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Letters

Exponential stability of a class of generalized neural networks with time-varying delays ☆

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Abstract

The dynamics of a class of generalized neural networks with time-varying delays are analyzed. Without constructing a Lyapunov function, general sufficient conditions for the existence, uniqueness and exponential stability of an equilibrium of the neural networks are obtained by the nonlinear Lipschitz measure approach. The new criteria are mild, independent of the delays and do not require the boundedness, differentiability or monotonicity assumption of the activation functions. Moreover, the proposed results extend and improve existing ones.

Keywords: Neural networks; Time-varying delay; Exponential stability; Exponential decay; Nonlinear Lipschitz measure

1. Introduction

In this letter, we consider a generalized neural networks model with time-varying delays (GNND) described by the following differential equations:

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -b_i(u_i(t)) + \sum_{j=1}^{n} w_{ij} f_j(v_j u_j(t)) + \sum_{j=1}^{n} w_{ij}^{\tau} f_j^{\tau}(v_j^{\tau} u_j(t-\tau_{ij}(t))) + I_i, \quad i = 1, 2, \ldots, n, \tag{1}$$

where $n \geqslant 2$ is the number of neurons in the networks, $u_i(t)$ denotes the neuron state vector, b_i is an appropriately behaved function, $W = (w_{ij})_{n\times n}$ and $W^{\tau} = (w_{ij}^{\tau})_{n\times n}$, respectively, denote the normal and the delayed connection weight matrix, f_j and f_j^{τ}, respectively, denote the normal and the delayed activation function, $v_j > 0$ and $v_j^{\tau} > 0$, respectively, denote the normal and the delayed amplifier gain, $\tau_{ij}(t) \geqslant 0$ is the time-varying delay caused during the switching and transmission processes, and I_i denotes the constant external input. The initial conditions associated with system (1) are of the form

$$u_i(s) = \phi_i(s) \in C([t_0 - \tau, t_0], R), \quad s \in [t_0 - \tau, t_0], \quad i = 1, 2, \ldots, n, \tag{2}$$

where $\tau = \sup\{\tau_{ij}(t) : 1 \leqslant i, j \leqslant n,\ t \in R\} \in [0, +\infty)$ and $C([t_0 - \tau, t_0], R)$ denotes the set of all continuous functions from $[t_0 - \tau, t_0]$ to R. It can be easily seen that system (1) includes many famous neural networks models as its special cases, for example,

(1) Hopfield-type neural networks with time-varying delays (HNND) [5]:

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -c_i u_i(t) + \sum_{j=1}^{n} w_{ij} f_j(u_j(t - \tau_{ij}(t))) + I_i, \quad i = 1, 2, \ldots, n. \tag{3}$$

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(2) Cellular neural networks with time-varying delays (CNND) [8–11]:

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -c_i u_i(t) + \sum_{j=1}^{n} w_{ij} f_j(u_j(t)) + \sum_{j=1}^{n} w_{ij}^{\tau} f_j(u_j(t-\tau_{ij}(t))) + I_i, \quad i = 1, 2, \ldots, n. \tag{4}$$

(3) Bi-directional associative memory neural networks with discrete delays (BAMD) [2]:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -a_i x_i(t) + \sum_{j=1}^{p} w_{ji} f_j(y_j(t-\tau_{ji})) + I_i, \quad i = 1, 2, \ldots, n,$$
$$\frac{\mathrm{d}y_j(t)}{\mathrm{d}t} = -b_j y_j(t) + \sum_{i=1}^{n} \tilde{w}_{ij} f_i(x_i(t-\sigma_{ij})) + J_j, \quad j = 1, 2, \ldots, p. \tag{5}$$

Clearly, the above systems HNND and CNND take as special cases those systems when $\tau_{ij}(t)$ is in particular discrete delay τ_{ij} [1,6,7].

As is well-known, the qualitative analysis of the stability of neural networks is a prerequisite step for the practical design and application of the networks. Actually, when networks are applied as associative memories, the equilibrium points of networks represent the stored patterns, and their stability means that the stored patterns can be retrieved even in the presence of noise; when networks are applied as optimization solvers, the equilibrium points of networks characterize all possible optimal solutions of the optimization problem, and stability of the networks then ensures the convergence to the optimal solutions, and in particular, the global stability ensures the convergence to a unique optimal solution [5]. Therefore, the stability analysis of neural networks has attracted extensive interest in the scientific community [1–11].

