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Permanence and stability in non-autonomous predator–prey Lotka–Volterra systems with feedback controls[☆]

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ABSTRACT

The main purpose of this article is considering whether or not the feedback controls have an influence on a non-autonomous predator–prey Lotka–Volterra type system. General criteria on permanence are established, which is described by an integral form and independent of some feedback controls. By constructing suitable Lyapunov functionals, a set of easily verifiable sufficient conditions are derived for the global stability of any positive solution to the model.

1. Introduction

Traditional two species autonomous or non-autonomous predator–prey Lotka–Volterra systems take the form

$$\begin{cases} \dfrac{dx_1(t)}{dt} = x_1(t)\big[b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t)\big], \\ \dfrac{dx_2(t)}{dt} = x_2(t)\big[-b_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t)\big], \end{cases} \tag{1.1}$$

where $x_1(t)$ is the prey population density and $x_2(t)$ is the predator population density, $b_1(t), a_{11}(t)$, the intrinsic growth rate and density-dependent coefficient of the prey, respectively; $b_2(t)$, $a_{22}(t)$, the intrinsic growth rate and density-dependent coefficient of the predator, respectively; $a_{12}(t)$ the capturing rate of the predator and $a_{21}(t)$ the rate of conversion of nutrients into the reproduction of the predator.

In the last decades, system (1.1) has been studied extensively, for example [1–9] and the references therein. Some sufficient conditions are obtained for the permanence, existence and uniqueness, and asymptotic stability of periodic solution for system (1.1).

However, we note that ecosystems in the real world are continuously distributed by unpredictable forces which can result in changes in the biological parameters such as survival rates. In ecology, we know that the practical question of interest is just whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of

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time. In the language of control variables, we call the disturbance functions control variables. Whereas, the control variables discussed in much of the literature are constants or time dependent [10–12].

Recently, many scholars have done works on the ecosystem with feedback controls (see [13–20] and the references cited therein). In particular, Gopalsamy and Weng [21] discussed the asymptotic behavior of solutions in Logistic systems with feedback controls, Weng [22] considered a class of periodic integro-differential systems with feedback controls, Xiao [23] considered a two species competitive system with feedback controls, Chen [24] considered a non-autonomous Lotka–Volterra competitive system with feedback controls. These motivate us to consider the following non-autonomous predator–prey Lotka–Volterra system with feedback controls

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = x_1(t)\big[b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) + c_1(t)u_1(t)\big] \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = x_2(t)\big[-b_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - c_2(t)u_2(t)\big] \\ \dfrac{\mathrm{d}u_1(t)}{\mathrm{d}t} = f_1(t) - e_1(t)u_1(t) - d_1(t)x_1(t) \\ \dfrac{\mathrm{d}u_2(t)}{\mathrm{d}t} = -e_2(t)u_2(t) + d_2(t)x_2(t). \end{cases} \tag{1.2}$$

In this paper, we study whether or not the feedback controls have an influence on the permanence of a positive solution of the general non-autonomous predator–prey Lotka–Volterra type systems, and establish the general criteria on the permanence of system (1.2), which is independent of some feedback controls. In additional, by constructing a suitable Lyapunov function, some sufficient conditions are obtained for the global stability of any positive solution to system (1.2).

This paper is organized as follows. In the next section, two useful lemmas, several basic assumptions for system (1.2) and the definitions of permanence are presented. We state and prove the sufficient conditions on the ultimately bounded and permanence of positive solutions for system (1.2), which is described by integrable form and independent of some feedback controls in Section 3. In the last section, a set of easily verifiable sufficient conditions are derived for the global stability of any positive solution of system (1.2).

2. Preliminaries

Let $R_+ = (0, \infty)$ and $R_{+0} = [0, \infty)$. In this section, we consider the following first order linear differential equations with a parameter

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = g(t, \beta) - d(t)v(t), \tag{2.1}$$

where $g(t, \beta)$ is a continuous function defined on $(t, \beta) \in R_{+0} \times [0, \beta_0]$ and β_0 is a constant, $d(t)$ is a continuous function defined on R_{+0}. For system (2.1) we introduce the following assumptions.

(A_1) Function $g(t, \beta)$ is a non-negative bounded on $R_{+0} \times [0, \beta_0]$ and satisfies the Lipschitz condition with $\beta \in [0, \beta_0]$, i.e., there is a constant $L = L(\beta_0) > 0$ such that $|g(t, \beta_1) - g(t, \beta_2)| \le L|\beta_1 - \beta_2|$ for all $t \in R$, $\beta_1, \beta_2 \in [0, \beta_0]$.
(A_2) Function $d(t)$ is non-negative bounded on R_{+0} and there is a constant $\omega_1 > 0$ such that $\liminf_{t\to\infty} \int_t^{t+\omega_1} d(s)\,\mathrm{d}s > 0$.

From assumptions (A_1) and (A_2), it is easy to proved that for any $(t_0, v_0) \in R_{+0} \times R_+$ and $\beta \in [0, \beta_0]$, system (2.1) has a unique solution $v_\beta(t)$ satisfying $v_\beta(t_0) = v_0$.

In system (2.1), when parameter $\beta = 0$ we obtain the following system

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = g(t, 0) - d(t)v(t). \tag{2.2}$$

Let $v^*_\beta(t)$ be a fixed solution of system (2.1) defined on R_{+0}. We say that $v^*_\beta(t)$ is globally uniformly attractive on R_{+0}, if for any constants $\eta > 1$ and $\varepsilon > 0$ there is a constant $T = T(\eta, \varepsilon) > 0$ such that for $t_0 \in R_{+0}$ and any solution $v_\beta(t)$ of system (2.1) with $v_\beta(t_0) \in [\eta^{-1}, \eta]$, one has $|v_\beta(t) - v^*_\beta(t)| < \varepsilon$ for all $t \ge t_0 + T$. By Lemma 4 given in [1], we have

Lemma 2.1. *Suppose that assumptions* (A_1) *and* (A_2) *hold. Then,*

(a) *there is a constant $M > 0$ such that $\limsup_{t\to\infty} v_\beta(t) \le M$ for any positive solution $v_\beta(t)$ of system (2.1).*
(b) *each fixed solution $u^*_\beta(t)$ of system (2.1) is globally uniformly attractive on R_{+0}.*
(c) *if there is a constant $\omega_2 > 0$ such that $\liminf_{t\to\infty} \int_t^{t+\omega_2} g(s, \beta)\,\mathrm{d}s > 0$ for all $\beta \in [0, \beta_0]$, then there is a constant $\eta > 1$ such that $\eta^{-1} \le \liminf_{t\to\infty} v_\beta(t) \le \limsup_{t\to\infty} v_\beta(t) \le \eta$ for any solution $v_\beta(t)$ of system (2.1).*

Let $v_0 \in R_+$, $t_0 \in R_{+0}$ and $\beta \in [0, \beta_0]$, and $v_\beta(t)$, $v_0(t)$ be the solutions of systems (2.1) and (2.2) with initial values $v_\beta(t_0) = v_0$ and $v_0(t_0) = u_0$, respectively. We can get the following result.

Lemma 2.2. *Suppose that assumptions* (A_1) *and* (A_2) *hold, then $v_\beta(t)$ converges to $v_0(t)$ uniformly for $t \in [t_0, \infty)$ as $\beta \to 0$.*

The proof of Lemma 2.2 is similar to that of Lemma 2.3 in [20], we therefore omit it here.

