally should be the one with the smallest global measure; i.e., the smallest deviation from the three basic spatial relationship properties.

Mathematically, the spatial relationship of any two points $x, y \in \Omega$ can be quantitatively measured by the following concept of "intrinsic spatial distance" (ISD):

$$d^{ISD}(x,y) = \min_{(x_0, \cdots, x_n) \in \Gamma_\varepsilon(x,y)} \sum_{i=1}^{n} d^\varepsilon(x_i, x_{i-1}). \tag{4}$$

We call the path along which $d^{ISD}(x,y)$ is attained the shortest spatial path (SSP) between x and y. The terms involved in equation (4) are graphically depicted in Figure 3 for ease of interpretation. Here, the attribute value of a point in the figure is indicated by its grey level. Γ_1, Γ_2, Γ_3 and Γ_4 are four ε-spatial-paths in $\Gamma_\varepsilon(A, B)$. Γ_4 represents the SSP along which the minimal value of $\sum_{i=1}^{n} d^\varepsilon(x_i, x_{i-1})$ is attained. The value of $\sum_{i=1}^{n} d^\varepsilon(x_i, x_{i-1})$ corresponds to the spatial relationship intrinsic to the geographical and attribute distances between A and B; i.e., $d^{ISD}(A, B)$. Correspondingly, the SSP between x and y intrinsically reveals the underlying path along which the two data points are spatially related through a series of interconnected relationships.

The smaller the ISD, the stronger the relationship between two points, and vice versa. Interestingly, by virtue of the ISD, Tobler's well-known law can be more precisely restated as: "Everything is related to everything else by the inter-point ISD, but things with a smaller ISD value are more strongly related than things with a larger ISD value." This, however, should not be misunderstood to mean "near things are more related to each other than distant things," because things near each other in geographical space might differ significantly in attribute space, resulting in large value of $d^{ISD}(x,y)$. The case depicted in Figure 2(b) is a typical example of this. Conceptually, we call the ISD a "distance." However, does the ISD satisfy the classical concept of distance in metric space? The answer is positive, with the corresponding theorem being as follows:

Theorem 1 *The ISD function $d^{ISD}(x,y)$ is a distance metric on the spatial data space Ω.*

(The proof is given in the Appendix.)

To make the concept operational, we need a method to calculate the ISD given a limited number of data points. The ISD calibration algorithm is formulated in the next section.

The algorithm for calculating the ISD

Given a spatial data set $X = \{x_i = (x_i^g, x_i^a)\}_{i=1}^l$, our aim is to obtain the ISD $d^{ISD}(x_i, x_j)$ $(1 \leq i, j \leq l)$ as defined by equation (4). By specifying the spatial data space Ω as the data set X, the ISD between any x_i and x_j $(1 \leq i, j \leq l)$ can be calculated directly with equation (4). Graph theory tools may be employed to achieve this task. In particular, by connecting the ε-NN neighbors[2] of all vertices X, a graph superimposed on the spatial data set X can be constructed. Thus, each path on this graph corresponds to an ε-spatial-path of the data. By weighing each ε-NN edge between x_i and x_j as the ε-spatial-distance $e^{cd^a(x_i, x_j)} - 1 + cd^g(x_i, x_j)$ previously defined, the length of an ε-spatial-path corresponds to the global spatial relationship measure for the path. By virtue of some standard shortest-path algorithms employed in graph theory, such as the well-known ones by Dijkstra (Dijkstra 1959) and Floyd (Floyd 1962; Silva and Tenenbaum 2003), the ISD can be computed efficiently.

The ISD algorithm

Step I *(Initialization)***:** Preset the constant $c > 0$; the neighborhood size ε (a positive real number).

Step II *(Data normalization)***:** Normalize the geographical and attribute vectors in X_l respectively as:
$x_i^g = x_i^g / (\max_{1 \leq j \leq l} \|x_j^g\|)$, $x_i^a = x_i^a / (\max_{1 \leq j \leq l} \|x_j^a\|)$, $i = 1, 2, \cdots, l$.

Step III *(Neighborhood graph construction)***:** Construct ε-NN graph $G = (V, E)$ superimposed on the spatial data set X, where the vertex set V corresponds to all data in X and the edge set E contains the ε-NN edges of all vertices. The weight for each edge in E between x_i and x_j is set as $e^{cd^a(x_i, x_j)} - 1 + cd^g(x_i, x_j)$.

Step IV *(Shortest path calculation)***:** Calculate the SSP $\Gamma_{i,j}$ between any data pair x_i and x_j in the graph G, and record the length of the calculated path as $d_{i,j}$.

Step V *(Exportation)***:** Output the ISD matrix $D = \{d_{i,j}\}_{l \times l}$, where $d_{i,j}$ is the estimated ISD between x_i and x_j and the shortest spatial path set $\Lambda = \{\Gamma_{i,j}\}_{1 \leq i,j \leq l}$.

In the algorithm, data normalization in Step II puts the geographical and attribute feature vectors on a similar scale so the same parameter c can be employed to measure both geographical and attribute features. Step III approximates the ε-spatial-distance $d^\varepsilon(x_i, x_j)$, as defined by equation (1), for all ε-NN data pairs. Step IV calculates the ISD value $d^{ISD}(x_i, x_j)$ as defined by equation (4).

The output of the ISD algorithm includes the estimated ISD matrix D and the SSP set Λ between all spatial data pairs in X. The results so obtained are of specific significance in the analysis of spatial

relationships. First, the ISD can be employed to describe a spatial relationship between any pair of spatial data in a much more elaborate way than currently available spatial relationship measures do, and, second, the path $\Gamma_{i,j}$ tends to reveal intrinsically the transition of relatedness between any two data points, and explicitly depicts how they are associated in both geographical and attribute space. We substantiate our theoretical arguments in the next section with simulation experiment results and real-life applications.

Simulation experiment results and real-life applications

To verify the effectiveness of the ISD method, and to demonstrate the ability of the ISD measure to evaluate spatial relationships (in the single variable cases), we employ four sets of spatial data: a synthetic data set, two climatic data sets (mean daily cloud coverage, and mean daily humidity), and a gross domestic product (GDP) data set. We utilize three classical measures of spatial autocorrelation for comparison: Moran's I (Moran 1950), a widely adopted global measure of spatial autocorrelation; local Moran's I (Anselin 1995), a widely used local measure of spatial autocorrelation; and the semivariogram (Cressie 1993; Wackernagel 2003), a common distance-based measure of spatial autocorrelation. All programs are implemented in Matlab 7.0.