To the best of the authors' knowledge, the most popular approach to stability analysis of delayed neural networks is based on Lyapunov's direct method (see, for example, [6,8–11]), however, the construction of a proper Lyapunov function is usually rather difficult. This letter aims to derive sufficient conditions for the local and global exponential stability of GNND (1) by means of the nonlinear Lipschitz measure approach [5].

Let R^n denote the n-dimensional real vector space with vector norm $\|\cdot\|$. Recall that an operator f from an open subset Ω of R^n to R^n is said to be Lipschitz continuous if there exists a constant $M \geqslant 0$ such that for any $s_1, s_2 \in \Omega$, $\|f(s_1) - f(s_2)\| \leqslant M\|s_1 - s_2\|$. In this letter, we make the following assumptions:

(H_1) Each $b_i(\cdot)$ is Lipschitz continuous, and there exists a constant $\lambda_i > 0$ such that for any $s_1, s_2 \in R$, $(s_1 - s_2)[b_i(s_1) - b_i(s_2)] \geqslant \lambda_i (s_1 - s_2)^2$.

(H_2) Both $f_i(\cdot)$ and $f_i^{\tau}(\cdot)$ are Lipschitz continuous.

It is clear that HNND, CNND and BAMD naturally satisfy condition (H_1). In recent investigations, Zhang et al. [8,9] analyzed the global asymptotic and exponential stability of CNND (4) with $\tau_{ij}(t) = \tau(t)$, but they additionally assumed that each f_j is bounded. Zhang et al. [10] discussed the global exponential stability of CNND (4) with $\tau_{ij}(t) = \tau(t)$, but they made the additional assumptions that each f_j is bounded and monotonically nondecreasing and $\tau(t)$ is differentiable with $\tau'(t) < 1$. Zhou et al. [11] investigated the global exponential stability of CNND (4) with $\tau_{ij}(t) = \tau_j(t)$, but they additionally required that each f_j is bounded. However, generally speaking, nonmonotonic activation functions might be better candidates for neuron activation in designing and implementing an artificial neural networks [4], on the other hand, the boundedness assumption on the activation functions does make the results inapplicable to some important engineering problems [3]. In contrast, we remove the boundedness, monotonicity and differentiability assumptions of the activation functions. Moreover, throughout this letter, we neither make the differentiability assumption of the time-varying delays, nor impose any restrictive condition on the connection weight matrices. As a consequence, a much broader connection topology for the networks is allowed.

2. Preliminaries

In this letter, we will always use the l^1-norm of R^n, i.e., for each $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^n$, $\|x\|_1 = \sum_{i=1}^{n} |x_i|$. Let Ω be an open subset of R^n. Consider the following system:

$$\frac{\mathrm{d}u(t)}{\mathrm{d}t} = F(u(t)) + G(u_{\tau}(t)), \quad t \geqslant t_0, \tag{6}$$

where F and G are both operators from Ω into R^n, $x(t) \in \Omega$, and $G(u_{\tau}(t))$ is defined as $G_i(u_{\tau}(t)) = G_i((u_1(t-\tau_{i1}(t)), u_2(t-\tau_{i2}(t)), \ldots, u_n(t-\tau_{in}(t)))^{\mathrm{T}})$, where $G(u) = (G_1(u), G_2(u), \ldots, G_n(u))^{\mathrm{T}}$.

Definition 1 (*Peng et al. [5]*). Suppose that Ω is an open subset of R^n and F is a Lipschitz operator from Ω to R^n. The constant

$$L(F) = \sup_{x,y \in \Omega,\ x \neq y} \|F(x) - F(y)\| / \|x - y\|$$

is called the minimal Lipschitz constant (MLC) of F.

Denote $L_1(F)$ the MLC of a Lipschitz operator F in the sense of l^1-norm.

Definition 2 (*Peng et al. [5]*). Suppose that Ω is an open subset of R^n and F is a Lipschitz operator from Ω to R^n. The constant

$$m(F) = \sup_{x,y \in \Omega,\ x \neq y} \langle F(x) - F(y), \mathrm{sign}(x - y) \rangle / \|x - y\|_1$$

is called the nonlinear Lipschitz measure (NLM) of F on Ω, where $\langle \cdot, \cdot \rangle$ denotes the inner product of vectors and $\mathrm{sign}(x) = (\mathrm{sign}(x_1), \mathrm{sign}(x_2), \ldots, \mathrm{sign}(x_n))^{\mathrm{T}}$ denotes the sign vector of $x \in R^n$, where $\mathrm{sign}(r)$ is the usual sign function of any $r \in R$.

