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Permanence and stability in multi-species non-autonomous Lotka–Volterra competitive systems with delays and feedback controls[☆]

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ABSTRACT

In this paper, we consider a multi-species Lotka–Volterra type competitive system with delays and feedback controls. A general criteria on the permanence is established, which is described by integral form and independent of feedback controls. By constructing suitable Lyapunov functionals, a set of easily verifiable sufficient conditions are derived for global stability of any positive solution to the model.

1. Introduction

In this paper, we consider the non-autonomous n-species Lotka–Volterra type competitive systems with delays and feedback controls

$$\begin{aligned}\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &= x_i(t)\left[b_i(t) - a_i(t)x_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t-\tau_{ij}(t)) - d_i(t)u_i(t-\delta_i(t))\right]\\ \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} &= r_i(t) - e_i(t)u_i(t) + c_i(t)x_i(t-\sigma_i(t)), \quad i = 1, 2, \ldots, n,\end{aligned} \tag{1.1}$$

where $x_i(t)$ represents the population density of the i-th species at time t, $u_i(t)$ denotes "indirect control" variable (see [1,14,16]); the functions $a_i(t)$, $b_i(t)$, $c_i(t)$, $d_i(t)$, $e_i(t)$, $r_i(t)$, $\delta_i(t)$, $\sigma_i(t)$, $\tau_{ij}(t)$ and $a_{ij}(t)$ $(i, j = 1, 2, \ldots, n)$ are defined on $R = (-\infty, \infty)$.

In particular, when the delays $\delta_i(t) \equiv 0$, $\sigma_i(t) \equiv 0$ and $\tau_{ij}(t) \equiv 0$ for all $t \in R$ and $i, j = 1, 2, \ldots, n$, then the system (1.1) will degenerate into the following non-delayed non-autonomous n-species Lotka–Volterra system

$$\begin{aligned}\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &= x_i(t)\left[b_i(t) - \sum_{j=1}^{n} b_{ij}(t)x_j(t) - d_i(t)u_i(t)\right]\\ \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} &= r_i(t) - e_i(t)u_i(t) + c_i(t)x_i(t), \quad i = 1, 2, \ldots, n,\end{aligned} \tag{1.2}$$

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where $b_{ii}(t) = a_i(t) + a_{ii}(t)$ and $b_{ij}(t) = a_{ij}(t)$ for $i, j = 1, 2, \ldots, n$ and $i \neq j$. We remark that the method we develop for (1.1) can be used to discuss the special system (1.2).

As we well know, systems like (1.1) and (1.2) without feedback controls are very important in the models of multi-species populations dynamics. Many important results on the permanence, extinction, global asymptotical stability for the two species or multi-species non-autonomous Lotka–Volterra systems and their special cases of periodic and almost periodic systems can be found in [3–5,11–13,16,19,23–27] and the references therein.

However, we note that ecosystems in the real world are continuously distributed by unpredictable forces which can result in changes in the biological parameters such as survival rates. In ecology, a question of practical interest is whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control theory, we call the disturbance functions as control variables. In 1993, Gopalsamy and Weng [1] introduced a feedback control variable into the delayed logistic model and discussed the asymptotic behavior of solutions in logistic models with feedback controls, in which the control variables satisfy certain differential equation. In the last decade, much work has been done on the ecosystem with feedback controls (see [1,2,6–8,10,15,17,21,22,28,29] and the references therein). In particular, Li and Liu [2], Lalli et al. [8], Li and Wang [10], Liu and Xu [21], Chen and Li [28], Tang and Zou [29] have studied delay equations with feedback controls.

The main purpose of this paper is to study the system (1.1) in the general non-autonomous case. We will establish a new general criteria for the permanence of system (1.1), which is described by integral form and independent of feedback controls. In addition, sufficient conditions are obtained for the global stability of any positive solution to system (1.1). The structure of this paper is as follows. In the next section, some useful lemmas are presented. In Section 3, we state and prove the sufficient conditions on the permanence and global stability of any positive solutions for system (1.1). We apply our criteria for some well-known special cases of system (1.1) to illustrate the generality of our results in Section 4.

2. Preliminaries

First we consider the following single-species non-autonomous logistic system

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)[a(t) - b(t)y(t)], \tag{2.1}$$

where $a(t)$ and $b(t)$ are bounded continuous functions defined on R. We note that for any $(t_0, y_0) \in R_{+0} \times R_+$, system (2.1) has a unique solution $y(t)$ satisfying $y(t_0) = y_0$, where $R_{+0} = [0, \infty)$ and $R_+ = (0, \infty)$. If $y(t) > 0$ on the interval of existence, then $y(t)$ is said to be a positive solution. It is easy to see that $y(t)$ is positive if and only if the initial value $y(t_0) > 0$.

Let $y^*(t)$ be a positive solution of system (2.1) defined on R_{+0}. We say that $y^*(t)$ is globally uniformly attractive on R_{+0}, if for any constants $\eta > 1$ and $\varepsilon > 0$, there is a constant $T(\eta, \varepsilon) > 0$ such that for any initial time $t_0 \in R_{+0}$ and any solution $y(t)$ of system (2.2) with $y(t_0) \in [\eta^{-1}, \eta]$, one has $|y(t) - y^*(t)| < \varepsilon$ for all $t \geq t_0 + T(\eta, \varepsilon)$. By Lemma 1 given in [9], we have the following result.

Lemma 2.1. *Suppose that* $\inf_{t\geq 0} b(t) \geq 0$ *and there are positive constants* ω_1 *and* ω_2 *such that* $\liminf_{t\to\infty} \int_t^{t+\omega_1} a(s)\,\mathrm{d}s > 0$ *and* $\liminf_{t\to\infty} \int_t^{t+\omega_2} b(s)\,\mathrm{d}s > 0$. *Then*

(a) *there is a constant* $M > 1$ *such that*

$$M^{-1} \leq \liminf_{t\to\infty} y(t) \leq \limsup_{t\to\infty} y(t) \leq M$$

for any positive solution $y(t)$ *of system* (2.1).

(b) *each fixed positive solution* $y^*(t)$ *of system* (2.1) *is globally uniformly attractive on* R_{+0}.

Now, we consider the following first order linear differential equation with a parameter

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = g(t, \beta) - c(t)v(t), \tag{2.2}$$

where $g(t, \beta)$ is a continuous bounded function defined on $(t, \beta) \in R_{+0} \times [0, \beta_0]$, β_0 a constant, and $c(t)$ a continuous function defined on R. For system (2.2) we introduce the following assumptions.

(A_1) Function $g(t, \beta)$ satisfies the Lipschitz condition with $\beta \in [0, \beta_0]$, i.e., there is a constant $L = L(\beta_0) > 0$ such that $|g(t, \beta_1) - g(t, \beta_2)| \leq L|\beta_1 - \beta_2|$ for all $t \in R$ and $\beta_1, \beta_2 \in [0, \beta_0]$.