Simulation results based on synthetic spatial data

The synthetic spatial data set consists of 1,000 spatial points $\{(x_i^g, x_i^a)\}_{i=1}^{1000}$, where x_i^g and x_i^a denote the location and attribute of the i-th point, respectively. Specifically, the two-dimensional (2-D) location $x_i^g = (x_1, x_2)$ of each point is randomly generated within the rectangle area $[-1.4, 1.4] \times [-1.4, 1.4]$, and the value of its attribute is obtained with $x_i^a = e^{(-2(x_1^2 + x_2^2 - 1)^2)}$, as portrayed by Figure 4(b). The distribution manifold of the data set with respect to location and attribute values is depicted by Figure 4(a).

The spatial autocorrelation information obtained by local Moran's I, Moran's I, and semivariogram measures are portrayed by Figures 4(d), (c) and (e), respectively. The ISD outputs corresponding to two randomly selected data points are depicted in Figures 5(b) and (c), respectively (the background spatial distribution is obtained by the kernel interpolation method (Nadaraya 1964) for all experiments). For each data point, we can obtain the corresponding ISD exhibiting its degree of intrinsic spatial relatedness to all other points of its parent data set. The transitional paths between four pairs of randomly selected data points

also are shown in Figure 5(a). The transitional path meticulously reveals how the two points are intrinsically related in both geographical and attribute space.

Figure 4 shows that Moran's I only gives a unique positive value for the spatial autocorrelation of the entire data set, indicating a strong degree of spatial relatedness among neighbors on a global scale. This measure is too coarse to reflect the fine details of spatial relationships within a data set. Local Moran's I, in contrast, better reveals the spatial autocorrelation around each point on a local scale. Neighboring points located on the ridge and at the bottom of the attribute manifold tend to be positively autocorrelated, whereas those on the inclines of the manifold tend to be negatively autocorrelated. Although this local measure is clearly a more detailed gauge of a spatial relationship, it only reveals relatedness isotropically in the neighborhood within a specific radius around each point. This spatial relationship yardstick again is predisposed to the first law, and does not measure spatial relatedness between any two points of the entire data set elaborately and globally, particularly those that defy the maxim "Near things are more related than distant things." To a certain extent, the semivariogram measure is also a global spatial autocorrelation measure. It measures spatial autocorrelation as a function of the distances between all known points (see Figure 4(e)). The limitation of the semivariogram measure is that its computation generally hinges on the assumptions of stationarity (mean and variance are not a function of location) and isotropy (no directional trends in the data) among spatial observations. Furthermore, it does not describe in detail the spatial relationship between any two locations.

The ISD measure capitalizes on two main aspects. First, the ISD measure can be quantitatively evaluated between each point and any other point. Thus the ISD reflects global spatial relationship information between all data points in an extremely elaborate way, and the evaluation it provides not only is not predisposed to the first law, but also does not require any spatial data assumptions such as stationarity and isotropy. This advantage of the ISD measure can easily be observed in Figures 5(b) and (c). In particular, a point located on the ridge of the underlying distribution manifold of the attribute (Figure 5(b)) is more related to points along the ridge according to the ISD measure, but is less related to points at the bottom of the ridge in terms of the ISD measure, although it is nearer in geographic space to many of the latter. Furthermore, the point located at the bottom of the inner part surrounded by the ridge of the manifold (Figure 5(c)) is more related to points located at the bottom part according to the ISD measure, while it is less related to those outside this part. The results agree completely with our intuitive understanding of a spatial relationship. The

proposed ISD measure quantitatively addresses the qualitative statement of the first law that "Everything is related to everything else," and precisely evaluates the extent to which two points are related with varying combinations of nearness in space and attributes.

The second advantage of the ISD measure is that for any pair of points, it can be used to construct a transitional path along which the points are intrinsically related in space. Such a path reveals the intrinsic spatial relationship between the two points, and meticulously reflects how they are spatially related to each other. Figure 5(a) depicts such transitional paths corresponding to the four situations described in the Introduction, respectively. Specifically, the triplet of attribute distance d^a, physical distance d^g, and ISD d^{ISD} for the four pairs of points (connecting the curves 1 to 4, respectively) in Figures 5(a) are $(0.0007, 0.0397, 0.8079)$ (near in both geographical and attribute distances), $(0.2438, 0.0297, 17.1990)$ (near in geographical distance but far in attribute distance), $(0.0126, 0.6918, 36.6081)$ (far in geographical distance and near in attribute distance), and $(0.7010, 0.4432, 52.2622)$ (far in both geographical and attribute distances), respectively. Although both point pairs connecting the curves 1 and 2 are near in geographic space, the latter pair has a much larger ISD value because of the significant difference in their attribute values. Despite the spatial distance between the points connecting the curve 3 being longer than that connecting the curve 4, the ISD for the former pair is smaller due to its smoother attribute variation along the corresponding ISD path, as can be observed in the figure. The points lying on the ridge of the manifold (residing on the curve 3) are spatially related along the ISD path on the ridge, but are not related along the shortest connection in space. These results provide a good level of agreement between reality and our intuition, and thus verify the validity supporting the conceptual arguments of the ISD evaluation method.

Climate data applications

To further demonstrate the appropriateness of the ISD-estimation algorithm, we also apply it to study spatial relationships in terms of daily cloud coverage and daily humidity data recorded at 641 stations in China from 1990 to 1999. The two attribute data sets are derived from the data files of the Key Laboratory of Regional Climate-Environment Research for Temperate East Asia, Institute of Atmospheric Physics, Chinese Academy of Sciences. Each station is identified by its latitude and longitude as depicted in Figures 6(a) and 7(a), respectively, and each attribute is measured in terms of its mean value. As a comparison,

Moran's I, local Moran's I, and the semivariogram also are calculated for both data sets to measure spatial autocorrelation.

