Lecture 02 Dimensionless number

Archimedes number (Ar)	*the ratio of buoyancy and inertial forces*	$Ar = \frac{gL^3\rho_l(\rho - \rho_l)}{\mu^2}$
Atwood number (A)	*instabilities in fluid mixtures due to density differences*	$A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$
Bingham number (Bn)	*Ratio of yield stress to viscous stress*	$Bn = \frac{\tau_y L}{\mu_\infty u_\infty}$
Bond number (Bo)	*a measure of surface tension forces compared to body forces*	$B_o = \frac{\rho g L^2}{\sigma}$
Capillary number (Ca)	*relative effect of viscous forces versus surface tension acting across an interface between between two immiscible liquids*	$Ca = \frac{\mu U}{\sigma}$
Eötvös number (Eo)	*proportional to buoyancy force divided by surface tension force*	$E_o = \frac{\Delta\rho g L^2}{\sigma}$
Froude number (Fr)	*the ratio of the inertial to gravity forces*	$Fr = \frac{U}{\sqrt{gL}}$
Knudsen number (Kn)	*The ratio of the mean free path length of the molecules of a fluid to a characteristic length*	$kn = \frac{\lambda}{L}$
Lewis number (Le)	*the ratio of thermal diffusivity to mass diffusivity*	$Le = \frac{\alpha}{D} = \frac{Sc}{Pr}$
Morton number (Mo)	*ratio of the gravitational acceleration to the molecular acceleration of a fluid*	$Mo = \frac{g\mu^4\Delta\rho}{\rho^2\sigma^3} = \frac{We^3}{Fr^2Re^4}$
Stefan Number (Ste)	*the ratio of sensible heat to latent heat*	$Ste = \frac{c_p\Delta T}{\lambda}$
Stokes number (St)	*the ratio of the characteristic time of a particle (or droplet) to a characteristic time of the flow or of an obstacle*	$St = \frac{\tau U_o}{d_c} = \frac{\rho_d d_d^2}{18\mu_g}$
Weber number (We)	*relative importance of the fluid's inertia compared to its surface tension*	$We = \frac{\rho u^2 L}{\sigma}$

Lecture 03 Macro-to-micro transition criteria

$$Co = \frac{1}{D}\sqrt{\frac{\sigma}{(\rho_l - \rho_g)g}} \quad \begin{cases} < 0.5 & macro\ scale \\ > 0.5 & microscale \end{cases}$$

$$B_o = \frac{\rho a L^2}{\sigma} = \frac{(\rho_l - \rho_g)gd^2}{\sigma} < 4$$

$$E_o = \frac{(\rho_l - \rho_g)gd^2}{8\sigma} < 0.2$$

$$Bo^{0.5} \times Re \leq 160$$

$$d_{crit} = 0.224 L_{cap} \text{ , where } \quad L_{cap} = \left(\frac{\sigma}{(\rho_l - \rho_g)g}\right)^{1/2}$$

Lecture 04 Gas-liquid two-phase flow

void fraction	$\alpha = \frac{\Delta z \int_{A_g} dA}{\int_A \, dA} = \frac{A_g}{A_g + A_l}$
total mass flow rate	$\dot{m} = \dot{m}_g + \dot{m}_l$
total volumetric flow rate	$\dot{Q} = \dot{Q}_g + \dot{Q}_l = \frac{\dot{m}_g}{\rho_g} + \frac{\dot{m}_l}{\rho_l}$
total mass flux	$G = \frac{\dot{m}}{A}$
phase velocity	$\langle \omega_g \rangle = \frac{\dot{Q}_g}{A_g} \quad \langle \omega_l \rangle = \frac{\dot{Q}_l}{A_l}$
superficial velocity	$j_g = \frac{\dot{Q}_g}{A} = \alpha \langle \omega_g \rangle \quad j_l = \frac{\dot{Q}_l}{A} = (1-\alpha)\langle \omega_l \rangle$
volumetric flow fraction	$\beta = \frac{\dot{Q}_g}{\dot{Q}_g + \dot{Q}_l} = \frac{j_g}{j_g + j_l}$
slip ratio	$s = \frac{\langle \omega_g \rangle}{\langle \omega_l \rangle}$
quality (dryness fraction)	$x = \frac{\dot{m}_g}{\dot{m}}$
volumetric quality	$\beta = \frac{\dot{Q}_g}{\dot{Q}} = \frac{\rho_g \dot{m}_g}{\rho_g \dot{m}_g + \rho_l \dot{m}_l}$
superficial mass flux or mass velocity	$G_g = \frac{\dot{m}_g}{A} = \rho_g \frac{\dot{Q}_g}{A} = \rho_g j_g = \rho_g \langle \omega_g \rangle \alpha$ $G_l = \frac{\dot{m}_l}{A} = \rho_l \frac{\dot{Q}_l}{A} = \rho_l j_l = \rho_l \langle \omega_l \rangle (1-\alpha)$
total mass flux	$\dot{m}" = G_g + G_l = \rho_g j_g + \rho_l j_l$
superficial momentum flux	$\rho_g j_g^2 = \rho_g \frac{G_g^2}{\rho_g^2} = \frac{x^2 G^2}{\rho_g}$ $\rho_l j_l^2 = \rho_l \frac{G_l^2}{\rho_l^2} = \frac{(1-x^2) G^2}{\rho_l}$