(A_2) Function $c(t)$ is non-negative bounded on R_{+0} and there is a constant $\omega_3 > 0$ such that $\liminf_{t\to\infty} \int_t^{t+\omega_3} c(s)\,\mathrm{d}s > 0$.

In system (2.2), when parameter $\beta = 0$ we obtain the following system

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = g(t, 0) - c(t)v(t). \tag{2.3}$$

For any $\theta > 0$, let $v(t)$ be a solution of system (2.3) with initial value $v(t_0) \in [0, \theta]$. It is easy to prove that for all $t \geq t_0$, $v(t) \geq 0$ if the initial value $v(t_0) \geq 0$, and $v(t) > 0$ if the initial value $v(t_0) > 0$. By Lemma 3 given in [9], we have the following results.

Lemma 2.2. *Suppose that conditions* (A_1) *and* (A_2) *hold. Then*

(a) *there is a constant* $M > 0$ *such that* $\limsup_{t\to\infty} v(t) \le M$ *for any positive solution* $v(t)$ *of system* (2.3).
(b) *if there is a constant* $\omega_5 > 0$ *such that* $\liminf_{t\to\infty} \int_t^{t+\omega_5} g(s,0)\,ds > 0$*, then there is a constant* $\eta > 1$ *such that* $\eta^{-1} \le \liminf_{t\to\infty} v(t) \le \limsup_{t\to\infty} v(t) \le \eta$ *for any positive solution* $v(t)$ *of system* (2.3).
(c) *if the condition of the* (b) *holds, then each fixed positive solution* $v^*(t)$ *of system* (2.3) *is globally uniformly attractive on* R_{+0}.

Let $v_0 \in R_+$, $t_0 \in R_{+0}$ and $\beta \in [0,\ \beta_0]$, and further let $v_\beta(t)$ and $v_0(t)$ be the solutions of systems (2.2) and (2.3) with initial value $v_\beta(t_0) = v_0$ and $v_0(t_0) = v_0$, respectively. So we can get the follow result.

Lemma 2.3. *Suppose that assumptions* (A_1) *and* (A_2) *hold, then* $v_\beta(t)$ *converges to* $v_0(t)$ *uniformly for* $t \in [t_0,\ \infty)$ *as* $\beta \to 0$.

Proof. Let $V(t) = |v_\beta(t) - v_0(t)|$. By assumptions (A_1) and (A_2) we obtain the Dini derivative

$$\begin{aligned} D^+V(t) &= \operatorname{sgn}(v_\beta(t) - v_0(t))\left\{-c(t)[v_\beta(t) - v_0(t)] + g(t,\beta) - g(t,0)\right\} \\ &\le -c(t)|v_\beta(t) - v_0(t)| + |g(t,\beta) - g(t,0)| \\ &\le -c(t)V(t) + \beta L \end{aligned} \tag{2.4}$$

for all $t \ge t_0$ and $\beta \in [0,\ \beta_0]$. From (A_2) and $v_\beta(t_0) = v_0(t_0) = v_0$, by the famous comparison theorem and the variation of constants formula of solutions for first-order linear differential equation, we can get from (2.4) that $V(t) \to 0$ uniformly for $t \in [t_0,\ \infty)$ as $\beta \to 0$, that is, $v_\beta(t)$ converges to $v_0(t)$ uniformly for $t \in [t_0,\ \infty)$ as $\beta \to 0$. This completes the proof. □

Remark 2.1. In system (2.3), if $g(t,0) \equiv 0$, then system (2.3) has a trivial equilibria $E = 0$, and E is globally asymptotically stable. For any $\Gamma > 0$ and $t_0 \in R_{+0}$, let $\beta \in [0,\ \beta_0]$ and $v_\beta(t)$ be the positive solution of system (2.2) with initial value $v_\beta(t_0) \in [0,\ \Gamma]$. By Lemmas 2.2 and 2.3, we further have the following result: the solution $v_\beta(t)$ converges to 0, as $\beta \to 0$ and $t \to \infty$, i.e., for any $\varepsilon > 0$, there are positive constants $T = T(\varepsilon, \Gamma)$ and $\delta = \delta(\varepsilon)$ such that $v_\beta(t) < \varepsilon$ for all $t \ge t_0 + T$ and $\beta < \delta$.

For the convenience of statement in the following of this paper, we introduce the definition on permanence.

Definition 2.1. System (1.1) is said to be permanent, if there are positive constants m and M such that

$$m \le \liminf_{t\to\infty} x_i(t) \le \limsup_{t\to\infty} x_i(t) \le M \quad (i = 1, 2, \ldots, n)$$

for any positive solution $(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_n(t))$ of system (1.1).

Remark 2.2. In system (1.1), $u_i(t)$ $(i = 1, 2, \ldots, n)$ are control variables, so we do not consider the permanence of control variables.

3. Main results

For system (1.1), we introduce the following assumptions. Let $t_0 \in R$ be a fixed initial time.

(H_1) Functions $a_i(t), b_i(t), c_i(t), d_i(t), e_i(t), r_i(t), \delta_i(t), \sigma_i(t), \tau_{ij}(t)$ and $a_{ij}(t)$ are bounded continuous on $[t_0,\ \infty)$, and $a_i(t)$, $c_i(t), d_i(t), e_i(t), r_i(t), \delta_i(t), \sigma_i(t), \tau_{ij}(t)$ and $a_{ij}(t)$ $(i, j = 1, 2, \ldots, n)$ are non-negative for all $t \ge t_0$.
(H_2) For each $1 \le i \le n$, there are positive constants ω_i and α_i such that

$$\liminf_{t\to\infty} \int_t^{t+\omega_i} a_i(s)\,ds > 0, \qquad \liminf_{t\to\infty} \int_t^{t+\alpha_i} e_i(s)\,ds > 0.$$

Let $\tau = \sup\{\tau_{ij}(t),\ \delta_i(t),\ \sigma_i(t) : t \ge t_0,\ i,\ j = 1, 2, \ldots, n\}$. We define $C^n[-\tau,\ 0]$ the Banach space of bounded continuous functions $\phi : [-\tau,\ 0] \to R^n$ with the supremum norm defined by

$$\|\phi\|_c = \sup_{-\tau\le s\le 0} |\phi(s)|,$$

where $\phi = (\phi_1, \phi_2, \ldots, \phi_n)$ and $|\phi(s)| = \sum_{i=1}^n |\phi_i(s)|$. By the fundamental theory of functional differential equations [13,16,18], we know that for any $(\phi, \psi) \in C^n[-\tau,\ 0] \times C^n[-\tau,\ 0]$, system (1.1) has a unique solution

$$X(t,\phi,\psi) = (x_1(t,\phi), x_2(t,\phi), \ldots, x_n(t,\phi), u_1(t,\psi), u_2(t,\psi), \ldots, u_n(t,\psi))$$

satisfying the initial condition $X_{t_0}(\cdot,\phi,\psi) = (\phi,\psi)$.