Figures 6(b) and 7(b) show the distributions of local Moran's I with respect to cloud coverage and humidity, respectively. The gray lines across the bars in Figures 6(c) and 7(c) indicate the global Moran's I of the two respective data sets, indicating strong global spatial autocorrelation of the two phenomena. Local Moran's I, however, gives further information concerning local variations of cloud coverage and humidity. Daily cloud coverage tends to be more autocorrelated in the northeast and southeast regions than in other areas of China, and daily humidity tends to be more autocorrelated in the northwest and southeast parts of China than in other places. Figures 6(d) and 7(d) depict the semivariogram curves, showing the global tendency of inter-point spatial relatedness for different distances.

By applying the ISD measure, we can obtain more complete information concerning a spatial relationship between any two locations. By way of illustration, we show the spatial relationship of a chosen point and all other points obtained by the ISD measure in both cases. The Mianyang station located west of the Sichuan Basin is chosen as the reference station, and the ISD results are shown in Figures 8(a), (b) and (c). These figures show that Mianyang is strongly related to places within the same basin, despite some being far away, but is weakly related to those to its west (i.e., the Tibetan Plateau), even though some are very near geographically. This result is as expected, because regions in the Sichuan Basin have a subtropical climate, and hence tend to have greater cloud cover, whereas those on the Tibetan Plateau have a high altitude climate, and hence lack cloud cover. These interesting relationships are fully reflected by the ISD results, but are not revealed by the other spatial autocorrelation methods. With respect to daily humidity, Figure 9(a) depicts spatial associations between the Henan station in Qinghai province in the northeastern part of the Tibetan Plateau and other stations. Figure 9(b) and (c) indicate that this station has stronger spatial relationships with places on the plateau, and weaker spatial relationships with places at lower elevations to its east. This outcome is due to the different climatic regions within which the stations are located. Places at higher elevations around the Henan station are in the arid region of northern China, whereas those at lower elevations east of the station are in the monsoon region of east China. These two regions have distinct humidity conditions, meaning places within the same region tend to be more closely related. The ISD outputs in both experiments reveal a high degree of spatial heterogeneity in the data sets and their embedded spatial autocorrelation.

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Our ISD-evaluation algorithm also was applied to both data sets to obtain inter-station transitional paths with respect to daily cloud coverage and humidity. In terms of cloud coverage, two stations (Huili and Yuanping) are randomly selected from the entire data set to obtain the transitional path connecting them, as shown in Figure 10(a). The path passes though the two stations at comparatively high elevations, while it just bypasses those at relatively low elevations. This outcome agrees with the tendency for stations at high elevations to be more closely related with each other because of their similarity in cloud coverage caused by topographical conditions. Thus, the transitional path reveals how the two stations are intrinsically related in attribute and geographical space. We also identify the transitional path between two arbitrarily chosen stations (Huaiyin and Shipu) in terms of humidity in Figure 10(b). This figure shows that the path connects the two stations along the East China seashore, meticulously circumventing the inland stations. This outcome matches the general pattern of humidity along the seashore being higher than that inland. These results further demonstrate the discriminating power of the ISD evaluation method in the study of complex spatial relationship.

Application to the study of spatial relationships for gross domestic product

The data set for this case study is drawn from the 2009 Statistical Yearbook of Chinese cities, and contains figures for the per capita gross domestic product (GDP) of 287 Chinese cities. Each city is identified by its latitude and longitude (Figure 11(a)). The ISD measure, Moran's I, local Moran's I, and the semivariogram are employed to measure spatial autocorrelation/relationships in terms of GDP.

Figure 11 depicts results obtained by applying the three spatial relationship measures currently in use. Moran's I detects positive global spatial autocorrelation among the cities (Figure 11(c)). Local Moran's I gives more local information, especially by revealing stronger autocorrelation in the Yangtze River Delta and the Pearl River Delta (Figure 11(b)). The semivariogram plot (Figure 11(d)) depicts the range of spatial autocorrelation.

In comparison, our ISD measure provides the spatial relationship and transitional path between any two cities, giving more detailed spatial association information than the other measures. For example, Figure 12(b) depicts spatial relationships between Xiamen and all other cities with respect to GDP. The per capita GDP of Xiamen tends to be more closely related to that of places along the seashore, from the Taiwan Strait

to the South China Sea. It reveals that GDP figures are more strongly related among coastal cities than among inland cities because of their economic linkages. Figure 12(a) depicts the transitional path between Baoshan and Jinzhou. This path closely bypasses less-developed areas at higher altitude with a lower GDP, but passes though cities at comparatively low altitudes where GDP is higher.

The preceding empirical results establish that the ISD estimation algorithm precisely measures spatial relationships in terms of attribute and physical distances. It substantially extends our ability to analyze spatial relationships in general.

Conclusion and discussion

In this paper, we develop the concept of intrinsic spatial distance (ISD) and present an associated ISD algorithm for the analysis of a spatial relationship between any two points in geographic space. The ISD is a distance measure that accounts for the nearness of two points with respect to their physical and attribute distances. We argue conceptually that the ISD measure of a spatial relationship naturally reflects how two points are related locally and globally. The transitional path linking two points vividly depicts how they are related in space. Unlike conventional spatial autocorrelation measures with respect to a single variable, the ISD is not predisposed in favor of the first law of geography, but directly accounts for the varying physical and attribute distances to measure the extent to which two points are spatially autocorrelated. We also point out that the information provided by ISD is more elaborate than that revealed by Moran's I, local Moran's I, and the semivariogram. Because ISD measures the relationship between any two points, the spatial relatedness of any single point with all other points over space can be evaluated. The distance matrix thus derived shows the intrinsic spatial relations between points. Our conceptual arguments are substantiated and validated by a series of experiments based on synthetic and real-world data sets.

As a point of interest, we now discuss the relationship between the ISD path – the shortest path in both physical and attribute space – and the least/minimum effort path. The ISD path accounts for both nearness in space and nearness in attribute distance. The shortest path connecting two spatial data points is constructed in either the physical or attribute space. This can be clearly observed in all of our experiments (e.g., the ISD paths in the cloud coverage experiment shown in Figure 10). The least effort path is defined as the physical or metaphorical pathway among a set of alternatives that requires the least effort (potential energy) for a

given object or entity to make forward motion. Given that the ISD path guarantees a smooth gradation of attribute values and is likely to be physically short, the gradient, with respect to either physical or attribute values, is always shallow along the ISD path. Because an abrupt change in attribute values along a physical path always implies a significant increase in potential energy (or effort), as can be seen in differential human behaviors in adjacent administrative areas under different political regimes, the ISD path is related to the least effort path in this sense. Thus, the proposed ISD measure might provide a new way of gauging the potential energy discussed by Warntz (1961). This potential application of the proposed ISD measure is worthy of investigation in further research.