Lemma 1 (*Peng et al.* [5]). *Suppose that* Ω *is an open subset of* R^n *and* F *is a Lipschitz operator from* Ω *to* R^n. *If* $m(F)<0$, *then* F *is one-to-one on* Ω. *If in addition* $\Omega = R^n$, *then* F *is a homeomorphism of* R^n.

Lemma 2 (*Peng et al.* [5]). *Suppose that* Ω *is a neighborhood of an equilibrium* u^* *of the delayed system* (6), F *and* G *are both Lipschitz operators from* Ω *to* R^n, *and the delays satisfy* $\tau = \sup\{\tau_{ij}(t) : 1 \leqslant i,\ j \leqslant n,\ t \in R\} < +\infty$. *If* $m(FA) + L_1(GA) < 0$ *for some matrix* $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$ *with* $a_i > 0$, *then system* (6) *is exponentially stable on* Ω. *Moreover, the exponential decay estimate is governed by*

$$\|x(t) - y(t)\|_1 \leqslant \mathrm{e}^{-\sigma(t-t_0)} \cdot \sup_{s \in [t_0-\tau, t_0]} \|x_0(s) - y_0(s)\|_1, \quad t \geqslant t_0, \tag{7}$$

where $x(t)$ *and* $y(t)$ *are the trajectories of system* (6) *respectively initiated from* $x_0(s), y_0(s) \in \Omega$ *for all* $s \in [t_0 - \sigma,\ t_0]$, *and* σ *is the unique positive solution of the equation*

$\sigma \cdot \min_{1 \leqslant i \leqslant n} a_i + m(FA) + L_1(GA)\,\mathrm{e}^{\tau\sigma} = 0.$

3. Main results

Theorem 1. *Suppose* Ω *is an open subset of* R^n. *If conditions* $(\mathrm{H}_1), (\mathrm{H}_2)$ *hold, and there exists a set of real numbers* $d_i > 0$ $(i = 1, 2, \ldots, n)$ *such that*

$$\max_{1 \leqslant i \leqslant n} \frac{1}{\lambda_i} \left\{ m_i v_i \sum_{j=1}^{n} \frac{d_j}{d_i} |w_{ji}| + m_i^\tau v_i^\tau \sum_{j=1}^{n} \frac{d_j}{d_i} |w_{ji}^\tau| \right\} < 1, \tag{8}$$

where m_i *and* m_i^τ *are respectively the* MLC *of* f_i *and* f_i^τ *on* Ω_i, *then for each set of external input* I_i, *the equilibrium point of system* (1) *is unique in* Ω. *Moreover, if* $\Omega = R^n$, *then system* (1) *has and only has one equilibrium point.*

Proof. Let $H(u) = (H_1(u), H_2(u), \ldots, H_n(u))^{\mathrm{T}} : R^n \to R^n$, where $H_i(u) = -b_i(u_i) + \sum_{j=1}^{n} w_{ij} f_j(v_j u_j) + \sum_{j=1}^{n} w_{ij}^\tau f_j^\tau(v_j^\tau u_j) + I_i$, $u = (u_1, u_2, \ldots, u_n)^{\mathrm{T}}$. Let $P = \mathrm{diag}(d_1, d_2, \ldots, d_n)$. Note that u^* is an equilibrium of system (1) if and only if $H(u^*) = 0$, which is equivalent to $PH(u^*) = 0$. Therefore, the equilibrium u^* of system (1) is unique in Ω if and only if the solution of $PH(u) = 0$ is unique in Ω, and thus we only need to prove PH is one-to-one on Ω. For all $y, z \in \Omega$, we deduce