Remark 2.1. In system (2.2), if function $g(t, 0) \equiv 0$, then system (2.2) has a trivial equilibrium $E = 0$, and E is globally asymptotically stable. For any $\Gamma > 1$ and $t_0 \in R_{+0}$, let $\beta \in [0, \beta_0]$, and $v_\beta(t)$ be the positive solution of systems (2.1) with initial value $|v_\beta(t_0)| \le \Gamma$. By Lemmas 2.1 and 2.2, we further have the result: the solution $v_\beta(t)$ converges to 0, as $\beta \to 0$ and $t \to \infty$, i.e., for any $\varepsilon > 0$, there are positive constants $T = T(\varepsilon, \Gamma)$ and $\delta = \delta(\varepsilon)$ such that $v_\beta(t) < \varepsilon$ for all $t \ge t_0 + T$ and $\beta < \delta$.

For system (1.2), we first introduce the basic assumptions.

(H_1) Functions $f_1(t), b_i(t), c_i(t), d_i(t), e_i(t)$ and $a_{ij}(t)$ are bounded and continuous on R_{+0}, and $f_1(t) \ge 0$, $b_2(t) \ge 0$ $c_i(t) \ge 0$, $d_i(t) \ge 0$, $e_i(t) \ge 0$ and $a_{ij}(t) \ge 0$ for all $t \in R_{+0}$, $i, j = 1, 2$.

(H_2) There is a constant $\lambda_i > 0$ such that $\liminf_{t\to\infty} \int_t^{t+\lambda_i} a_{ii}(s)\,\mathrm{d}s > 0$ $(i = 1, 2)$.

(H_3) There is a constant $\gamma_i > 0$ such that $\liminf_{t\to\infty} \int_t^{t+\gamma_i} e_i(s)\,\mathrm{d}s > 0$ $(i = 1, 2)$.

For the convenience of statements in this paper, we introduce the following definition on permanence.

Definition 2.1. System (1.2) is said to be permanent, if for any positive solution $(x_1(t), x_2(t), u_1(t), u_2(t))$ of system (1.2), there are positive constants m and M such that

$$m \le \liminf_{t\to\infty} x_i(t) \le \limsup_{t\to\infty} x_i(t) \le M \quad (i = 1, 2).$$

Remark 2.2. In system (1.2), $u_1(t)$ and $u_2(t)$ are control variables, so we do not consider the permanence of control variables.

3. Permanence

Let $R_+^4 = \{(x_1, x_2, x_3, x_4) : x_i > 0, i = 1, 2, 3, 4.\}$. For any $(t_0, X_0) \in R_{+0} \times R_+^4$, it is well known by the fundamental theory of differential equations that system (1.2) has a unique solution $X(t) = (x_1(t), x_2(t), u_1(t), u_2(t))$, which is through (t_0, X_0) and continuous. If $x_1(t) > 0$ and $x_2(t) > 0$ on the interval of existence, then $(x_1(t), x_2(t), u_1(t), u_2(t))$ is said to be a positive solution. It is easy to verify that solutions of system (1.2) are defined on $[0, \infty)$ and remain positive for all $t \ge 0$ if the initial value $(x_1(0), x_2(0)) \in R_+^2$.

First, on the ultimate boundedness of positive solution of system (1.2), we get the theorem.

Theorem 3.1. *Suppose that assumptions (H_1)–(H_3) hold, then system (1.2) is ultimately bounded, in the sense that there are positive constants M and T such that if $t > T$, then $x_i(t) \le M$ and $|u_i(t)| \le M$ $(i = 1, 2)$ for all positive solutions $(x_1(t), x_2(t), u_1(t), u_2(t))$ of system (1.2).*

Proof. Let $X(t) = (x_1(t), x_2(t), u_1(t), u_2(t))$ be any positive solution of system (1.2). We first prove that the component u_1 of system (1.2) is ultimately bounded. From assumption (H_1) and the third equation of system (1.2) we have

$$\frac{\mathrm{d}u_1(t)}{\mathrm{d}t} \le f_1(t) - e_1(t)u_1(t).$$

From (H_1) and (H_3), it is easy to verify that the comparison equation $\mathrm{d}x(t)/\mathrm{d}t = f_1(t) - e_1(t)x(t)$ satisfies all conditions of Lemma 2.1. So, by the comparison theorem and Lemma 2.1, we obtain there is a constant M_1 such that for any positive solution $X(t)$ of system (1.2), there is a $T_1 > 0$ such that $u_1(t) < M_1$ for all $t \ge T_1$. Further, from assumption (H_1) and the first equation of system (1.2) we have

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} \le x_1(t)[b_1(t) - a_{11}(t)x_1(t) + d_1(t)M_1]$$

for all $t \ge T_1$. It is proved in many papers, for example, see [9,25], that under assumptions (H_1) and (H_2) any positive solution $x(t)$ of the following non-autonomous logistic equation

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)[b_1(t) - a_{11}(t)x(t) + d_1(t)M_1]$$

is ultimately bounded on R_{+0}. Hence, using the comparison theorem, we further can obtain that there is a constant $M_2 > 0$ such that for any positive solution $X(t)$ of system (1.2) there is a $T_2 \ge T_1$ such that $x_1(t) < M_2$ for all $t \ge T_2$. Further, from assumption (H_1) and the third equation of system (1.2) we have that

$$\frac{\mathrm{d}u_1(t)}{\mathrm{d}t} \ge f_1(t) - e_1(t)u_1(t) - d_1(t)M_1.$$

It is easy verify that there are positive constant M_2, T_3 and $T_3 \ge T_2$ such that $u_1(t) > -M_2$ for all $t \ge T_3$.

From assumption (H_1) and the second equation of system (1.2) we have

$$\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} \le x_2(t)[b_2(t) + a_{12}(t)M_2 - a_{22}(t)x_2(t)]$$

for all $t \geq T_3$. Similar the above, we further can obtain that there is a constant $M_4 > 0$ such that for any positive solution $X(t)$ of system (1.2) there is a $T_4 \geq T_3$ such that $x_2(t) < M_4$ for all $t \geq T_4$. Therefore, from assumption (H_1) and the fourth equation of system (1.2) we have

$$\frac{du_2(t)}{dt} \leq -e_2(t)u_2(t) + d_1(t)M_3$$

for all $t \geq T_4$. Similarly, using the comparison theorem and Lemma 2.1, we can obtain that there is a constant $M_5 > 0$ such that for any positive solution $X(t)$ of system (1.2) there is a $T_5 \geq T_4$ such that $0 < u_2(t) < M_4$ for all $t \geq T_5$.

Now, we let $M = \max\{M_1, M_1, M_3, M_4, M_5\}$, then for all $t \geq T_5$

$$x_i(t) \leq M, \qquad |u_i(t)| \leq M, \qquad i = 1, 2.$$

Therefore, the solution $X(t)$ is ultimately bounded. We complete the proof. □

Next, we consider the auxiliary system

$$\frac{du_1(t)}{dt} = f_1(t) - e_1(t)u_1(t). \tag{3.1}$$

By assumptions (H_1) and (H_3), we note that system (3.1) satisfies all conditions of the (a) and (b) of Lemma 2.1. Then each positive solution of system (3.1) is globally uniformly attractive. Let $u_{10}(t)$ be some fixed positive solution of system (3.1), we assume that

(H_4) There is a constant $\lambda_3 > 0$ such that

$$\liminf_{t\to\infty} \int_t^{t+\lambda_3} \left[b_1(s) + c_1(s)u_{10}(s)\right] ds > 0.$$

Remark 3.1. If function $f_i(t) \equiv 0$, then system (3.1) has a trivial equilibrium $E = 0$, and E is globally asymptotically stable. In this case, we choose $u_{10}(t) = 0$.