In the present analysis, we employ the ISD to study spatial relationships with respect to a single variable. However, the definition in equation (4) is a general definition for the relatedness of two points by reference to a profile of variables. That is, it is also suitable for the analysis of multivariate spatial relationships. This conceptualization represents a natural extension warranting further examination.

Another line of research with potential is an extension of the framework proposed here to 3-dimensional (3-D) space. Such an extension would be very useful when it is necessary to evaluate a spatial relationship over a topographical profile in geographical research in general and in 3-D GIS research in particular. In addition to its application to spatial points, the proposed ISD method should be further extended to other spatial data types, such as curves, surfaces, and polygons.

To make the notion of spatial relationship a complete concept, the approach suggested here also could be extended to the temporal domain. Extending the proposed definition would allow us to construct a distance measure of relationship in space and time. The framework we provide posits that two points are related through a space-time transitional path that simultaneously accounts for the variation of attribute values. This is perhaps both an important issue in space-time integration and a building block for the study of spatio-temporal relationships.

The cartographic transformation method (Tobler 1979) sheds light on the evaluation of spatial relationships. By taking attribute values as elevations of observations, geographical-attribute characteristics can be viewed as an imaginary earth surface and relevant cartographic transformation techniques might be employed to measure spatial relationships. Furthermore, future investigations need to make a thorough comparison between the proposed method and other flexible spatial-weight-matrix-specification approaches, such as the generalized-moments estimation approach (Bell and Bockstael 2000).

The methodological and practical importance of the proposed ISD measure in solving problems and advancing theory in multiple knowledge domains suggests a number of promising research directions in addition to those highlighted here: (1) Instead of using Euclidean distance, the ISD can be employed to extend any distance-based aspatial data mining method to the spatial context; (2) Using the ISD measure in place of a Euclidean-distance-based weight matrix gives a more reliable and informative weight matrix for the analysis of many geographical problems involving the nearness of spatial entities in physical and attribute space, such as in geographically weighted regression; (3) Because the ISD path is the intrinsic least effort path between spatial data points, it can be used to extract intrinsic relations in data and explore the progression of feature relatedness from one place to another; and, (4) Give the status of image data as a specific type of spatial data, the ISD method can be applied to image processing, such as in evaluating the weights of intensity values under the well-known bilateral filter method.

Appendix: Proof of Theorem 1

Theorem 1 *The ISD function ($d^{ISD}(x,y)$) is a distance metric on the spatial data space Ω.*

Proof: The ISD function satisfies the conditions of a distance metric:

(i) ***Positive definiteness***: $d^{ISD}(x,y) \geq 0$ for any $x, y \in \Omega$, and $d^{ISD}(x,y) = 0$ if and only if $x = y$.

Because

$$d^{\varepsilon}(x,y) = \begin{cases} e^{cd^a(x,y)} - 1 + cd^g(x,y) & , \text{if } d^g(x,y) < \varepsilon, \\ \infty & , \text{otherwise}, \end{cases}$$

and

$$d^{ISD}(x,y) = \min_{(x_0,\cdots,x_n)\in\Gamma_\varepsilon(x,y)} \sum_{i=1}^{n} d^{\varepsilon}(x_i, x_{i-1}),$$

it is evident that $d^{ISD}(x,y) \geq 0$ for any $x, y \in \Omega$.

If $d^{ISD}(x,y) = 0$, then for each x_i along the shortest spatial path between x and y, $d^{\varepsilon}(x_i, x_{i-1}) = 0$. Therefore, $d^a(x_i, x_{i-1}) = d^g(x_i, x_{i-1}) = 0$ holds. We then have $x_i = x_{i-1}$ for any $i = 1, 2, \cdots, n$. Thus, it is apparent that $x = x_0 = x_1 = \cdots = x_{n-1} = x_n = y$. The positive definiteness property of $d^{ISD}(x,y)$ is then verified.

(ii) ***Symmetry***: $d^{ISD}(x,y) = d^{ISD}(y,x)$ for any $x,y \in \Omega$.

Because $(x_0,\cdots,x_n) \in \Gamma_\varepsilon(x,y)$ implies that $(x_n,\cdots,x_0) \in \Gamma_\varepsilon(y,x)$, it is easy to obtain this property based on the definition of the ISD.

(iii) ***Triangle inequality***: $d^{ISD}(x,z) \le d^{ISD}(x,y) + d^{ISD}(y,z)$ for any $x,y,z \in \Omega$.

Let the shortest spatial paths between x and y and y and z be $(x_0,\cdots,x_n)$ and $(\bar{x}_0,\cdots,\bar{x}_{\bar{n}})$, respectively. Based on the definitions of the SSP and the ISD, $x_0 = x$, $x_n = \bar{x}_0 = y$, $\bar{x}_{\bar{n}} = z$, and along the two paths, the respective ISD values $d^{ISD}(x,y)$ and $d^{ISD}(y,z)$ can be calculated precisely.

Construct a new path as $x_0,\cdots,x_n,\bar{x}_1,\cdots,\bar{x}_{\bar{n}}$ connecting points x and z. The value of the global measure along this path is:

$$\sum_{i=1}^{n} d^\varepsilon(x_i,x_{i-1}) + \sum_{i=1}^{\bar{n}} d^\varepsilon(\bar{x}_i,\bar{x}_{i-1}) = d^{ISD}(x,y) + d^{ISD}(y,z).$$

Thus,

$$d^{ISD}(x,z) = \min_{(x_0,\cdots,x_n) \in \Gamma_\varepsilon(x,z)} \sum_{i=1}^{n} d^\varepsilon(x_i,x_{i-1}) \le d^{ISD}(x,y) + d^{ISD}(y,z).$$

Therefore, the triangle inequality property of the ISD measure is satisfied.

This completes the proof of the theorem.

Acknowledgement

Notes

[1]In this paper, we mainly focus on single attribute cases, while throughout the paper, all our analysis can be easily generalized to multiple attribute cases by setting $x^a \in R^d$, where d is the number of variables involved in a problem.