Defined $C_+^n[-\tau,\ 0] = \{\phi = (\phi_1, \phi_2, \ldots, \phi_n) \in C^n[-\tau,\ 0] : \phi_i(s) \ge 0$ and $\phi_i(0) > 0$ for all $s \in [-\tau,\ 0]$ and $i = 1, 2, \ldots, n\}$. Motivated by the biological background of system (1.1), in this paper we are only concerned with positive solutions of system (1.1). It is not difficult to see that the solution $X(t,\phi,\psi)$ of system (1.1) is positive, if the initial function $(\phi,\psi) \in C_+^n[-\tau,\ 0] \times C_+^n[-\tau,\ 0]$.

First, we have the theorem on the ultimate boundedness of all positive solutions of system (1.1).

Theorem 3.1. *Suppose that assumptions* (H_1) *and* (H_2) *hold, then system* (1.1) *is ultimately bounded, in the sense that there are positive constants* M *and* T *such that if* $t > T$, *then* $x_i(t) \leq M$ *and* $u_i(t) \leq M$ $(i = 1, 2, \ldots, n)$ *for all positive solutions* $X(t) = (x(t), u(t))$ *of system* (1.1).

Proof. Let $X(t) = (x(t), u(t))$ be any positive solution of system (1.1). We first prove that the components $x_i (i = 1, 2, \ldots, n)$ of system (1.1) is ultimately bounded. From assumption (H_1) and the i-th equation of system (1.1) we have

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} \leq x_i(t)[b_i(t) - a_i(t)x_i(t)].$$

It has been shown (see, for example [30,31]) that under assumptions (H_1) and (H_2) any positive solution $y(t)$ of the following nonautonomous system

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)[b_i(t) - a_i(t)y(t)]$$

is ultimately bounded on R_{+0}. Hence, by the comparison theorem, we can obtain a constant M_{i1} such that for any positive solution $(x(t), u(t))$ of system (1.1), there is a $T_{i1} > 0$ such that $x_i(t) < M_{i1}$ for all $t \geq T_{i1}$. Let $M_1 = \max_{1\leq i\leq n}\{M_{i1}\}$ and $T_1 = \max_{1\leq i\leq n}\{T_{i1}\}$, we have

$$x_i(t) \leq M_1 \quad \text{for all } t \geq T_1,\ i = 1, 2, \ldots, n.$$

Further, from the above, assumption (H_1) and the $(n + i)$-th equation of system (1.1) we have

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} \leq r_i(t) - e_i(t)u_i(t) + c_i(t)M_1$$

for all $t \geq T_1 + \tau$. From assumptions (H_1) and (H_2), we have that $r_i(t)$, $e_i(t)c_i(t)$ are non-negative and $\liminf_{t\to\infty} \int_t^{t+\alpha_i} e_i(s)\,\mathrm{d}s > 0$. Hence, using the comparison theorem and the conclusion (a) of Lemma 2.2; further, we can get a constant $M_{i2} > 0$ such that for any positive solution $(x(t), u(t))$ of system (1.1) there is a $T_{i2} \geq T_1 + \tau$ such that $u_i(t) < M_{i2}$ for all $t \geq T_{i2}$.

Now, we let $M = \max\{M_1, M_{12}, \ldots, M_{n2}\}$ and $T = \max\{T_1, T_{12}, \ldots, T_{n2}\}$, then for all $t \geq T$

$$x_i(t) \leq M, \qquad u_i(t) \leq M, \quad i = 1, 2, \ldots, n.$$

Therefore, the solution $X(t) = (x(t), u(t))$ is ultimately bounded. This completes the proof. □

In order to obtain the permanence of system (1.1), we consider the following two auxiliary systems

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = x_i(t)[b_i(t) - a_i(t)x_i(t)] \tag{3.1}$$

and

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = r_i(t) - e_i(t)u_i(t), \tag{3.2}$$

where $i = 1, 2, \ldots, n$. Let $x_{i0}(t)$ and $u_{i0}(t)$ be some positive solutions of systems (3.1) and (3.2), respectively.

Remark 3.1. If $r_i(t) \equiv 0$, then system (3.2) has a trivial equilibria $E = 0$, and E is globally asymptotically stable. In this case, we let $u_{i0}(t) = 0$ $(i = 1, 2, \ldots, n)$.

Now, we state and prove our result on the permanence of system (1.1).

Theorem 3.2. *Suppose that assumptions* (H_1) *and* (H_2) *hold. If there exist positive constants* γ_i *and* λ_i *such that for each* $1 \leq i \leq n$,

$$\liminf_{t\to\infty} \int_t^{t+\gamma_i} \left[b_i(s) - \sum_{j=1, j\neq i}^{n} a_{ij}(s)x_{j0}(s - \tau_{ij}(s)) - d_i(s)u_{i0}(s - \delta_i(s)) \right] \mathrm{d}s > 0 \tag{3.3}$$

and

$$\liminf_{t\to\infty} \int_t^{t+\lambda_i} r_i(s)\,\mathrm{d}s > 0, \tag{3.4}$$

then system (1.1) *is permanent.*

Proof. Let $X(t) = (x(t), u(t))$ be any positive solution of system (1.1). From Theorem 3.1, there is a constant $M > 0$ satisfying for any positive solution $X(t)$ of system (1.1), there is a $T_1 \geq t_0$

$$0 < x_i(t) < M, \qquad 0 < u_i(t) < M, \quad \text{for all } t \geq T_1,\ i = 1, 2, \ldots, n.$$

Therefore, from assumption (H_1) and the i-th equation of system (1.1) we have

$$\begin{aligned}\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &\geq x_i(t)\left[b_i(t) - a_i(t)M - \sum_{j=1}^{n} a_{ij}(t)M - d_i(t)M\right] \\ &\geq -\alpha_i x(t)\end{aligned} \tag{3.5}$$

for all $t \geq T_1 + \tau$, where $\alpha_i = \sup_{t\geq t_0}\{|b_i(t) - a_i(t)M - \sum_{j=1}^{n} a_{ij}(t)M - d_i(t)M|\}$. For any $t \geq T_1 + \tau$, integrating (3.5) from $t - \sigma_i(t)$ to t we obtain

$$x_i(t - \sigma_i(t)) \leq x_i(t)\exp(\alpha_i\sigma_i(t)) \leq x_i(t)\exp(\alpha_i\tau) \leq x_i(t)\exp(\alpha\tau), \tag{3.6}$$

where $\alpha = \max_{1\leq i\leq n}\{\alpha_i\}$ and $i = 1, 2, \ldots, n$.