On the the permanence of component x_1 of system (1.2), we can get:

Theorem 3.2. *Suppose that assumptions (H_1)–(H_4) hold, then component x_1 of system (1.2) is permanent.*

Proof. In fact, by assumptions (H_1), (H_2) and (H_4), we can choose positive constants ε_1, ε_2, δ_1 and T_0 such that for any continuous function $u(t)$ defined on R_{+0}, satisfying $|u(t) - u_{10}(t)| < \varepsilon_1$, we have

$$\int_t^{t+\lambda_3} \left[b_1(s) - a_{11}(s)\varepsilon_1 - a_{12}(s)\varepsilon_2 \exp(\theta_1\lambda_2) + c_1(s)u(s)\right] ds > \delta_1 \tag{3.2}$$

and

$$\int_t^{t+\lambda_2} \left[-b_2(s) + a_{21}(s)\varepsilon_1 - a_{22}(s)\varepsilon_2\right] ds < -\delta_1 \tag{3.3}$$

for all $t \geq T_0$, where $\theta_1 = \sup_{t\geq 0}\{b_2(t) + a_{21}(t)\varepsilon_1\}$.

For any t_0, t^* and $t^* \geq t_0 \geq 0$, integrating directly with system (1.2) we have

$$x_1(t^*) = x_1(t_0) \exp \int_{t_0}^{t^*} \left[b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) + c_1(t)u_1(t)\right] dt \tag{3.4}$$

and

$$x_2(t^*) = x_2(t_0) \exp \int_{t_0}^{t^*} \left[-b_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - c_2(t)u_2(t)\right] dt. \tag{3.5}$$

Claim 3.1. *There is a constant $\beta_1 > 0$ such that $\limsup_{t\to\infty} x_1(t) > \beta_1$ for any positive solution $X(t)$ of system (1.2).*

Now, we consider the following system with one parameter

$$\frac{du_1(t)}{dt} = f_1(t) - e_1(t)u_1(t) - d_1(t)\beta, \tag{3.6}$$

where $\beta \in [0, \beta_0]$ is a parameter. Let $u_{1\beta}(t)$ be the solution of system (3.6) with the initial value $u_{1\beta}(0) = u_{10}(0)$. By assumptions (H_1), (H_3) and Lemmas 2.1 and 2.2, we obtain that $u_{1\beta}(t)$ is globally asymptotically stable and which converges to $u_{10}(t)$ uniformly for $t \in R_+$ as $\beta \to 0$. Hence, there is a constant $\beta_1 > 0$ and $\beta_1 < \varepsilon_1$ such that

$$u_{1\beta_1}(t) > u_{10}(t) - \frac{\varepsilon_1}{2} \quad \text{for all} \quad t \in R_{+0}. \tag{3.7}$$

If Claim 3.1 is not true, then there is a positive solution $X(t)$ of system (1.2) such that

$$\limsup_{t\to\infty} x_1(t) < \beta_1. \tag{3.8}$$

From Theorem 3.1, there exist positive constants M, T_1 and $T_1 \geq T_0$ such that $0 < x_i(t) < M$ and $|u_i(t)| < M$ $(i = 1, 2)$ for all $t \geq T_1$. Further, by (3.8), we obtain that there is a constant $T_2 \geq T_1$ such that

$$x_1(t) < \beta_1 \quad \text{for all } t \geq T_2. \tag{3.9}$$

From assumption (H_1), (3.9) and the third equation of system (1.2) we have

$$\frac{\mathrm{d}u_1(t)}{\mathrm{d}t} \geq f_1(t) - e_1(t)u_1(t) - d_1(t)\beta_1 \quad \text{for all } t \geq T_2.$$

Using the comparison theorem and globally asymptotically stable of solution $u_{1\beta_1}(t)$, we obtain there is a $T_3 \geq T_2$ such that

$$u_1(t) > u_{1\beta_1}(t) - \frac{\varepsilon_1}{2} \quad \text{for all } t \geq T_3. \tag{3.10}$$

From assumption (H_1) we have

$$\frac{\mathrm{d}u_1(t)}{\mathrm{d}t} \leq f_1(t) - e_1(t)u_1(t) \quad \text{for all } t \geq 0.$$

Since $u_{10}(t)$ is the globally asymptotically stable positive solution of system (3.1), by the comparison theorem, we obtain that there is a constant $T_4 > T_3$ such that

$$u_1(t) \leq u_{10}(t) + \varepsilon_1 \quad \text{for all } t \geq T_4. \tag{3.11}$$

So, from this and (3.7), (3.10) we have

$$|u_1(t) - u_{10}(t)| < \varepsilon_1 \quad \text{for all } t \geq T_4. \tag{3.12}$$

On the other hand, If $x_2(t) \geq \varepsilon_2$ for all $t \geq T_4$, then by assumption (H_1) and (3.5), (3.9) we have

$$x_2(t) \leq x_2(T_4) \exp \int_{T_4}^{t} [-b_2(s) + a_{21}(s)\varepsilon_1 - a_{22}(s)\varepsilon_2]\, \mathrm{d}s \quad \text{for all } t \geq T_4.$$

From this and (3.3) it follows $\lim_{t\to\infty} x_2(t) = 0$ which leads to a contradiction. Then there is a $T_5 \geq T_4$ such that $x_2(T_5) < \varepsilon_2$.

In the following, we prove that

$$x_2(t) \leq \varepsilon_2 \exp(\theta_1\lambda_2) \quad \text{for all } t \geq T_5. \tag{3.13}$$

If there is a $t_1 > T_5$ such that $x_2(t_1) > \varepsilon_2 \exp(\theta_1\lambda_2)$, then there is a $t_2 \in (T_5, t_1)$ such that $x_2(t_2) = \varepsilon_2$ and $x_2(t) > \varepsilon_2$ for all $t \in (t_2, t_1]$. Choose an integer $n \geq 0$ such that $t_1 \in [t_2 + n\lambda_2, t_2 + (n+1)\lambda_2]$, then by assumption (H_1), (3.3), (3.5) and (3.9) we have

$$\begin{aligned} x_2(t_1) &\leq \varepsilon_2 \exp\Big[\int_{t_2}^{t_2+\lambda_2} + \cdots + \int_{t_2+(n-1)\lambda_2}^{t_2+n\lambda_2} + \int_{t_2+n\lambda_2}^{t_1}\Big]\big[-b_2(t) + a_{21}(t)\varepsilon_1 - a_{22}(t)\varepsilon_2\big]\,\mathrm{d}t \\ &\leq \varepsilon_2 \exp \int_{t_2+n\lambda_2}^{t_1} \big[-b_2(t) + a_{21}(t)\varepsilon_1 - a_{22}(t)\varepsilon_2\big]\,\mathrm{d}t \\ &\leq \varepsilon_2 \exp(\theta_1\lambda_2) \end{aligned}$$

which is a contradiction and (3.13) is true.

Finally, by assumption (H_1), (3.4), (3.9), (3.12) and (3.13) we obtain that for all $t \geq T_5$

$$x_1(t) \geq x_1(T_5) \exp \int_{T_5}^{t} \big[b_1(s) - a_{11}(s)\varepsilon_1 - a_{12}(s)\varepsilon_2 \exp(\theta_1\lambda_2) + c_1(s)u_1(s)\big]\,\mathrm{d}s.$$

From this and by (3.2) it follows $\lim_{t\to\infty} x_1(t) = \infty$ which leads to a contradiction. Therefore, Claim 3.1 is true.