[2]ε-NN defines the neighbors of a vertex as those with distances from the vertex smaller than the threshold ε (Roweis and Saul 2000; Tenenbaum et al. 2000).

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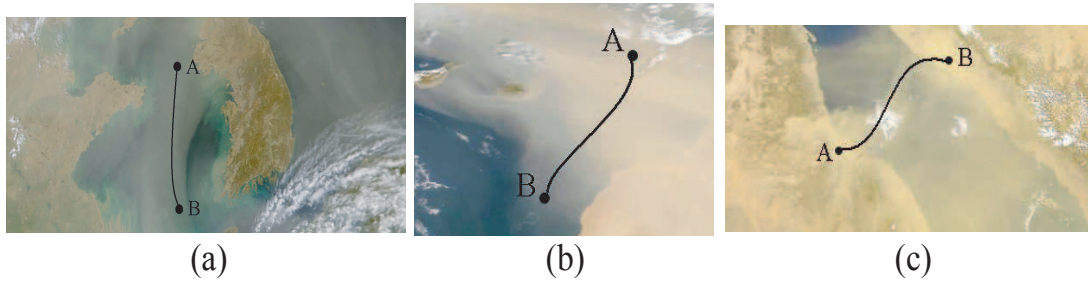
Figure 1. Sandstorm maps downloaded from "http://www.cpus.gov.cn/ZLG/shachenbao/img". In each sub-figure, the curve depicts the path between points possessing the properties A(i), A(ii) and A(iii).
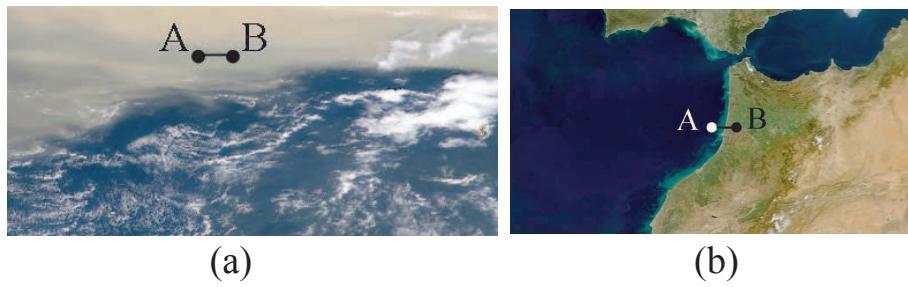
Figure 2. Two sandstorm maps downloaded from "http://www.cpus.gov.cn/ZLG/shachenbao/img". (a) Neighboring locations with highly similar attribute values; (b) Neighboring locations with significantly different attribute values.
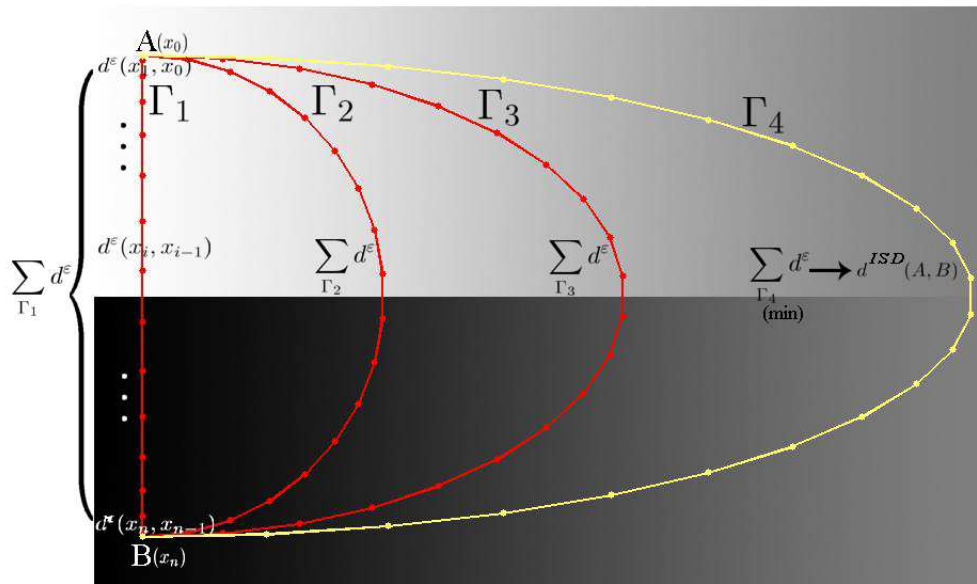
Figure 3. An example of the ISD concept defined by equation (4). Red lines indicate ε-spatial-paths between A and B, and the yellow line corresponds to the ISD path between the two points.
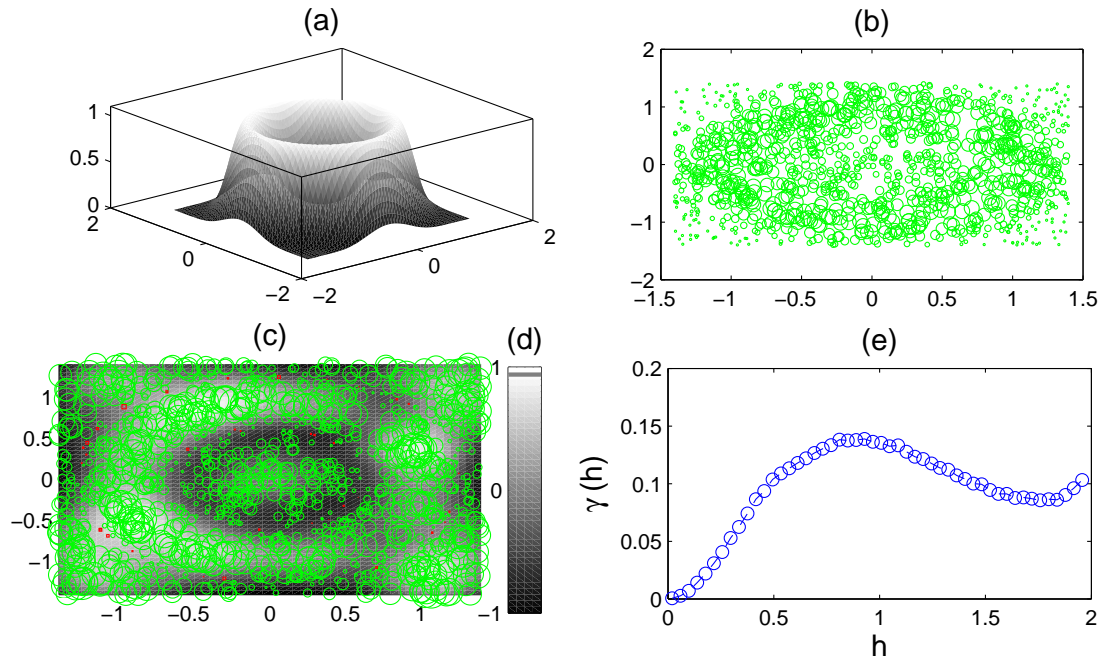
Figure 4.   (a) The distribution manifold (function) of an attribute. (b) Distribution of $1,000$ synthetic points. The circles are locations of the points in the coordinated space. Each point is represented by a circle with a radius drawn in proportion to the value of its attribute. (c) Distribution of local Moran's I (LISA) with radii drawn in proportion to LISA values. The background gray values are obtained by applying the kernel interpolation method to LISA values. Larger LISA values are indicated by a lighter gray value. (d) Global Moran's I, the gray line across the bar, for the entire data set. (e) The semivariogram of the data with respect to different neighborhood distances.