$$\begin{aligned}
&\langle PH(y) - PH(z), \mathrm{sign}(y - z)\rangle \\
&\quad = \sum_{i=1}^{n} \Bigg\{ \mathrm{sign}(y_i - z_i) d_i \Bigg[-(b_i(y_i) - b_i(z_i)) \\
&\qquad + \sum_{j=1}^{n} w_{ij}(f_j(v_j y_j) - f_j(v_j z_j)) \\
&\qquad + \sum_{j=1}^{n} w_{ij}^\tau (f_j^\tau(v_j^\tau y_j) - f_j^\tau(v_j^\tau z_j)) \Bigg] \Bigg\} \\
&\quad \leqslant \sum_{i=1}^{n} d_i \Bigg\{ -|b_i(y_i) - b_i(z_i)| + \sum_{j=1}^{n} (|w_{ij}||f_j(v_j y_j) - f_j(v_j z_j)| \\
&\qquad + |w_{ij}^\tau||f_j^\tau(v_j^\tau y_j) - f_j^\tau(v_j^\tau z_j)|) \Bigg\} \\
&\quad \leqslant -\sum_{i=1}^{n} d_i \lambda_i |y_i - z_i| + \sum_{j=1}^{n} \sum_{i=1}^{n} d_i (|w_{ij}| \cdot m_j v_j |y_j - z_j| \\
&\qquad + |w_{ij}^\tau| \cdot m_j^\tau v_j^\tau |y_j - z_j|) \\
&\quad \leqslant -\sum_{i=1}^{n} d_i \lambda_i |y_i - z_i| + \sum_{j=1}^{n} \Bigg\{ m_j v_j |y_j - z_j| \sum_{i=1}^{n} d_i |w_{ij}| \\
&\qquad + m_j^\tau v_j^\tau |y_j - z_j| \sum_{i=1}^{n} d_i |w_{ij}^\tau| \Bigg\} \\
&\quad \leqslant -\sum_{i=1}^{n} \Bigg\{ d_i \Bigg(\lambda_i - m_i v_i \sum_{j=1}^{n} \frac{d_j}{d_i} |w_{ji}| \\
&\qquad - m_i^\tau v_i^\tau \sum_{j=1}^{n} \frac{d_j}{d_i} |w_{ji}^\tau| \Bigg) \Bigg\} |y_i - z_i|.
\end{aligned}$$

Then it follows directly from condition (8) that $m(PH) < 0$. By Lemma 1, we infer that PH is one-to-one on Ω. Therefore, the equilibrium point of system (1) is unique in Ω. Moreover, if $\Omega = R^n$, then PH is a homeomorphism of R^n, and therefore, there exists a unique equilibrium point of system (1).

Theorem 2. *Suppose that* u^* *is an equilibrium point of system* (1) *and* Ω *is a neighborhood of* u^*. *If* $(\mathrm{H}_1), (\mathrm{H}_2)$ *hold and* (8) *holds on* Ω, *then for each set of external input* I_i, *system* (1) *is exponentially stable on* Ω, *and there exists a constant* $\sigma > 0$ *such that the exponential decay estimate of any solution initiated from* $\phi(s) \in \Omega$, $s \in [t_0 - \tau, t_0]$ *satisfies*

$$\|u(t) - u^*\|_1 \leqslant \frac{\max d_i}{\min d_i} \mathrm{e}^{-\sigma(t-t_0)} \sup_{s \in [t_0-\tau, t_0]} \|\phi(s) - u^*\|_1, \quad t \geqslant t_0, \tag{9}$$

where σ *is the unique positive solution of* $\sigma \cdot \min_{1 \leqslant i \leqslant n} 1/c_i - 1 + b\,\mathrm{e}^{\tau\sigma} = 0$ *with*

$$c_i = \lambda_i - m_i v_i \sum_{j=1}^{n} \frac{d_j}{d_i} |w_{ji}| \quad \text{and}$$

$$b = \max_{1 \leqslant i \leqslant n} \left\{ c_i^{-1} m_i^\tau v_i^\tau \sum_{j=1}^{n} \frac{d_j}{d_i} |w_{ji}^\tau| \right\}.$$

Particularly, if (8) *holds on the whole space* R^n, *then for each set of external input* I_i, *system* (1) *is globally exponentially stable, and the exponential decay estimate of any solution initiated from* $\phi(s) \in R^n$, $s \in [t_0 - \tau, t_0]$ *satisfies* (9).

Proof. It follows directly from Theorem 1 that the equilibrium point $u^* = (u_1^*, u_2^*, \ldots, u_n^*)^{\mathrm{T}} \in \Omega$ of system (1) is unique in Ω.

Define $F : R^n \longrightarrow R^n$ by $F_i(u) = -b_i(u_i) + \sum_{j=1}^{n} w_{ij} f_j(v_j u_j)$. Define $G : R^n \longrightarrow R^n$ by $G_i(u) = \sum_{j=1}^{n} w_{ij}^\tau f_j^\tau(v_j^\tau u_j) + I_i$. Then system (1) can be equivalently rewritten as the delayed system of type (6).