On the other hand, from assumption (H_1) and the i-th equation of system (1.1) we have

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} \leq x_i(t)[b_i(t) - a_i(t)x_i(t)].$$

The comparison equation is system (3.1). From assumptions (H_1) and (H_2) we have $a_i(t) \geq 0$ and $\liminf_{t\to\infty}\int_t^{t+\omega_i} a_i(t)\,\mathrm{d}t > 0$. Further, by (3.3) it follows that $\liminf_{t\to\infty}\int_t^{t+\gamma_i} b_i(t)\,\mathrm{d}t > 0$. So, the system (3.1) satisfies all conditions of Lemma 2.1. Hence, by Lemma 2.1, each positive solution of system (3.1) is globally asymptotically stable. Therefore, by the comparison theorem, and since $x_{i0}(t)$ is be globally uniformly attractive positive solution of system (3.1), we obtain for any $\epsilon > 0$ that there is a constant $t_{i1} = t_{i1}(\epsilon) > 0$ such that

$$x_i(t) \leq x_{i0}(t) + \epsilon \quad \text{for all } t \geq t_{i1}.$$

Let $t_1 = \max_{1\leq i\leq n}\{t_{i1}\}$, then for all $t \geq t_1$

$$x_i(t) \leq x_{i0}(t) + \epsilon, \quad i = 1, 2, \ldots, n. \tag{3.7}$$

For any t_2, t_3 and $t_3 \geq t_2 \geq 0$, integrating directly system (1.1) we have

$$x_i(t_3) = x_i(t_2)\exp\int_{t_2}^{t_3}\left[b_i(t) - a_i(t)x_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t - \tau_{ij}(t)) - d_i(t)u_i(t - \delta_i(t))\right]\mathrm{d}t. \tag{3.8}$$

Claim 3.1. *There is a constant $\beta > 0$ such that $\limsup_{t\to\infty} x_i(t) > \beta$ $(i = 1, 2, \ldots, n)$ for any positive solution $X(t) = (x(t), u(t))$ of system (1.1).*

In fact, by (3.3), we can choose positive constants $T_2 \geq T_1$, ε and δ such that

$$\int_t^{t+\gamma_i}\left\{b_i(s) - [a_i(s) + a_{ii}(s)]\,\varepsilon - \sum_{j=1, j\neq i}^{n} a_{ij}(s)\left[x_{j0}(s - \tau_{ij}(s)) + \varepsilon\right] - d_i(s)\left[u_{i0}(s - \delta_i(s)) + \varepsilon\right]\right\}\mathrm{d}s > \delta. \tag{3.9}$$

for all $t \geq T_2$ and $i = 1, 2, \ldots, n$.

By assumptions (H_1), (H_2) and (3.4), we have $r_i(t)$ and $e_i(t)$ are non-negative and $\liminf_{t\to\infty}\int_t^{t+\lambda_i} r_i(t)\,\mathrm{d}t > 0$, $\liminf_{t\to\infty}\int_t^{t+\alpha_i} e_i(t)\,\mathrm{d}t > 0$ $(i = 1, 2, \ldots, n)$. Therefore, system (3.2) satisfies all conditions of the (b) and (c) of Lemma 2.2. So, by Lemma 2.2, each positive solution of system (3.2) is globally asymptotically stable.

Consider the following system with one parameter

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = r_i(t) - e_i(t)u_i(t) + c_i(t)\beta\exp(\alpha\tau), \tag{3.10}$$

where $\beta \in [0, \ \beta_0]$ is the parameter and $i = 1, 2, \ldots, n$. Let $u_{i\beta}(t)$ be a solution of system (3.10). Similar, by assumptions (H_1), (H_2) and (3.4), we see that system (3.10) satisfies all conditions of the (c) of Lemmas 2.2 and 2.3. By the conclusion (c) of Lemma 2.2, $u_{i\beta}(t)$ is globally asymptotically stable. Further, by Lemma 2.3 we obtain that $u_{i\beta}(t)$ uniformly for $t \in R_+$ converges to $u_{i0}(t)$, as $\beta \to 0$. Hence, there is a constant $\beta > 0$ and $\beta < \varepsilon$ such that for all $t \geq T_2$

$$u_{i\beta}(t) \leq u_{i0}(t) + \frac{\varepsilon}{2}, \quad i = 1, 2, \ldots, n. \tag{3.11}$$

If Claim 3.1 is not true, then there is an integer $k \in \{1, 2, \ldots, n\}$ and a positive solution $(x(t), u(t))$ of system (1.1) such that $\limsup_{t\to\infty} x_k(t) < \beta$. Hence, there is a constant $T_3 > T_2$ such that $x_k(t) < \beta$ for all $t \geq T_3$. From assumption (H_1), (3.6) and the $(n + k)$-th equation of system (1.1) we obtain

$$\frac{\mathrm{d}u_k(t)}{\mathrm{d}t} \le r_k(t) - e_k(t)u_k(t) + c_k(t)\beta\exp(\alpha\tau) \quad \text{for all } t \ge T_3.$$

Using the comparison theorem and globally asymptotically stability of solution $u_{k\beta}(t)$, we obtain that there is a $T_4 \ge T_3$ such that

$$u_k(t) \le u_{k\beta}(t) + \frac{\varepsilon}{2} \quad \text{for all } t \ge T_4. \tag{3.12}$$

Therefore, from (3.11) and (3.12) it follows that

$$u_k(t) \le u_{k0}(t) + \varepsilon \quad \text{for all } t \ge T_4. \tag{3.13}$$

On the other hand, by (3.7) there is a $T_5 \ge T_4$ such that

$$x_i(t) \le x_{i0}(t) + \varepsilon \quad \text{for all } t \ge T_5, \tag{3.14}$$

where $i = 1, 2, \ldots, n$ and $i \ne k$.

By (3.8), (3.13) and (3.14) and assumption (H_1) we obtain

$$\begin{aligned} x_k(t) &= x_k(T_5+\tau)\exp\int_{T_5+\tau}^{t}\left[b_i(s) - a_i(s)x_i(s) - \sum_{j=1}^{n} a_{ij}(s)x_j(s-\tau_{ij}(s)) - d_i(s)u_i(s-\delta_i(s))\right]\mathrm{d}s \\ &\ge x_k(T_5+\tau)\exp\int_{T_5+\tau}^{t}\Bigg\{b_i(t) - [a_i(t)+a_{ii}(t)]\,\varepsilon \\ &\quad - \sum_{j=1, j\ne i}^{n} a_{ij}(s)\left[x_{j0}(s-\tau_{ij}(s))+\varepsilon\right] - d_i(s)\left[u_{i0}(s-\delta_i(s))+\varepsilon\right]\Bigg\}\,\mathrm{d}s \end{aligned}$$

for all $t \ge T_5 + \tau$. Thus from (3.9) we finally obtain $\lim_{t\to\infty} x_k(t) = \infty$, which leads to a contradiction. Therefore, Claim 3.1 is true.