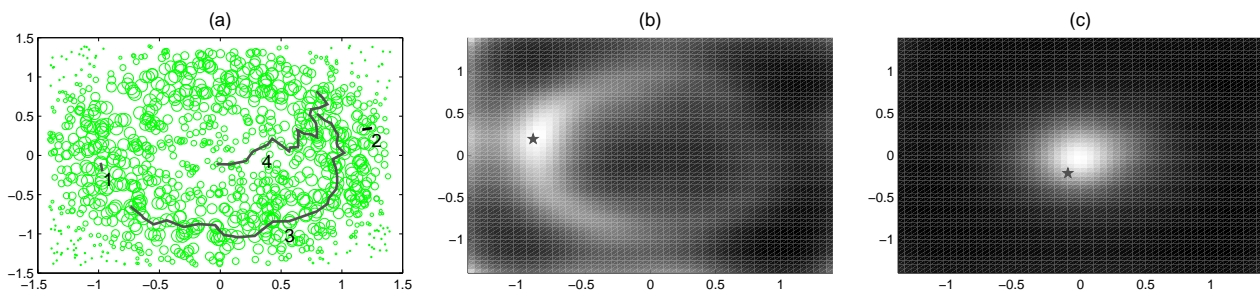
Figure 5. (a): The transitional paths (denoted as 1, 2, 3, and 4) between four pairs of points of the synthetic data set obtained by the ISD estimation algorithm, corresponding to the four situations described in the Introduction, respectively. The circles are locations of data points, with each point being represented by a circle with a radius drawn in proportion to its attribute value. (b)(c): The ISD outputs of two randomly selected spatial points (the stars) in the synthetic data set. The background gray scale is obtained by applying the kernel interpolation method to the ISD values of these points. The brighter the spatial location, the more related the corresponding point is to the selected point.
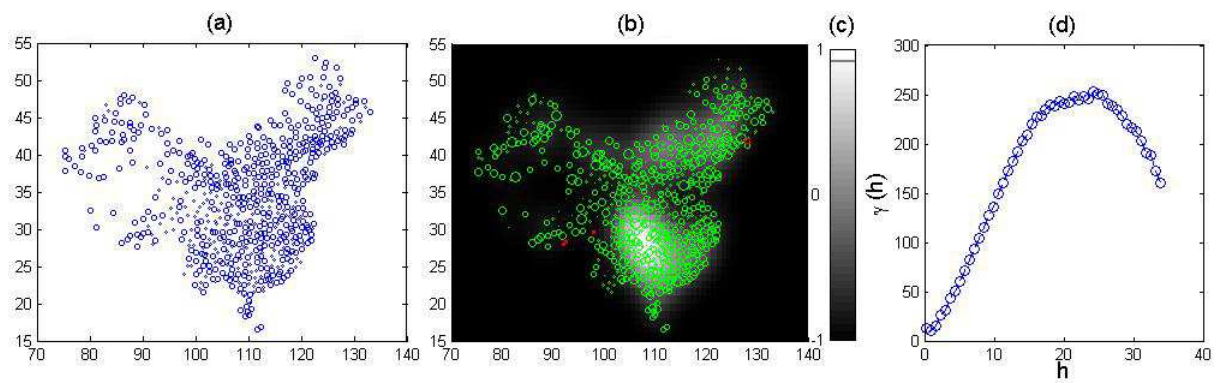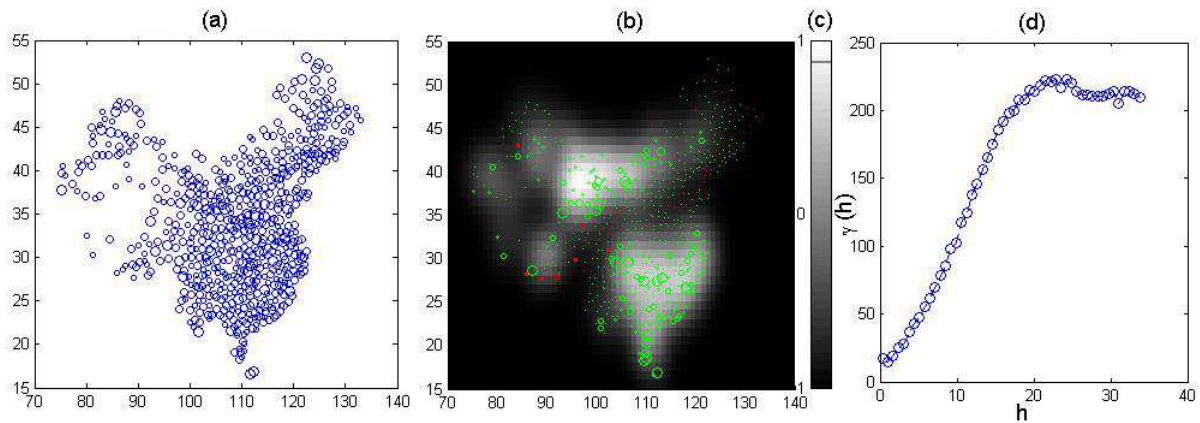
Figure 6. (a) The spatial distribution of mean daily cloud coverage observations from 641 stations with the circle radius of each point scaled to its attribute value. (b) The spatial distribution of local Moran's I (LISA) for each of the 641 stations. The circles represent the locations of the points and the circle radius of each point is scaled to its LISA value. The background gray scale is obtained by applying the kernel interpolation method to the LISA values of these points. (c) The gray line across the bar is the global Moran's I measure of the entire data set. (d) The semivariogram plot for the data.