Claim 3.2. *There is a constant $\beta_2 > 0$ such that $\liminf_{t\to\infty} x_1(t) > \beta_2$ for any positive solution $X(t)$ of system (1.2).*

In fact, from (3.2) and (3.3) there is a constant $L > 0$ such that

$$\int_{t}^{t+a} \big[b_1(s) - a_{11}(s)\varepsilon_1 - a_{12}(s)\varepsilon_2 \exp(\theta_1\lambda_2) + c_1(s)u(s)\big]\,\mathrm{d}s > \varepsilon_1, \tag{3.14}$$

and

$$M \exp \int_t^{t+a} [-b_2(s) + a_{21}(s)\varepsilon_1 - a_{22}(s)\varepsilon_2]\,ds < \varepsilon_2 \tag{3.15}$$

for all $t \geq T_0$ and $a \geq L$, where constant M is given in the above. If Claim 3.2 is not true, then there is a sequence of initial values $\{X_m\} \subset R_+^4$ such that for the solution $X(t, X_m)$ of system (1.2),

$$\liminf_{t\to\infty} x_1(t, X_m) < \frac{\beta_1}{m^2}, \quad m = 1, 2, \ldots,$$

where constant β_1 is given in Claim 3.1. By Claim 3.1, for every m there are two time sequences $\{s_q^{(m)}\}$ and $\{t_q^{(m)}\}$, satisfying $0 < s_1^{(m)} < t_1^{(m)} < s_2^{(m)} < t_2^{(m)} < \cdots < s_q^{(m)} < t_q^{(m)} < \cdots$ and $\lim_{q\to\infty} s_q^{(m)} = \infty$, such that

$$x_1(s_q^{(m)}, X_m) = \frac{\beta_1}{m}, \qquad x_1(t_q^{(m)}, X_m) = \frac{\beta_1}{m^2}, \tag{3.16}$$

and

$$\frac{\beta_1}{m^2} < x_1(t, X_m) < \frac{\beta_1}{m} \quad \text{for all } t \in (s_q^{(m)}, t_q^{(m)}). \tag{3.17}$$

From (3.11) and the ultimate boundedness of system (1.2), we can choose a positive constant $T^{(m)}$ for every m such that

$$u_1(t, X_m) < u_{10}(t) + \varepsilon_1 \tag{3.18}$$

and $x_i(t, X_m) < M$, $|u_i(t, X_m)| < M$ for all $t > T^{(m)}$ and $i = 1, 2$. Further, there is an integer $K_1^{(m)} > 0$ such that $s_q^{(m)} > T^{(m)}$ for all $q > K_1^{(m)}$. Let $q > K_1^{(m)}$, for any $t \in [s_q^{(m)}, t_q^{(m)}]$, by assumption (H_1) we have

$$\frac{dx_1(t, X_m)}{dt} \geq x_1(t, X_m)[b_1(t) - a_{11}(t)M - a_{12}(t)M - c_1(t)M] \geq -L_1 x_1(t, X_m),$$

where $L_1 = \sup_{t\geq 0}\{|b_1(t) - a_{11}(t)M - a_{12}(t)M - c_1(t)M|\}$. Integrating the above inequality from $s_q^{(m)}$ to $t_q^{(m)}$, we further have

$$x_1(t_q^{(m)}, X_m) \geq x_1(s_q^{(m)}, X_m) \exp[-L_1(t_q^{(m)} - s_q^{(m)})].$$

Consequently, by (3.16)

$$\frac{\beta_1}{m^2} \geq \frac{\beta_1}{m} \exp[-L_1(t_q^{(m)} - s_q^{(m)})].$$

Hence,

$$t_q^{(m)} - s_q^{(m)} \geq \frac{\ln m}{L_1} \quad \text{for all } q > K_1^{(m)}. \tag{3.19}$$

Let $\widetilde{u}_{1\beta_1}(t)$ be the solution of system (3.6) with the initial condition $\widetilde{u}_{1\beta_1}(s_q^{(m)}) = u_1(s_q^{(m)}, X_m)$. By Lemma 2.1, the solution $u_{1\beta_1}(t)$ of system (3.6) is globally uniformly attractive on R_{+0}, and so there is a constant $T_1 \geq T_0$, and T_1 is independent of any m and $q \geq K^{(m)}$, such that

$$\widetilde{u}_{1\beta_1}(t) \geq u_{1\beta_1}(t) - \frac{\varepsilon_1}{2} \quad \text{for all } t \geq s_q^{(m)} + T_1. \tag{3.20}$$

By (3.17) and assumption (H_1), we have

$$\frac{du_1(t, X_m)}{dt} \geq f_1(t) - e_1(t)u_1(t, X_m) - d_1(t)\beta_1$$

for any m, q and $t \in [s_q^{(m)}, t_q^{(m)}]$. Using the comparison theorem it follows that

$$u_1(t, X_m) \geq \widetilde{u}_{1\beta_1}(t) \quad \text{for all } t \in [s_q^{(m)}, t_q^{(m)}]. \tag{3.21}$$

On the other hand, by (3.19) there is an integer $N > 0$ such that for all $m \geq N$ and $q \geq K^{(m)}$

$$t_q^{(m)} - s_q^{(m)} > 2P_0,$$

where $P_0 = \max\{L, T_1, \lambda_2\}$. Further, by (3.7), (3.18), (3.20) and (3.21) we obtain

$$|u_1(t, X_m) - u_{10}(t)| < \varepsilon_1 \quad \text{for all } t \in [s_q^{(m)} + P_0, t_q^{(m)}]. \tag{3.22}$$

For any $m \geq N$ and $q \geq K^{(m)}$, if $x_2(t, X_m) > \varepsilon_2$ for all $t \in [s_q^{(m)}, s_q^{(m)} + P_0]$, then by assumption (H_1), (3.5), (3.15) and (3.17) we have

$$\begin{aligned} x_2(s_q^{(m)} + P_0, X_m) &= x_2(s_q^{(m)}, X_m) \exp \int_{s_q^{(m)}}^{s_q^{(m)}+P_0} \left[-b_2(t) + a_{21}(t)x_1(t, X_m) - a_{22}(t)x_2(t, X_m) - d_2(t)u_2(t, X_m)\right] \mathrm{d}t \\ &\leq M \exp \int_{s_q^{(m)}}^{s_q^{(m)}+P_0} \left[-b_2(t) + a_{21}(t)\varepsilon_1 - a_{22}(t)\varepsilon_2\right] \mathrm{d}t \\ &< \varepsilon_2. \end{aligned}$$

This leads to a contradiction. Hence, there is a $t_1 \in [s_q^{(m)}, s_q^{(m)} + P_0]$ such that $x_2(t_1, X_m) < \varepsilon_2$. Similarly, according to the proof of (3.12), we can obtain that

$$x_2(t, X_m) \leq \varepsilon_2 \exp(\theta_1 \lambda_2) \quad \text{for all } t \in [t_1, t_q^{(m)}]. \tag{3.23}$$

Finally, when $m \geq N$ and $q \geq K^{(m)}$, by assumption (H_1), (3.4), (3.14), (3.16), (3.17), (3.22) and (3.23) we have

$$\begin{aligned} x_1(t_q^{(m)}, X_m) &= x_1(s_q^{(m)} + P_0, X_m) \exp \int_{s_q^{(m)}+P_0}^{t_q^{(n)}} \left[b_1(t) - a_{11}(t)x_1(t, X_m) - a_{22}(t)x_2(t, X_m) + d_1(t)u_1(t, X_m)\right] \mathrm{d}t \\ &\geq x_1(s_q^{(m)} + P_0, X_m) \exp \int_{s_q^{(m)}+P_0}^{t_q^{(n)}} \left[b_1(t) - a_{11}(t)\varepsilon_1 - a_{12}(t)\varepsilon_2 \exp(\theta_1\lambda_2) + d_1(t)u_1(t, X_m)\right] \mathrm{d}t \\ &> \frac{\beta_1}{m^2} \end{aligned}$$

which leads to a contradiction. Then Claim 3.2 is true.

From Claims 3.1 and 3.2 we complete the proof of this theorem. □

In order to obtain the permanence of component x_2 of system (1.2), we next consider the single-species logistic system with feedback control, which is a subsystem of system (1.2)

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = x_1(t)[b_1(t) - a_{11}(t)x_1(t) + c_1(t)u_1(t)] \\ \dfrac{\mathrm{d}u_1(t)}{\mathrm{d}t} = f_1(t) - e_1(t)u_1(t) - d_1(t)x_1(t). \end{cases} \tag{3.24}$$

On system (1.2), we further introduce the following assumption.