Claim 3.2. *There is a constant $\gamma > 0$ such that $\liminf_{t\to\infty} x_i(t) > \gamma$ $(i = 1, 2, \ldots, n)$ for any positive solution $X(t)$ of system (1.1).*

In fact, if Claim 3.2 is not true, then there is an integer $k \in \{1, 2, \ldots, n\}$ and a sequence of the initial value $\{X_m = (\phi^{(m)}, \psi^{(m)})\} \subset C_+^n \times C_+^n$ such that, for the solution $(x(t, X_m), u(t, X_m))$ of system (1.1),

$$\liminf_{t\to\infty} x_k(t, X_m) < \frac{\beta}{m^2}, \quad m = 1, 2, \ldots,$$

where constant β is given in Claim 3.1. By Claim 3.1, for every m there are two time sequences $\{s_q^{(m)}\}$ and $\{t_q^{(m)}\}$, satisfying $0 < s_1^{(m)} < t_1^{(m)} < s_2^{(m)} < t_2^{(m)} < \cdots < s_q^{(m)} < t_q^{(m)} < \cdots$ and $\lim_{q\to\infty} s_q^{(m)} = \infty$, such that

$$x_k(s_q^{(m)}, X_m) = \frac{\beta}{m}, \qquad x_k(t_q^{(m)}, X_m) = \frac{\beta}{m^2}, \tag{3.15}$$

and

$$\frac{\beta}{m^2} < x_k(t, X_m) < \frac{\beta}{m} \quad \text{for all } t \in (s_q^{(m)},\ t_q^{(m)}). \tag{3.16}$$

From Theorem 3.1, we obtain that system system (1.1) is ultimate boundedness. Therefore, we can choose a positive constant $T^{(m)}$ for every m such that $x_i(t, X_m) < M$ and $u_i(t, X_m) < M$ for all $t > T^{(m)}$ and $i = 1, 2, \ldots, n$. Further, there is an integer $K_1^{(m)} > 0$ such that $s_q^{(m)} > T^{(m)} + \tau$ for all $q > K_1^{(m)}$. Let $q > K_1^{(m)}$, for any $t \in [s_q^{(m)},\ t_q^{(m)}]$, by assumption (H_1) we have

$$\begin{aligned} \frac{\mathrm{d}x_k(t, X_m)}{\mathrm{d}t} &\ge x_k(t, X_m)\left[b_k(t) - a_k(t)M - \sum_{j=1}^{n} a_{kj}(t)M - d_k(t)M\right] \\ &\ge -\gamma_0 x_k(t, X_m), \end{aligned}$$

where $\gamma_0 = \sup_{t\ge t_0}\{|b_k(t) - a_k(t)M - \sum_{j=1}^{n} a_{kj}(t)M - d_k(t)M|\}$. Integrating the above inequality from $s_q^{(m)}$ to $t_q^{(m)}$, we further have

$$x_k(t_q^{(m)}, X_m) \ge x_k(s_q^{(m)}, X_m)\exp[-\gamma_0(t_q^{(m)} - s_q^{(m)})].$$

Consequently, by (3.15)

$$\frac{\beta}{m^2} \ge \frac{\beta}{m}\exp[-\gamma_0(t_q^{(m)} - s_q^{(m)})].$$

Then,

$$t_q^{(m)} - s_q^{(m)} \geq \frac{\ln m}{\gamma_0} \quad \text{for all } q > K_1^{(m)}.$$

By (3.9), there are positive constants P and ϱ such that

$$\int_t^{t+\kappa} \left\{ b_i(t) - [a_i(t) + a_{ii}(t)]\,\varepsilon - \sum_{j=1, j\neq i}^{n} a_{ij}(s) \left[x_{j0}(s - \tau_{ij}(s)) + \varepsilon \right] - d_i(s) \left[u_{i0}(s - \delta_i(s)) + \varepsilon \right] \right\} \mathrm{d}s > \varrho.$$

for all $t \geq 0, \kappa \geq P$ and $i = 1, 2, \ldots, n$.

Let $\widetilde{u}_{k\beta}(t)$ be the solution of system (3.10) with the initial condition $\widetilde{u}_{k\beta}(t) = u_k(s_q^{(m)}, X_m)$. By (3.6) and (3.16) and assumption (H_1), we have

$$\frac{\mathrm{d}u_k(t, X_m)}{\mathrm{d}t} \leq r_k(t) - e_k(t) u_k(t, X_m) + c_k(t) \beta \exp(\alpha \tau)$$

for any m, q and $t \in [s_q^{(m)},\ t_q^{(m)}]$. Using the comparison theorem it follows that

$$u_k(t, X_m) \leq \widetilde{u}_{k\beta}(t) \quad \text{for all } t \in [s_q^{(m)},\ t_q^{(m)}]. \tag{3.17}$$

By Lemma 2.2, the solution $u_{k\beta}(t)$ of system (3.10) is globally uniformly attractive on R_{+0}, and so there is a constant $T_2 \geq P$, which is independent of any m and $q \geq K^{(m)}$, such that

$$\widetilde{u}_{k\beta}(t) \leq u_{k\beta}(t) + \frac{\varepsilon}{2} \quad \text{for all } t \geq s_q^{(m)} + T_2. \tag{3.18}$$

On the other hand, by (3.6) there is a $T_3 \geq T_2$ which is independent of any m and $q \geq K^{(m)}$, such that

$$x_i(t, X_m) \leq x_{i0}(t) + \varepsilon \quad \text{for all } t \geq T_3, \tag{3.19}$$

where $i = 1, 2, \ldots, n$ and $i \neq k$. Choose an integer $N_0 > 0$ such that for $m \geq N_0$ and $q \geq K^{(m)}$,

$$t_q^{(m)} - s_q^{(m)} > T_3 + P.$$

Further, from (3.11), (3.17) and (3.18) we obtain

$$u_k(t, X_m) \leq u_{k0}(t) + \varepsilon \quad \text{for all } t \in [s_q^{(m)} + T_3,\ t_q^{(m)}]. \tag{3.20}$$

So, when $m \geq N_0$ and $q \geq K^{(m)}$, by (3.8), (3.19) and (3.20) and assumption (H_1) it follows

$$\begin{aligned}
\frac{\beta}{m^2} &= x_k(s_q^{(m)} + T_3, X_m) \exp \int_{s_q^{(m)}+T_3}^{t_q^{(n)}} \Bigg[b_k(t) - a_k(t) x_k(t, X_m) \\
&\quad - \sum_{j=1}^{n} a_{kj}(t) x_j(t - \tau_{kj}(t), X_m) - d_k(t) u_k(t - \delta_k(t), X_m) \Bigg] \mathrm{d}t \\
&\geq x_k(s_q^{(m)} + T_3, X_m) \exp \int_{s_q^{(m)}+T_3}^{t_q^{(n)}} \Bigg\{ b_k(t) - [a_k(t) + a_{kk}(t)]\,\varepsilon \\
&\quad - \sum_{j=1, j\neq k}^{n} a_{kj}(s) \left[x_{j0}(s - \tau_{kj}(s)) + \varepsilon \right] - d_k(s) \left[u_{k0}(s - \delta_k(s)) + \varepsilon \right] \Bigg\} \mathrm{d}t \\
&> \frac{\beta}{m^2}
\end{aligned}$$

which leads to a contradiction. Therefore, Claim 3.2 is true.