Figure 7. (a) The spatial distribution of mean daily humidity observations from 641 stations with the circle radius of each point scaled to its attribute value. (b) The spatial distribution of local Moran's I (LISA) for each of the 641 stations. The circles represent the locations of the points, and the circle radius of each point is scaled to its LISA value. The background gray scale is obtained by applying the kernel interpolation method to the LISA values of these points. (c) The gray line across the bar is the global Moran's I measure of the entire data set. (d) The semivariogram plot for the data.
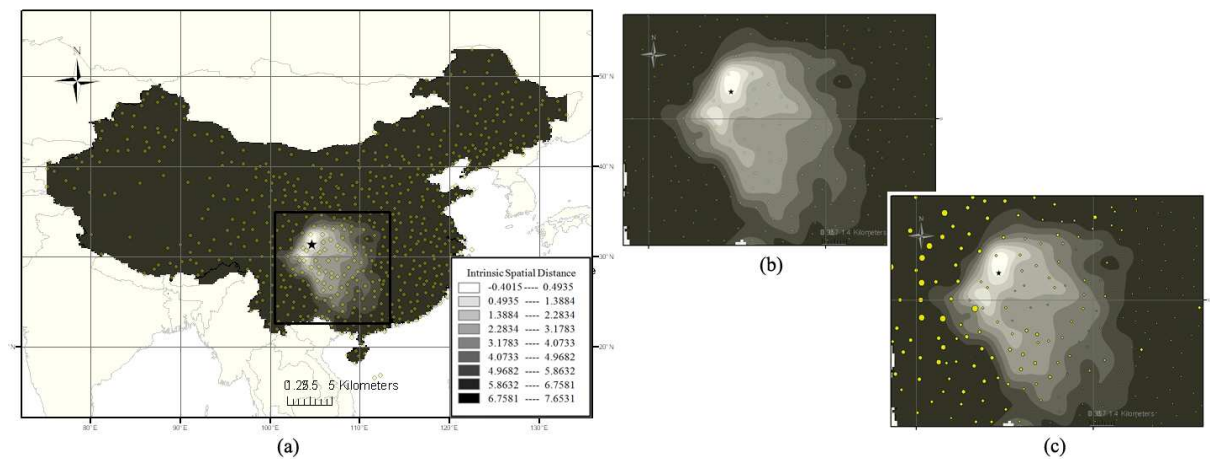
Figure 8. (a) Spatial relatedness as measured by the ISD between the Mianyang station (denoted by the star) and 640 other stations with respect to daily cloud coverage. The background gray scale is obtained by applying the kernel interpolation method to the ISD values of these stations. (b)(c) The demarcated area of (a) on a larger scale. In (c), the circle radius of each station is scaled to its elevation.
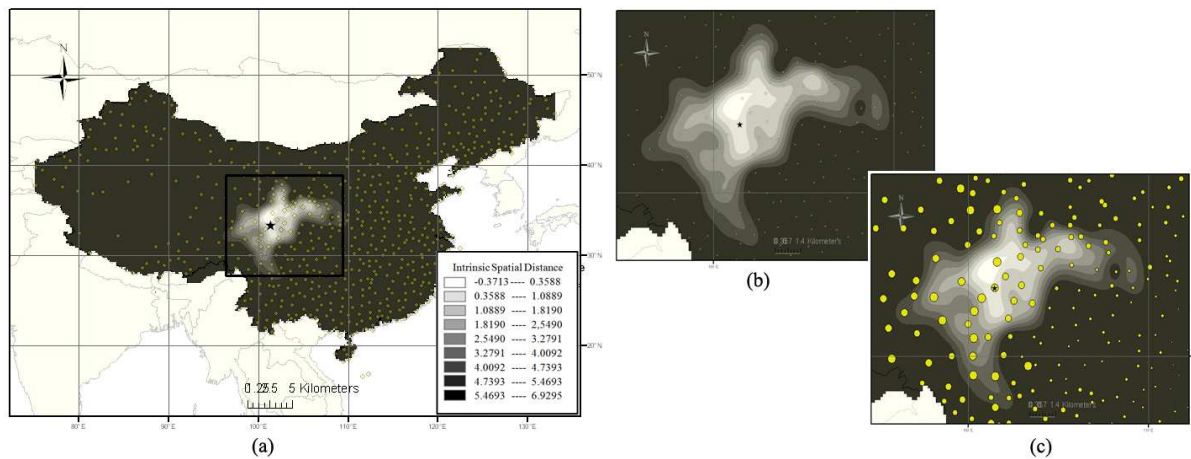
Figure 9. (a) Spatial relatedness as measured by the ISD between the Henan station (denoted by the star) and 640 other stations with respect to daily humidity. The background gray scale is obtained by applying the kernel interpolation method to the ISD values of these stations. (b)(c) The demarcated area of (a) on a larger scale. In (c), the circle radius of each station is scaled to its elevation.
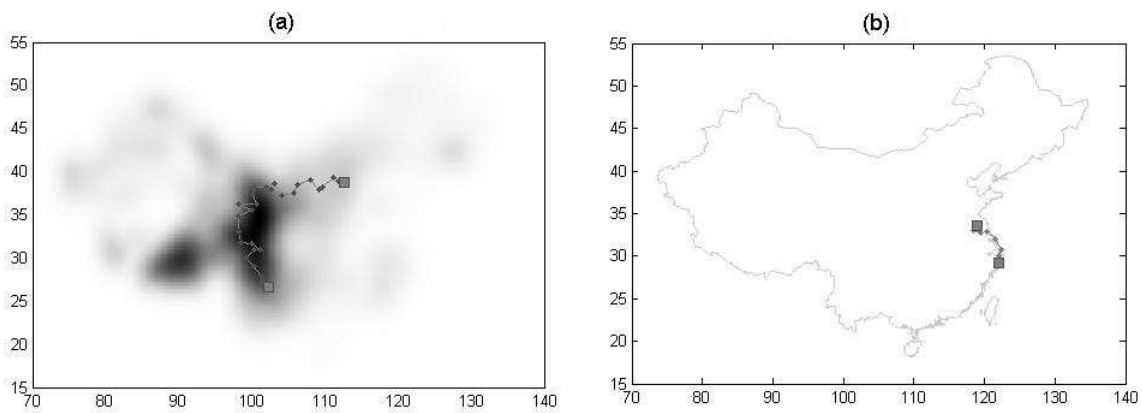
Figure 10. (a) The ISD transitional path, with respect to cloud coverage, between the Huili and Yuanping stations. The stations on the path include Huili, Yanyuan, Muli, Daocheng, Batang, Xinlong, Daofu, Ganzi, Dege, Shiqu, Maduo, Xinghai, Dulan, Qiabuqia, Gangcha, Qilian, Gansu, Yongchang, Minqin, Wuwei, Jingdai, Zhongning, Yinchuan, Etuokeqi, Hengshan, Yulin, Hequ, Wusai and Yuanping. The background gray scale indicates elevation: the brighter the location, the lower the elevation. (b) The ISD transitional path between the Huaiyin and Shipu stations (denoted by the squares) with respect to humidity. The stations on the path include Huanyin, Xuyi, Gaoyou, Dongtai, Lvsi, Chengsi, Dinghai and Shipu.