(H_5) There are positive constants k_1, k_2 and λ_4 such that $\inf_{t\geq 0} A_i(t) \geq 0$ and $\liminf_{t\to\infty} \int_t^{t+\lambda_4} A_i(s)\,\mathrm{d}s > 0$ $(i = 1, 2)$, where $A_1(t) = k_1 a_{11}(t) - k_2 d_1(t)$ and $A_2(t) = k_2 e_1(t) - k_1 c_1(t)$.

For the system (3.24), we have the result.

Lemma 3.1. *Suppose that assumptions* (H_1)–(H_4) *hold. Then,*

(a) *there is a constant* $M > 1$ *such that*

$$M^{-1} \leq \liminf_{t\to\infty} x_{10}^*(t) \leq \limsup_{t\to\infty} x_{10}^*(t) \leq M, \quad \limsup_{t\to\infty} u_{10}^*(t) < M$$

for any positive solution $(x_{10}^*(t), u_{10}^*(t))$ *of system* (3.24).

(b) *if assumption* (H_5) *holds, then each fixed solution* $(x_{10}^*(t), u_{10}^*(t))$ *of system* (3.24) *is globally uniformly attractive on* R_{+0}.

Proof. On the basis of (H_1)–(H_4), conclusion (a) can be proved by using a similar argument as in Theorem 3.2.

Here, we prove conclusion (b). For any constant $\eta > 1$, $t_0 \in R_{+0}$ and solution $(x_{10}^*(t), u_{10}^*(t))$, let $(x_{10}(t), u_{10}(t))$ be a solution of system (3.24) with initial values $x_{10}(t_0), u_{10}(t_0) \in [\eta^{-1}, \eta]$. by Theorem 3.1, there is a constant $M > 1$ such that

$$M^{-1} < x_{10}^*(t), x_{10}(t) < M, \quad \text{and} \quad |u_{10}^*(t)|, |u_{10}(t)| < M \tag{3.25}$$

for all $t \in R_{+0}$. Consider the Lyapunov function $V(t) = k_1|\ln x_{10}(t) - \ln x_{10}^*(t)| + k_2|u_{10}(t) - u_{10}^*(t)|$, calculating the Dini derivative of $V(t)$ along system (3.24), it follows that

$$\begin{aligned} D^+V(t) &= k_1 \mathrm{sgn}(x_{10}(t) - x_{10}^*(t)(t))\left\{-a_{11}(t)[x_{10}(t) - x_{10}^*(t)] + c_1(t)[u_{10}(t) - u_{10}^*(t)]\right\} \\ &\quad + k_2 \mathrm{sgn}(u_{10}(t) - u_{10}^*(t))\left\{-e_1(t)[u_{10}(t) - u_{10}^*(t)] - d_1(t)[x_{10}(t) - x_{10}^*(t)]\right\} \\ &\leq -[k_1 a_{11}(t) - k_2 d_1(t)]|x_{10}(t) - x_{10}^*(t)| - [k_2 e_1(t) - k_1 c_1(t)]|u_{10}(t) - u_{10}^*(t)|. \end{aligned}$$

Using the mean value theorem, we have

$$|x_{10}(t)-x_{10}^*(t)| = |e^{\ln x_{10}(t)} - e^{\ln x_{10}^*(t)}| = e^{\xi(t)}|\ln x_{10}(t) - \ln x_{10}^*(t)|, \tag{3.26}$$

where $\xi(t)$ lies between $\ln x_{10}(t)$ and $\ln x_{10}^*(t)$. By (3.25) we have $\ln M^{-1} < \xi_1(t) < \ln M$ for all $t \geq t_0$. From this and by (3.26) we obtain that

$$\begin{aligned} D^+V(t) &\leq -[k_1a_{11}(t) - k_2d_1(t)]e^{\xi(t)}|\ln x_1(t) - \ln x_{10}(t)| - [k_2e_1(t) - k_1c_1(t)]|u_1(t) - u_{10}(t)| \\ &\leq -\gamma(t)V(t) \end{aligned} \tag{3.27}$$

for all $t \geq t_0$, where

$$\gamma(t) = \min_{t\geq t_0}\left\{\frac{k_1a_{11}(t) - k_2d_1(t)}{k_1M}, \frac{k_2e_1(t) - k_1c_1(t)}{k_2}\right\}. \tag{3.28}$$

By assumption (H_5), we have that $\gamma(t) \geq 0$ and $\liminf_{t\to\infty}\int_t^{t+\lambda_4}\gamma(s)\,ds > 0$. Therefore, we can choose positive constants δ and T_0 such that

$$\int_t^{t+\lambda_4}\gamma(s)\,ds > \delta \quad \text{for all } t \geq T_0.$$

Let $T_1 = t_0 + T_0$. For any $t \geq T_1$, there is an integer $n_t \geq 0$ such that $t \in [T_1 + n_t\lambda_4, T_1 + (n_t+1)\lambda_4]$. Integrating (3.33) from T_1 to t, we have

$$\begin{aligned} V(t) &\leq V(T_1)\exp\int_{T_1}^{t}(-\gamma(s))\,ds \\ &= V(T_1)\exp\left[\int_{T_1}^{T_1+\lambda_4} + \cdots + \int_{T_1+(n_t-1)\lambda_4}^{T_1+n_t\lambda_4} + \int_{T_1+n_t\lambda_4}^{t}\right](-\gamma(s))\,ds \\ &\leq V(T_1)\exp(-\delta n_t). \end{aligned}$$

Since $V(T_1) \leq V(t_0) \leq k_1\ln(\eta M) + k_2(\eta + M)$, we further have

$$\begin{aligned} V(t) &\leq [k_1\ln(\eta M) + k_2(\eta+M)]\exp[-\delta\lambda_4^{-1}(t - T_1 - \lambda_4)] \\ &= M(\eta)\exp[-\delta\lambda_4^{-1}(t - t_0)], \end{aligned} \tag{3.29}$$

where $M(\eta) = [k_1\ln(\eta M) + k_2(\eta+M)]\exp[\delta(1 + T_0/\lambda_4)]$.

On the other hand, by (3.26), it follows that for all $t \geq t_0$

$$|x_{10}(t) - x_{10}^*(t)| + |u_{10}(t) - u_{10}^*(t)| \leq M_0V(t),$$

where $M_0 = \max\{M/k_1, 1/k_2\}$. For any $\varepsilon > 0$, from (3.29), there is a large enough $T = T(\eta, \varepsilon) \geq T_0$ such that

$$V(t) < M_0^{-1}\varepsilon \quad \text{for all } t \geq t_0 + T.$$

Therefore, $|x_{10}(t) - x_{10}^*(t)| < \varepsilon$ and $|u_{10}(t) - u_{10}^*(t)| < \varepsilon$ for all $t \geq t_0 + T$. This shows that solution $(x_{10}^*(t), u_{10}^*(t))$ is globally uniformly attractive on R_{+0}. This completes the proof. □

Let $(x_{10}^*(t), u_{10}^*(t))$ be a fixed solution of system (3.24) defined on R_{+0}. On the permanence of system (1.2), we have the following result.

Theorem 3.3. *Suppose that assumptions* (H_1)–(H_5) *hold. If there is a constant* $\lambda_5 > 0$ *such that*

$$\liminf_{t\to\infty}\int_t^{t+\lambda_5}[-b_2(s) + a_{21}(s)x_{10}^*(s)]\,ds > 0. \tag{3.30}$$

Then system (1.2) *is permanent.*

Proof. By Theorems 3.1 and 3.2, we have that the component x_1 of system (1.2) is permanent. So, we only need to show the component x_2 of system (1.2) is permanent.