From Claims 3.1 and 3.2 we complete the proof of this theorem. $\square$

Remark 3.2. From the proof of Theorem 3.2, we note that $u_0(t)$ be some fixed positive solution of system (3.2), which is independent of the feedback controls. So, the feedback controls have no influence on the permanence of system (1.1).

In system (1.1), when functions $r_i(t) \equiv 0$ $(i = 1, 2, \ldots, n)$, we have the following system

$$\begin{aligned}
\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &= x_i(t) \left[b_i(t) - a_i(t) x_i(t) - \sum_{j=1}^{n} a_{ij}(t) x_j(t - \tau_{ij}(t)) - d_i(t) u_i(t - \delta_i(t)) \right] \\
\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} &= -e_i(t) u_i(t) + c_i(t) x_i(t - \sigma_i(t)), \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{3.21}$$

In this case, let $u_{i0}(t) = 0$ $(i = 1, 2, \ldots, n)$. Similar to the proof of Theorems 3.1 and 3.2 we have the result.

Theorem 3.3. *Suppose that assumptions* (H_1) *and* (H_2) *hold. If there exist a constant* $\gamma_i > 0$ *such that*

$$\liminf_{t\to\infty} \int_t^{t+\gamma_i} \left[b_i(s) - \sum_{j=1,j\neq i}^{n} a_{ij}(s)x_{j0}(s-\tau_{ij}(s)) \right] \mathrm{d}s > 0$$

for each $1 \le i \le n$*, then system* (3.21) *is permanent.*

Using Theorem 1 given by Teng and Chen in [20] on the existence of positive periodic solutions for the general n-species periodic Kolmogorov type systems with delays, we have the theorem on the existence of positive periodic solutions for periodic system (1.1).

Theorem 3.4. *Suppose assumptions* (H_1) *and* (H_2) *hold and system* (1.1) *is* ω*-periodic. If* (3.3) *hold and there is a constant* $\nu_i > 0$ $(i = 1, 2, \ldots, n)$ *such that* $\liminf_{t\to\infty} \int_t^{t+\nu_i} r_i(s)\,\mathrm{d}s > 0$ *or* $\liminf_{t\to\infty} \int_t^{t+\nu_i} c_i(s)\,\mathrm{d}s > 0$*, then system* (1.1) *has at least a positive* ω*-periodic solution.*

Proof. By the assumptions (H_1) and (H_2) and Theorems 3.1 and 3.2, we get that the component x_i of system (1.1) is permanent, that is, there are positive constants m_1, M_1 and T_1 such that for any positive solution $X(t)$ of system (1.1) we have $m_1 \le x_i(t) \le M_1$ for all $t \ge T_1$ and $i = 1, 2, \ldots, n$. Further, from this and the $(n+i)$-th equation of system (1.1) we have

$$r_i(t) - e_i(t)u_i(t) + c_i(t)m_1 \le \frac{\mathrm{d}u(t)}{\mathrm{d}t} \le r_i(t) - e_i(t)u_i(t) + c_i(t)M_1$$

for all $t \ge T_1 + \tau$. It is easy to test that the conditions of the (b) of Lemma 2.2 are satisfied. So, using the comparison theorem and the conclusion (b) of Lemma 2.2, we further can obtain that there are positive constants m_2, M_2 and $T_2 \ge T_1 + \tau$ such that for any positive solution $X(t)$ of system (1.1) we have $m_2 \le u_i(t) \le M_2$ for all $t \ge T_2$ and $i = 1, 2, \ldots, n$. Now, we let $m = \min\{m_1, m_2\}$ and $M = \max\{M_1, M_2\}$, then for all $t \ge T_2$

$$m \le x_i(t) \le M, \qquad m \le u_i(t) \le M \quad (i = 1, 2, \ldots, n).$$

Finally, by Theorem 1 in [18], it follows that system has at least a positive ω-periodic solution and this completes the proof. $\square$

We proceed to the discussion on the global stability of any positive solution to system (1.1). We first introduce an assumption.

(H_3) Functions $\tau_{ij}(t)$, $\delta_i(t)$ and $\sigma_i(t)$ are differentiable on $t \in R_+$ and the derivatives $\dot{\tau}_{ij}(t) < 1$, $\dot{\delta}_i(t) < 1$ and $\dot{\sigma}_i(t) < 1$ for all $t \in R_+$.

On the global stability of any positive solution to system (1.1), we get the following result.

Theorem 3.5. *Let* $(x^*(t), u^*(t))$ *be some fixed positive solution of system* (1.1)*. Suppose assumptions* (H_1)–(H_3) *and* (3.3) *hold. If there are positive constants* μ_i *and* κ_i *such that*

$$\liminf_{t\to\infty} A_i(t) > 0, \qquad \liminf_{t\to\infty} B_i(t) > 0 \tag{3.22}$$

where

$$\begin{aligned} A_i(t) &= \mu_i a_i(t) - \kappa_i \frac{c_i(\beta_i^{-1}(t))}{1-\dot{\sigma}_i(\beta_i^{-1}(t))} - \sum_{j=1}^{n} \mu_j a_{ji}(t) \frac{a_{ji}(\theta_{ji}^{-1}(t))}{1-\dot{\tau}_{ji}(\theta_j^{-1}(t))} \\ B_i(t) &= \kappa_i e_i(t) - \mu_i \frac{d_i(\gamma_i^{-1}(t))}{1-\dot{\delta}_i(\gamma_i^{-1}(t))} \end{aligned} \tag{3.23}$$

in which $i, j = 1, 2, \ldots, n$, $\theta_{ij}^{-1}(t)$, $\beta_i^{-1}(t)$ *and* $\gamma_i^{-1}(t)$ *denote the inverse function of* $t-\tau_{ij}(t)$, $t-\delta_i(t)$ *and* $t-\sigma_i(t)$*, respectively, then* $(x^*(t), u^*(t))$ *is globally asymptotically stable.*