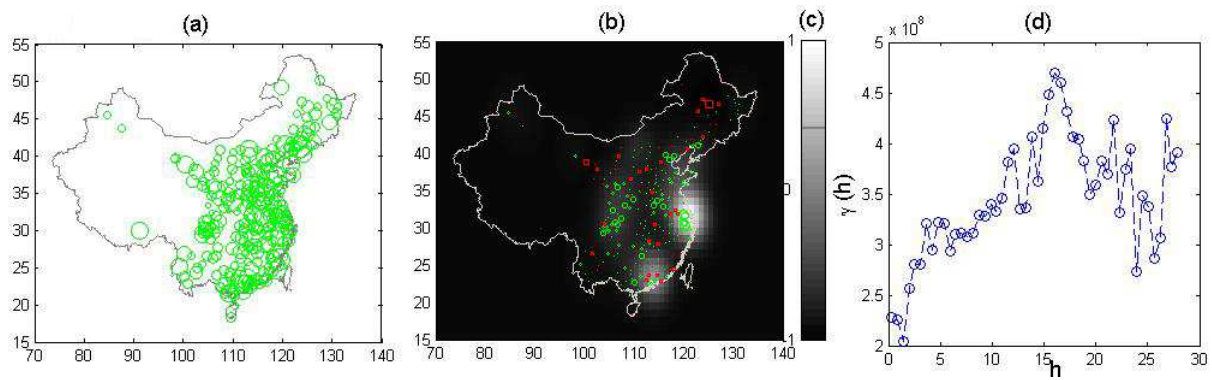
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Figure 11. (a) The spatial distribution of per capita GDP of 287 Chinese cities, with the circle radius of each point scaled to its GDP value. (b) The spatial distribution of the local Moran's I (LISA) for each of the 287 cities. The circles represent the locations of the cities, and the circle radius of each point is scaled to its LISA value. The background gray scale is obtained by applying the kernel interpolation method to the LISA values of these cities. (c) The gray line across the bar indicates the global Moran's I of the entire data set. (d) The semivariogram plot for the data.
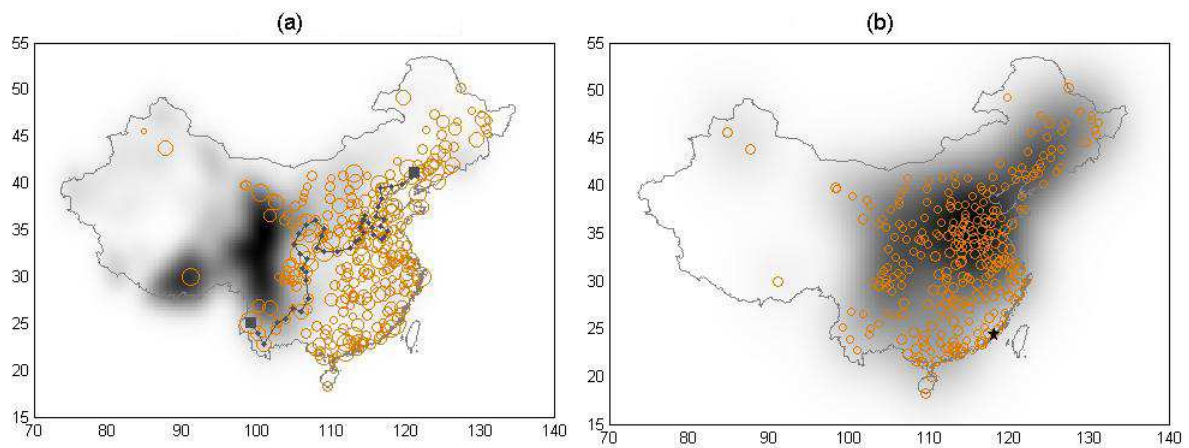
Figure 12. (a) The transitional path, with respect to per capita GDP, between Baoding and Jinzhou (denoted by the squares) obtained by applying the ISD estimation algorithm. The cities along the path include Baoshan, Linchuang, Simao, Kunming, Qujing, Liupanshui, Anshui, Zunyi, Chongqing, Guangan, Nanchong, Bazhong, Guangyuan, Longnan, Tianshui, Pingliang, Qingyang, Tongchuan, Xianyang, Hanzhong, Ankang, Shiyan, Nanyang, Pingdingshan, Xuchang, Kaifeng, Xinxiang, Hebi, Anyang, Puyang, Heze, Shangqiu, Huaibei, Xuzhou Zaozhuang, Jining, Taian, Liaocheng, Dezhou, Cangzhou, Langfang, Tangshan, Qinhuangdao, Huludao and Jinzhou. The gray scale in the background corresponds to the elevation. (b) Spatial relationships measured by ISD between Xiamen and all other cities. The circles represent the locations of the cities, with the radius of each point being scaled to its GDP value. The gray scale in the background is obtained by applying the kernel interpolation method to the ISD values of these cities.