In fact, by (3.30), there are positive constants ε_0, δ and T_0 such that for all $t \geq T_0$

$$\int_t^{t+\lambda_5}\left\{-b_2(s) + a_{21}(s)\left[x_{10}^*(s) - \varepsilon_0\right] - a_{22}(s)\varepsilon_0 - d_2(s)\varepsilon_0\right\}ds > \delta. \tag{3.31}$$

On the other hand, let $(x_1(t), x_2(t), u_1(t), u_2(t))$ be any positive solution of system (1.2) with $x_1(t_0) = x_{10}^*(t_0)$ and $u_1(t_0) = u_{10}^*(t_0)$, respectively. By Theorems 3.1 and 3.2, there exist positive constants $M > 1$ and such that $0 < x_2(t) < M$,

$0 < u_2(t) < M$ and

$$M^{-1} \le x_1(t), x_{10}^*(t) \le M, \qquad |u_1(t)|, |u_{10}^*(t)| \le M \tag{3.32}$$

for all $t \ge t_0$. Consider the Lyapunov function $V(t) = k_1|\ln x_1(t) - \ln x_{10}^*(t)| + k_2|u_1(t) - u_{10}^*(t)|$, calculating the Dini derivative of $V(t)$ along system (1.2) and (3.24), it follows that

$$D^+V(t) \le -[k_1a_{11}(t) - k_2d_1(t)]|x_1(t) - x_{10}^*(t)| - [k_2e_1(t) - k_1c_1(t)]|u_1(t) - u_{10}^*(t)| + k_1a_{12}(t)x_2(t).$$

Just as the discussion in Lemma 3.1, we have

$$\begin{aligned} D^+V(t) &\le -\gamma(t)V(t) + k_1mx_2(t), \\ &\le -\gamma(t)V(t) + k_1mx_2^u \end{aligned} \tag{3.33}$$

where $m = \sup_{t\ge0}\{a_{12}(t)\}, x_2^u = \sup_{t\ge0}\{x_2(t)\}$ and $\gamma(t)$ is given in (3.28).

By assumption (H_5), we have that $\gamma(t) \ge 0$ and $\liminf_{t\to\infty} \int_t^{t+\lambda_4} \gamma(s)\,ds > 0$. Hence, by the comparison theorem and the variation of constants formula of solutions for first-order linear differential equations, we can obtain from (3.33) that $V \to 0$ uniformly for $t \in [t_0, \infty)$ as $x_2^u \to 0$. On the other hand, we note that for all $t \ge t_0$

$$|x_1(t) - x_{10}^*(t)| + |u_1(t) - u_{10}^*(t)| \le M_0V(t),$$

where $M_0 = \max\{M/k_1, 1/k_2\}$. Therefore, we obtain that there is a constant $\beta_3' > 0$ such that

$$x_1(t) \ge x_{10}^*(t) - \frac{\varepsilon_0}{2} \quad \text{for all } t \ge t_0 \quad \text{and} \quad x_2(t) \in [0, \beta_3']. \tag{3.34}$$

Claim 3.3. *There is a constant $\beta_3 > 0$ such that $\limsup_{t\to\infty} x_2(t) > \beta_3$ for any positive solution $X(t)$ of system (1.2).*

We now consider the following system with a parameter

$$\frac{du_2(t)}{dt} = -e_2(t)u_2(t) + d_2(t)\beta, \tag{3.35}$$

where $\beta \in [0, \beta_0]$ is a parameter and β_0 is a constant. By Remark 2.1, for given in above ε_0 and M, there exist positive constants $\beta_3, T_1 = T_1(M)$ and $\beta_3 < \min\{\varepsilon_0, \beta_3'\}, T_1 \ge T_0$ such that for any $t_0 \in R_{+0}$ and $0 < u_2(t_0) \le M$, we have

$$u_{2\beta}(t) \le \frac{\varepsilon_0}{2} \quad \text{for all } t \ge T_1 \quad \text{and} \quad \beta \in [0, \beta_3]. \tag{3.36}$$

If Claim 3.3 is not true, then there is a positive solution $(\widetilde{x}_1(t), \widetilde{x}_2(t), \widetilde{u}_1(t), \widetilde{u}_2(t))$ of system (1.2) such that

$$\limsup_{t\to\infty} \widetilde{x}_2(t) < \beta_3. \tag{3.37}$$

Let $(\widetilde{x}_{10}(t), \widetilde{u}_{10}(t))$ be a positive solution of system (3.24) with $\widetilde{x}_1(t_0) = \widetilde{x}_{10}(t_0)$ and $\widetilde{u}_1(t_0) = \widetilde{u}_{10}(t_0)$. From Theorem 3.1, there exists a constant $T_2 \ge T_1$ such that $0 < \widetilde{x}_i(t), \widetilde{x}_{10}(t) < M$ and $|\widetilde{u}_i(t)|, |\widetilde{u}_{10}(t)| < M$ $(i = 1, 2)$ for all $t \ge T_2$. Further, by (3.37), we obtain that there is a constant $T_3 \ge T_2$ such that

$$\widetilde{x}_2(t) < \beta_3 \quad \text{for all } t \ge T_3. \tag{3.38}$$

From (3.38) and assumption (H_1) we have

$$\frac{d\widetilde{u}_2(t)}{dt} \le -e_2(t)\widetilde{u}_2(t) + d_2(t)\beta_3 \quad \text{for all } t \ge T_3.$$

Using the comparison theorem and the globally uniformly attractive of solution $u_{2\beta_3}$, we obtain that there is a constant $T_4 \ge T_3$ such that

$$\widetilde{u}_2(t) < u_{2\beta_3}(t) + \frac{\varepsilon_0}{2} \quad \text{for all} \quad t \ge T_4.$$

From this and by (3.36) it follows that

$$\widetilde{u}_2(t) < \varepsilon_0 \quad \text{for all } t \ge T_4. \tag{3.39}$$

On the other hand, from (3.34), we have

$$\widetilde{x}_1(t) \ge \widetilde{x}_{10}(t) - \frac{\varepsilon_0}{2} \quad \text{for all } t \ge T_4. \tag{3.40}$$

Since solution $(x_{10}^*(t), u_{10}^*(t))$ of system (3.24) is globally uniformly attractive on R_{+0}. So, there is a constant $T_5 \ge T_4$ such that

$$\widetilde{x}_{10}(t) \ge x_{10}^*(t) - \frac{\varepsilon_0}{2}.$$

From this and (3.40) we have that

$$\widetilde{x}_1(t) \geq x_{10}^*(t) - \varepsilon_0 \quad \text{for all} \quad t \geq T_5 \quad \text{and} \quad |x_2(t)| \in [0, \beta_3] \tag{3.41}$$

By (3.5), (3.39), (3.41) and assumption (H_1) we have

$$\begin{aligned}\widetilde{x}_2(t) &= \widetilde{x}_2(T_5) \exp \int_{T_5}^{t} \Big[-b_2(s) + a_{21}(s)\widetilde{x}_1(s) - a_{22}(s)\widetilde{x}_2(s) - d_2(s)\widetilde{u}_2(s)\Big] \mathrm{d}s \\ &\geq x_2(T_5) \exp \int_{T_6}^{t} \Big\{-b_2(s) + a_{21}(s)\big[x_{10}^*(s) - \varepsilon_0\big] - a_{22}(s)\varepsilon_0 - d_2(s)\varepsilon_0\Big\} \mathrm{d}s.\end{aligned}$$

Thus, from (3.31) we finally obtain that $\lim_{t\to\infty} \widetilde{x}_2(t) = \infty$ which leads to a contradiction. Therefore, Claim 3.3 is true.