Proof. Let $(x(t), u(t))$ be any positive solution of system (1.1). It follows from Theorem 3.1 that there exist positive constants T and M such that for all $t \ge T$

$$0 < x_i^*(t), x_i(t) \le M, \qquad 0 < u_i^*(t), u_i(t) \le M, \quad i = 1, 2, \ldots, n.$$

We define a Lyapunov functional $V_{1i}(t)$ as

$$V_{1i}(t) = \mu_i |\ln x_i(t) - \ln x_i^*(t)| + \kappa_i |u_i(t) - u_i^*(t)|. \tag{3.24}$$

Calculating the upper right derivative of $V_{1i}(t)$ along solutions of system (1.1), it follows that

$$\begin{aligned} D^+V_{1i}(t) = {} & \mu_i \operatorname{sgn}(x_i(t)-x_i^*(t)) \Bigg\{ -a_i(t)\left[x_i(t)-x_i^*(t)\right] - \sum_{j=1}^n a_{ij}(t)\left[x_j(t-\tau_{ij}(t)) - x_j^*(t-\tau_{ij}(t))\right] \\ & - d_i(t)\left[u_i(t-\delta_i(t)) - u_i^*(t-\delta_i(t))\right] \Bigg\} + \kappa_i \operatorname{sgn}(u_i(t) \\ & - u_i^*(t)) \left\{ -e_i(t)\left[u_i(t)-u_i^*(t)\right] - c_i(t)\left[x_i(t-\sigma_i(t)) - x_i^*(t-\sigma_i(t))\right]\right\} \\ \le {} & -\mu_i a_i(t)\left|x_1(t)-x_1^*(t)\right| - \kappa_i e_i(t)\left|u_i(t)-u_i^*(t)\right| + \sum_{j=1}^n \mu_i a_{ij}(t)\left|x_j(t-\tau_{ij}(t)) - x_j^*(t-\tau_{ij}(t))\right| \\ & + \mu_i d_i(t)\left|u_i(t-\delta_i(t)) - u_i^*(t-\delta_i(t))\right| + \kappa_i c_i(t)\left|x_i(t-\sigma_i(t)) - x_i^*(t-\sigma_i(t))\right|. \end{aligned} \tag{3.25}$$

Define

$$V_i(t) = V_{1i}(t) + V_{2i}(t) + V_{3i}(t), \tag{3.26}$$

where

$$V_{2i} = \mu_i \int_{t-\tau_{ij}(t)}^t \sum_{j=1}^n \frac{a_{ij}(\theta_{ij}^{-1}(s))}{1-\dot{\tau}_{ij}(\theta_{ij}^{-1}(s))}\left|x_j(s)-x_j^*(s)\right| \, \mathrm{d}s + \kappa_i \int_{t-\sigma_i(t)}^t \frac{c_i(\beta_i^{-1}(s))}{1-\dot{\sigma}_i(\beta_i^{-1}(s))}\left|x_i(s)-x_i^*(s)\right| \, \mathrm{d}s \tag{3.27}$$

and

$$V_{3i} = \mu_i \int_{t-\delta_i(t)}^t \frac{d_i(\gamma_i^{-1}(s))}{1-\dot{\delta}_i(\gamma_i^{-1}(s))}\left|u_i(s)-u_i^*(s)\right| \, \mathrm{d}s. \tag{3.28}$$

From (3.24)–(3.28), we have

$$\begin{aligned} D^+V_i(t) = {} & D^+V_{1i}(t) + \dot{V}_{2i}(t) + \dot{V}_{3i}(t) \\ \le {} & -\mu_i a_i(t)\left|x_i(t)-x_i^*(t)\right| - \kappa_i e_i(t)\left|u_i(t)-u_i^*(t)\right| + \sum_{j=1}^n \mu_i a_{ij}(t)\frac{a_{ij}(\theta_{ij}^{-1}(t))}{1-\dot{\tau}_{ij}(\theta_{ij}^{-1}(t))}\left|x_j(t)-x_j^*(t)\right| \\ & + \kappa_i \frac{c_i(\beta_i^{-1}(t))}{1-\dot{\sigma}_i(\beta_i^{-1}(t))}\left|x_i(t)-x_i^*(t)\right| + \mu_i \frac{d_i(\gamma_i^{-1}(t))}{1-\dot{\delta}_i(\gamma_i^{-1}(t))}\left|u_i(t)-u_i^*(t)\right| \\ = {} & -\left[\mu_i a_i(t) - \kappa_i \frac{c_i(\beta_i^{-1}(t))}{1-\dot{\sigma}_i(\beta_i^{-1}(t))}\right]\left|x_1(t)-x_1^*(t)\right| - \left[\kappa_i e_i(t) - \mu_i \frac{d_i(\gamma_i^{-1}(t))}{1-\dot{\delta}_i(\gamma_i^{-1}(t))}\right]\left|u_i(t)-u_i^*(t)\right| \\ & + \sum_{j=1}^n \mu_i a_{ij}(t)\frac{a_{ij}(\theta_{ij}^{-1}(t))}{1-\dot{\tau}_{ij}(\theta_{ij}^{-1}(t))}\left|x_j(t)-x_j^*(t)\right|. \end{aligned} \tag{3.29}$$

We now define a Lyapunov function $V(t)$ as

$$V(t) = \sum_{i=1}^n V_i(t). \tag{3.30}$$

So, from (3.26)–(3.30), we get that

$$\begin{aligned} D^+V(t) = {} & \sum_{i=1}^n D^+V_i(t) \\ \le {} & -\sum_{i=1}^n \Bigg\{ \left[\mu_i a_i(t) - \kappa_i \frac{c_i(\beta_i^{-1}(t))}{1-\dot{\sigma}_i(\beta_i^{-1}(t))} - \sum_{j=1}^n \mu_j a_{ji}(t)\frac{a_{ji}(\theta_{ji}^{-1}(t))}{1-\dot{\tau}_{ji}(\theta_{ji}^{-1}(t))}\right]\left|x_i(t)-x_i^*(t)\right| \\ & + \left[\kappa_i e_i(t) - \mu_i \frac{d_i(\gamma_i^{-1}(t))}{1-\dot{\delta}_i(\gamma_i^{-1}(t))}\right]\left|u_i(t)-u_i^*(t)\right| \Bigg\}. \end{aligned}$$

From (3.23), we get that

$$D^+V(t) \le -\sum_{i=1}^{n} \left[A_i(t)\left|x_i(t)-x_i^*(t)\right| + B_i(t)\left|u_i(t)-u_i^*(t)\right|\right], \tag{3.31}$$

where $A_i(t)$ and $B_i(t)$ $(i = 1, 2, \ldots, n)$ are defined in (3.23).