Lecture 05 Theoretical models for gas-liquid two-phase flow

Bankoff variable density model

$$\frac{u}{u_{CL}} = \left(\frac{y}{R}\right)^{\frac{1}{m}} \quad \frac{\alpha}{\alpha_{CL}} = \left(\frac{y}{R}\right)^{\frac{1}{n}}$$

$$Q_g = \int_0^R \alpha u 2\pi r dr = \int_0^R \alpha_{CL} u_{CL} \left(\frac{y}{R}\right)^{\frac{1}{m}} \left(\frac{y}{R}\right)^{\frac{1}{n}} 2\pi(R-y)dy$$

$$Q_g = \int_0^R \alpha u 2\pi r dr = \int_0^R \alpha_{CL} u_{CL} \left(\frac{y}{R}\right)^{\frac{1}{m}} \left(\frac{y}{R}\right)^{\frac{1}{n}} 2\pi(R-y)dy$$

$$Q = \int_0^R u 2\pi r dr = \frac{2\pi R^2 u_{CL} m^2 n^2}{(1+m)(1+2m)}$$

$$\beta = \frac{\alpha_{CL}(1+m)(1+2m)n^2}{(m+n+mn)(m+n+2mn)}$$

$$\bar{\alpha} = \frac{1}{\pi R^2}\int_0^R \alpha 2\pi r dr = \int_0^R \alpha_{CL} \left(\frac{y}{R}\right)^{\frac{1}{n}} 2\pi r dr = \frac{2\alpha_{CL} n^2}{(1+n)(1+2n)}$$

$$\frac{\alpha}{\beta} = \frac{2(m+n+mn)(m+n+2mn)}{(1+m)(1+2m)(1+n)(1+2n)} = C_A$$

Lockhart-Martinelli model

Lockhart-Martinelli parameter $\chi^2 = \frac{(dp/dz)_{f,l}}{(dp/dz)_{f,g}}$

Multipliers $\phi_l^2 = \frac{(dp/dz)_f}{(dp/dz)_{f,l}}$ $\phi_g^2 = \frac{(dp/dz)_f}{(dp/dz)_{f,g}}$

Flow Condition	X
turbulent-turbulent	$X_{tt}^2 = \left(\frac{1-x}{x}\right)^{1.8}\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right)^{0.2}$
laminar-turbulent	$X_{lt}^2 = \mathrm{Re}_g^{-0.8}\left(\frac{C_l}{C_g}\right)\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right)$
turbulent- laminar	$X_{tl}^2 = \mathrm{Re}_l^{0.8}\left(\frac{C_l}{C_g}\right)\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right)$
laminar-laminar	$X_{ll}^2 = \left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right)$

Turner model $\left(\frac{1}{\phi_l^2}\right)^{\frac{1}{n}} + \left(\frac{1}{\phi_g^2}\right)^{\frac{1}{n}} = 1$

Chisholm model $\Phi^2 = 1 + \frac{C}{X} + \frac{1}{X^2}$ $C = \left[\left(\frac{\rho_l}{\rho_g}\right)^{0.5} + \left(\frac{\rho_l}{\rho_g}\right)^{0.5}\right]$

$$\phi_l^2 = \left(\frac{dp}{dz}\right)_{f,tp} = \left(\frac{dp}{dz}\right)_{f,l} + C\left[\left(\frac{dp}{dz}\right)_{f,l}\left(\frac{dp}{dz}\right)_{f,g}\right]^{0.5} + \left(\frac{dp}{dz}\right)_{f,g}$$

The drift flux model

Drift velocity $u_{gj} = u_g - j$

Volume flux = Conc. × average volume flux + relative velocity correction

$$j_g = \alpha j + j_{gl}$$