Claim 3.4. *There is a constant $\beta_4 > 0$ such that $\liminf_{t\to\infty} x_2(t) > \beta_4$ for any positive solution $X(t)$ of system (1.2).*

In fact, by (3.31) there are positive constants P and γ such that

$$\int_t^{t+\lambda} \Big[-b_2(s) + a_{12}(s)(x_{10}^*(s) - \varepsilon_0) - a_{22}(s)\varepsilon_0 - d_2(s)\varepsilon_0\Big] \geq \gamma \tag{3.42}$$

for all $t \geq T_0$ and $\lambda \geq P$.

If Claim 3.4 is not true, then there is a sequence of initial values $\{X_n\} \subset R_+^4$ such that, for the solution $X(t, X_n)$ of system (1.2),

$$\liminf_{t\to\infty} x_2(t, X_n) < \frac{\beta_3}{n^2}, \quad n = 1, 2, \ldots,$$

where constant β_3 is given in Claim 3.3. By Claim 3.3, for every n there are two time sequences $\{s_q^{(n)}\}$ and $\{t_q^{(n)}\}$, satisfying $0 < s_1^{(n)} < t_1^{(n)} < s_2^{(n)} < t_2^{(n)} < \cdots < s_q^{(n)} < t_q^{(n)} < \cdots$ and $\lim_{q\to\infty} s_q^{(n)} = \infty$, such that

$$x_2(s_q^{(n)}, X_n) = \frac{\beta_3}{n}, \qquad x_2(t_q^{(n)}, X_n) = \frac{\beta_3}{n^2} \tag{3.43}$$

and

$$\frac{\beta_3}{n^2} < x_2(t, X_n) < \frac{\beta_3}{n} \quad \text{for all } t \in (s_q^{(n)}, t_q^{(n)}). \tag{3.44}$$

From the ultimate boundedness of system (1.2), we can choose a positive constant $T^{(n)}$ for every n such that $x_i(t, X_n) < M$ and $u_i(t, X_n) < M$ for all $t > T^{(n)}$ and $i = 1, 2$. Further, there is an integer $K_1^{(n)} > 0$ such that $s_q^{(n)} > T^{(n)}$ for all $q > K_1^{(n)}$. Let $q > K_1^{(n)}$, for any $t \in [s_q^{(n)}, t_q^{(n)}]$, by assumption ($H_1$) we have

$$\frac{\mathrm{d}x_2(t, X_n)}{\mathrm{d}t} \geq x_2(t, X_n)[-b_2(t) - a_{22}(t)M - d_2(t)M] \geq -L_2 x_2(t, X_n),$$

where $L_2 = \sup_{t\geq 0}\{b_2(t) + a_{22}(t)M + d_2(t)M\}$. Integrating the above inequality from $s_q^{(n)}$ to $t_q^{(n)}$, we further have

$$x_2(t_q^{(n)}, X_n) \geq x_2(s_q^{(n)}, X_n) \exp[-L_2(t_q^{(n)} - s_q^{(n)})].$$

Therefore, by (3.43)

$$\frac{\beta_3}{n^2} \geq \frac{\beta_3}{n} \exp[-L_2(t_q^{(n)} - s_q^{(n)})].$$

Hence,

$$t_q^{(n)} - s_q^{(n)} \geq \frac{\ln n}{L_2} \quad \text{for all } q > K_1^{(n)}.$$

Let $\widetilde{u}_{2\beta_3}(t)$ be the solution of system (3.35) with the initial condition $\widetilde{u}_{2\beta_3}(t) = u_2(s_q^{(n)}, X_n)$. By (3.44) and assumption (H_1), we have

$$\frac{\mathrm{d}u_2(t, X_n)}{\mathrm{d}t} \leq -e_2(t)u_2(t, X_n) + d_2(t)\beta_3$$

for any n, q and $t \in [s_q^{(n)}, t_q^{(n)}]$. Using the comparison theorem it follows that

$$u_2(t, X_n) \leq \widetilde{u}_{2\beta_3}(t) \quad \text{for all } t \in [s_q^{(n)}, t_q^{(n)}]. \tag{3.45}$$

By Lemma 2.1, the solution $u_{2\beta_3}(t)$ of system (3.35) is globally uniformly attractive on R_{+0}, we obtain that there is a constant $T_1 \geq \max\{P, T_0\}$, and T_1 is independent of any n and $q \geq K_1^{(n)}$, such that

$$\widetilde{u}_{2\beta_3}(t) \leq u_{2\beta_3}(t) + \frac{\varepsilon_0}{2} \quad \text{for all } t \geq s_q^{(n)} + T_0. \tag{3.46}$$

Let $(\widetilde{x}_{10}(t), \widetilde{u}_{10}(t))$ be a solution of system (3.24) with $\widetilde{x}_{10}(s_q^{(n)}) = x_1(s_q^{(n)}, X_n)$ and $\widetilde{u}_{10}(s_q^{(n)}) = u_1(s_q^{(n)}, X_n)$. From (3.34), for all $q > K_1^{(n)}$, $t \in [s_q^{(n)}, t_q^{(n)}]$ and $|x_2(t)| \in [0, \beta_3]$, we obtain that

$$x_1(t, X_n) \geq \widetilde{x}_{10}(t) - \frac{\varepsilon_0}{2}. \tag{3.47}$$

By Lemma 3.1, the solution $(x_{10}^*(t), u_{10}^*(t))$ of system (3.24) is globally uniformly attractive on R_{+0}, we obtain that there is a constant $T^* \geq P$, and T^* is independent of any n and $q \geq K_1^{(n)}$, such that

$$\widetilde{x}_{10}(t) \geq x_{10}^*(t) - \frac{\varepsilon_0}{2} \quad \text{for all } t \geq s_q^{(n)} + T^*. \tag{3.48}$$

Choose an integer N_0 such that when $n \geq N_0$ and $q \geq K_1^{(n)}$

$$t_q^{(n)} - s_q^{(n)} > 2P_0,$$

where $P_0 = \max\{T_1, T^*, \lambda_5\}$. Further, from (3.36), (3.45) and (3.46), we obtain that

$$u_2(t, X_n) < \varepsilon_0 \quad \text{for all } t \in [s_q^{(n)} + P_0, t_q^{(n)}]. \tag{3.49}$$

On the other hand, by (3.47) and (3.48) we have

$$x_1(t, X_n) > x_{10}^*(t) - \varepsilon_0 \quad \text{for all } t \in [s_q^{(n)} + P_0, t_q^{(n)}]. \tag{3.50}$$

Finally, by assumption (H_2), (3.5), (3.42), (3.49) and (3.50) we have

$$\begin{aligned}
\frac{\beta_3}{n^2} &= x_2(s_q^{(n)} + P_0, X_n) \exp \int_{s_q^{(n)}+P_0}^{t_q^{(n)}} \Big[-b_2(t) + a_{21}(t)x_1(t, X_n) - a_{22}(t)x_2(t, X_n) - d_2(t)u_2(t, X_n)\Big]\, \mathrm{d}t \\
&\geq x_2(s_q^{(n)} + P_0, X_n) \exp \int_{s_q^{(n)}+P_0}^{t_q^{(n)}} \Big\{-b_2(t) + a_{21}(t)\big[x_{10}^*(t) - \varepsilon_0\big] - a_{22}(t)\varepsilon_0 - d_2\varepsilon_0\Big\}\, \mathrm{d}t \\
&> \frac{\beta_3}{n^2}
\end{aligned}$$

which leads to a contradiction. Therefore, Claim 3.4 is true.

Finally, from Claims 3.3 and 3.4 we see that the component x_2 of system (1.2) is permanent and this completes the proof. □

Remark 3.2. From Theorems 3.2 and 3.3, we note that control variable u_1 has an influence on the permanence of system (1.2) and control variable u_2 has no influence on the permanence of system (1.2).