For each $1 \le i \le n$, by (3.22), there exist positive constants α_i, η_i and $T^* \ge T$ such that

$$A_i(t) \ge \alpha_i > 0, \qquad B_i(t) \ge \eta_i > 0, \quad \text{for all } t \ge T^*. \tag{3.32}$$

Integrating both sides of (3.31) on interval $[T^*,\ t]$, yields that

$$V(T^*) \ge V(t) + \sum_{i=1}^{n} \int_{T^*}^{t} \left[A_i(s)\left|x_i(s)-x_i^*(s)\right| + B_i(t)\left|u_i(t)-u_i^*(t)\right|\right] \mathrm{d}s. \tag{3.33}$$

From (3.32) and (3.33), we have that

$$V(T^*) \ge V(t) + \sum_{i=1}^{n} \int_{T^*}^{t} \left[\alpha_i\left|x_i(s)-x_i^*(s)\right| + \eta_i\left|u_i(t)-u_i^*(t)\right|\right] \mathrm{d}s.$$

Therefore, $V(t)$ is bounded on $[T^*,\ \infty]$ and

$$\int_{T^*}^{\infty} \left|x_i(s)-x_i^*(s)\right| \mathrm{d}s < \infty, \qquad \int_{T^*}^{\infty} \left|u_i(s)-u_i^*(s)\right| \mathrm{d}s < \infty, \quad i = 1, 2, \ldots, n.$$

By Theorem 3.1, $|x_i(s)-x_i^*(s)|$ and $|u_i(s)-u_i^*(s)|$ $(i = 1, 2)$ are bounded on $[T^*,\ \infty]$.

On the other hand, it is easy to see that $\dot{x}_i(t)$, $\dot{x}_i^*(t)$, $\dot{u}_i(t)$, $\dot{u}_i^*(t)$ $(i = 1, 2, \ldots, n)$ are bounded for $t \ge T^*$. Therefore, $|x_i(t)-x_i^*(t)|$ and $|u_i(t)-u_i^*(t)|$ $(i = 1, 2, \ldots, n)$ are uniformly continuous on $[T^*,\ \infty]$. By Barbalat's Lemma [13, Lemmas 1.2.2 and 1.2.3], we conclude that

$$\lim_{t\to\infty} \left|x_i(t)-x_i^*(t)\right| = 0, \qquad \lim_{t\to\infty} \left|u_i(t)-u_i^*(t)\right| = 0, \quad i = 1, 2, \ldots, n.$$

This completes the proof. □

4. Applications

In this section, we will apply the results given in the last section to particular competition systems with delays and feedback controls or without delays and feedback controls, which have been studied extensively in the literature. The following two applications show that the derived sufficient conditions are easily verifiable and more general than those given in the literature, thus we improve and generalize some well-known results.

Example 4.1. We consider the Lotka–Volterra competition system

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t)\right], \quad i = 1, 2, \ldots, n, \tag{4.1}$$

where the functions $b_i(t)$ and $a_{ij}(t)$ $(i, j = 1, 2, \ldots, n)$ are defined on R_{+0} and are bounded continuous on R_{+0}, and $a_{ij}(t) \ge 0$ for all $t \in R_{+0}$ and $i, j = 1, 2, \ldots, n$.

On the permanence of system (4.1), applying Theorems 3.1 and 3.2 we have

Theorem 4.1. *Suppose that there exist positive constants ω_i and λ_i such that for each $i = 1, 2, \ldots, n$,*

$$\liminf_{t\to\infty} \int_t^{t+\omega_i} a_{ii}(s)\,\mathrm{d}s > 0$$

and

$$\liminf_{t\to\infty} \int_t^{t+\lambda_i} \left[b_i(s) - \sum_{j=1, j\ne i}^{n} a_{ij}(s)x_{j0}(s)\right] \mathrm{d}s > 0$$

where $x_{i0}(t)$ is some positive solution of system $\mathrm{d}x_i(t)/\mathrm{d}t = x_i(t)[b_i(t) - a_{ii}(t)x_i(t)]$, then system (4.1) is permanent.

Remark 4.1. When system (1.1) degenerates into system (4.1), the Theorem 4.1 is exactly the result of [23, Theorem 2], so our result is a generalization of the result of [23].

Example 4.2. Consider the permanence and global stability of any positive solutions to system (1.2).

In system (1.2), we assume that

(B_1) Functions $b_i(t)$, $b_{ij}(t)$, $d_i(t)$, $r_i(t)$, $e_i(t)$ and $c_i(t)$ are defined on $R_+ = [0, \infty)$ and bounded continuous functions on R_+. $b_{ij}(t)$, $d_i(t)$, $r_i(t)$ and $e_i(t)$ are non-negative for all $t \in R_+$ and $i, j = 1, 2, \dots, n$.

(B_2) For each $1 \le i \le n$, there are positive constants ω_i and θ_i such that

$$\liminf_{t\to\infty} \int_t^{t+\omega_i} r_i(s)\,\mathrm{d}s > 0, \qquad \liminf_{t\to\infty} \int_t^{t+\theta_i} e_i(t) > 0.$$

Theorem 4.2. *Suppose assumptions* (B_1) *and* (B_2) *hold. If there are positive constants μ_i and ν_i such that for each $i = 1, 2, \dots, n$,*

$$\liminf_{t\to\infty} \int_t^{t+\nu_i} \left[b_i(s) - \sum_{j=1, j\neq i}^{n} b_{ij}(s) x_{j0}(s) - d_i(s) u_{i0}(s) \right] \mathrm{d}s > 0$$

and

$$\liminf_{t\to\infty} \int_t^{t+\mu_i} b_{ii}(s)\,\mathrm{d}s > 0,$$

where $x_{i0}(t)$, $u_{i0}(t)$ denote some positive solution of systems $\mathrm{d}x_i(t)/\mathrm{d}t = x_i(t)[b_i(t) - b_{ii}(t)x_i(t)]$ and $\mathrm{d}u_i(t)/\mathrm{d}t = r_i(t) - e_i(t)u_i(t)$, respectively, then

(a) *system (1.2) is permanent.*

(b) *if there exist positive constants α_i and β_i $(i = 1, 2, \dots, n)$ such that*

$$\liminf_{t\to\infty} A_i(t) > 0 \quad \text{and} \quad \liminf_{t\to\infty} B_i(t) > 0,$$

where

$$A_i(t) = \alpha_i b_{ii}(t) - \beta_i c_i(t) - \sum_{j=1, j\neq i}^{n} \alpha_j b_{ji}(t),$$
$$B_i(t) = \beta_i e_i(t) - \alpha_i d_i(t),$$

then each positive solution of system (1.2) is globally asymptotically stable.

Remark 4.2. Similar results have been obtained by Chen in [17, Theorem 2.1]. However, it is obvious to see that the method in [17] is totally different from ours adopted in this paper, and our results can be more easily checked.

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