As consequences of Theorems 3.2 and 3.3, we can obtain the following result.

Corollary 3.1. *Suppose that assumptions* (H_1)–(H_3) *hold. If there are positive constants* ϑ_1 *and* ϑ_2 *such that*

$$\liminf_{t\to\infty} \int_t^{t+\vartheta_1} b_1(s)\, \mathrm{d}s > 0$$

and

$$\liminf_{t\to\infty} \int_t^{t+\vartheta_2} \big[-b_2(s) + a_{21}(s)x_{10}(s)\big]\, \mathrm{d}s > 0,$$

where $x_{10}(s)$ *is some positive solution of system* $\mathrm{d}x_1(t)/\mathrm{d}t = x_1(t)[b_1(t) - a_{11}(t)x_1(t)]$. *Then system* (1.2) *is permanent.*

Remark 3.3. In Corollary 3.1, we note that the conditions have nothing to do with feedback controls, and which are easily proved, although they are more strong than the conditions in Theorems 3.2 and 3.3.

4. Global stability

We finally proceed to the discussion global stability of any positive solution of system (1.2).

Theorem 4.1. *Let $(x_1^*(t), x_2^*(t), u_1^*(t), u_2^*(t))$ be some fixed positive solution of system (1.2). Suppose that assumptions* (H_1)–(H_5) *and (3.30) hold. If there are positive constants k_i $(i = 1, 2, 3, 4)$ such that*

$$\liminf_{t\to\infty} A_i(t) > 0, \tag{4.1}$$

where

$$\begin{aligned} A_1(t) &= k_1a_{11}(t) - k_2a_{21}(t) - k_3d_1(t), \qquad A_3(t) = k_3e_1(t) - k_1c_1(t), \\ A_2(t) &= k_2a_{22}(t) - k_1a_{12}(t) - k_4d_2(t), \qquad A_4(t) = k_4e_2(t) - k_2c_2(t). \end{aligned} \tag{4.2}$$

Then $(x_1^*(t), x_2^*(t), u_1^*(t), u_2^*(t))$ is globally asymptotically stable.

Proof. Let $(x_1(t), x_2(t), u_1(t), u_2(t))$ be any positive solution of system (1.2), it follows from Theorem 3.1 that there exist positive constants M and T_1 such that for all $t \geq T_1$

$$0 < x_i(t), x_i^*(t) < M, \qquad -M < u_i(t), u_i^*(t) < M \quad (i = 1, 2).$$

We define a Lyapunov functional $V(t)$ as

$$V(t) = \sum_{i=1}^{2} k_i|\ln x_i(t) - \ln x_i^*(t)| + k_{i+2}|u_i(t) - u_i^*(t)|.$$

Calculating the upper right derivative of $V(t)$ along solutions of system (1.2), it follows that

$$\begin{aligned} D^+V(t) &= k_1\mathrm{sgn}(x_1(t) - x_1^*(t))\left[\dot{x}_1(t) - \dot{x}_1^*(t)\right] + k_2\mathrm{sgn}(x_2(t) - x_2^*(t))\left[\dot{x}_2(t) - \dot{x}_2^*(t)\right] \\ &\quad + k_3\mathrm{sgn}(u_1(t) - u_1^*(t))\left[\dot{u}_1(t) - \dot{u}_1^*(t)\right] + k_4\mathrm{sgn}(u_2(t) - u_2^*(t))\left[\dot{u}_2(t) - \dot{u}_2^*(t)\right] \\ &= k_1\mathrm{sgn}(x_1(t) - x_1^*(t))\left\{-a_{11}(t)[x_1(t) - x_1^*(t)] - a_{12}(t)[x_2(t) - x_2^*(t)]\right. \\ &\quad \left. + c_1(t)[u_1(t) - u_1^*(t)]\right\} + k_2\mathrm{sgn}(x_2(t) - x_2^*(t))\left\{a_{21}(t)[x_1(t) - x_1^*(t)]\right. \\ &\quad \left. - a_{22}(t)[x_2(t) - x_2^*(t)] - d_2(t)[u_2(t) - u_2^*(t)]\right\} \\ &\quad + k_3\mathrm{sgn}(u_1(t) - u_1^*(t))\left\{-e_1(t)[u_1(t) - u_1^*(t)] - d_1(t)[x_1(t) - x_1^*(t)]\right\} \\ &\quad + k_4\mathrm{sgn}(u_2(t) - u_2^*(t))\left\{-e_2(t)[u_2(t) - u_2^*(t)] + d_2(t)[x_2(t) - x_2^*(t)]\right\} \\ &= \left[-k_1a_{11}(t) + k_2a_{21}(t) + k_3d_1(t)\right]\left|x_1(t) - x_1^*(t)\right| + \left[k_1a_{12}(t) - k_2a_{22}(t) + k_4d_2(t)\right]\left|x_2(t) - x_2^*(t)\right| \\ &\quad + \left[k_1c_1(t) - k_3e_1(t)\right]\left|u_1(t) - u_1^*(t)\right| + \left[k_2c_2(t) - k_4e_2(t)\right]\left|u_2(t) - u_2^*(t)\right|. \end{aligned}$$

It follows from (4.2) that

$$D^+V(t) \leq -\sum_{i=1}^{2} A_i(t)\left|x_i(t) - x_i^*(t)\right| - \sum_{i=1}^{2} A_{i+2}(t)\left|u_i(t) - u_i^*(t)\right|. \tag{4.3}$$

By (4.1), there are positive constants α and T^* such that

$$A_i(t) \geq \alpha > 0 \quad \text{for all } t \geq T^* \quad \text{and} \quad i = 1, 2, 3, 4. \tag{4.4}$$

Integrating both sides of (4.3) on interval $[T^*, t]$,

$$V(T^*) \geq V(t) + \sum_{i=1}^{2}\int_{T^*}^{t} A_i(s)\left|x_i(s) - x_i^*(s)\right| \mathrm{d}s + \sum_{i=1}^{2}\int_{T^*}^{t} A_{i+2}(t)\left|u_i(t) - u_i^*(t)\right| \mathrm{d}s. \tag{4.5}$$

It follows from (4.4) and (4.5) that

$$V(T^*) \geq V(t) + \sum_{i=1}^{2}\int_{T^*}^{t}\left\{\alpha\left|x_i(s) - x_i^*(s)\right| + \alpha\left|u_i(t) - u_i^*(t)\right|\right\}\mathrm{d}s.$$

Therefore, $V(t)$ is bounded on $[T^*, \infty]$ and

$$\int_{T^*}^{\infty}\left|x_i(s) - x_i^*(s)\right| \mathrm{d}s < \infty, \qquad \int_{T^*}^{\infty}\left|u_i(s) - u_i^*(s)\right| \mathrm{d}s < \infty \quad i = 1, 2.$$

By Theorem 3.1, $|x_i(s) - x_i^*(s)|$ and $|u_i(s) - u_i^*(s)|$ $(i = 1, 2)$ are bounded on $[T^*, \infty]$.

On the other hand, it is easy see that $\dot{x}_i(t), \dot{x}_i^*(t), \dot{u}_i(t)$ and $\dot{u}_i^*(t)$ $(i = 1, 2)$ are bounded for $t \geq T^*$. Therefore, $|x_i(t) - x_i^*(t)|$ and $|u_i(t) - u_i^*(t)|$ $(i = 1, 2)$ are uniformly continuous on $[T^*, \infty]$. By Barbalat's Lemma ([3], Lemmas 1.2.2 and 1.2.3), we conclude that

$$\lim_{t\to\infty} \left|x_i(t) - x_i^*(t)\right| = 0, \qquad \lim_{t\to\infty} \left|u_i(t) - u_i^*(t)\right| = 0, \quad i = 1, 2.$$

This completes the proof. □

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