Let $P=\mathrm{diag}(d_1^{-1},d_2^{-1},\ldots,d_n^{-1})$ and $A=\mathrm{diag}(c_1^{-1},c_2^{-1},\ldots,c_n^{-1})$. It follows from condition (8) that $c_i=\lambda_i-m_iv_i\sum_{j=1}^n d_j/d_i|w_{ji}|>0$, $i=1,2,\ldots,n$. Analogous to the proof of Theorem 1, we deduce that for all $y,z\in A^{-1}P^{-1}(\Omega)$,

$$\begin{aligned}
&\langle P^{-1}F(PAy)-P^{-1}F(PAz),\mathrm{sign}(y-z)\rangle\\
&\leqslant \sum_{i=1}^n d_i\Bigg\{-|b_i(d_i^{-1}c_i^{-1}y_i)-b_i(d_i^{-1}c_i^{-1}z_i)|\\
&\quad+\sum_{j=1}^n |w_{ij}||f_j(v_jd_j^{-1}c_j^{-1}y_j)-f_j(v_jd_j^{-1}c_j^{-1}z_j)|\Bigg\}\\
&\leqslant \sum_{i=1}^n d_i\Bigg\{-d_i^{-1}c_i^{-1}\lambda_i|y_i-z_i|\\
&\quad+\sum_{j=1}^n |w_{ij}|\cdot m_jv_jd_j^{-1}c_j^{-1}|y_j-z_j|\Bigg\}\\
&=-\sum_{j=1}^n c_j^{-1}\left(\lambda_j-m_jv_j\sum_{i=1}^n\frac{d_i}{d_j}|w_{ij}|\right)|y_j-z_j|\\
&=-\|y-z\|_1,
\end{aligned}$$

and therefore, we derive $m(P^{-1}FPA)\leqslant -1$.

For all $y,z\in A^{-1}P^{-1}(\Omega)$, we deduce that

$$\begin{aligned}
&\|P^{-1}G(PAy)-P^{-1}G(PAz)\|_1\\
&=\sum_{i=1}^n\left|d_i\sum_{j=1}^n w_{ij}^\tau(f_j^\tau(v_j^\tau d_j^{-1}c_j^{-1}y_j)-f_j^\tau(v_j^\tau d_j^{-1}c_j^{-1}z_j))\right|\\
&\leqslant\sum_{j=1}^n\left(\sum_{i=1}^n d_i|w_{ij}^\tau|\cdot m_j^\tau v_j^\tau d_j^{-1}c_j^{-1}\right)|y_j-z_j|,
\end{aligned}$$

and thus, $L_1(P^{-1}GPA)\leqslant\max_{1\leqslant j\leqslant n}\left\{c_j^{-1}m_j^\tau v_j^\tau\sum_{i=1}^n d_i/d_j|w_{ij}^\tau|\right\}=b$. In view of condition (8), we have $m_j^\tau v_j^\tau\sum_{i=1}^n d_i/d_j|w_{ij}^\tau|+m_jv_j\sum_{i=1}^n d_i/d_j|w_{ij}|<\lambda_j$, $j=1,2,\ldots,n$, and therefore

$$\begin{aligned}
&m(P^{-1}FPA)+L_1(P^{-1}GPA)\\
&\leqslant -1+\max_{1\leqslant j\leqslant n}\frac{m_j^\tau v_j^\tau\sum_{i=1}^n d_i/d_j|w_{ij}^\tau|}{c_j}\\
&=\max_{1\leqslant j\leqslant n}\left\{\frac{-\lambda_j+m_jv_j\sum_{i=1}^n d_i/d_j|w_{ij}|+m_j^\tau v_j^\tau\sum_{i=1}^n d_i/d_j|w_{ij}^\tau|}{\lambda_j-m_jv_j\sum_{i=1}^n d_i/d_j|w_{ij}|}\right\}<0.
\end{aligned}$$

Therefore, by Lemma 2 we infer that the trajectory of the following time-varying delayed system

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t}=P^{-1}FP(x(t))+P^{-1}GP(x_\tau(t)),\quad t\geqslant t_0 \tag{10}$$

satisfies

$\|x(t)-P^{-1}u^*\|_1\leqslant \mathrm{e}^{-\sigma(t-t_0)}\sup_{s\in[t_0-\tau,t_0]}\|x_0(s)-P^{-1}u^*\|_1$, $t\geqslant t_0$.

Observing that $x(t)=P^{-1}u(t)$ is the solution of (10) whenever $u(t)$ is a solution of (1), we derive that the exponential decay estimate (9) holds.

When in particular (8) holds on the whole space R^n, it follows from Theorem 1 that there exists a unique equilibrium point u^* of system (1), and then the above proof directly yields the global exponential stability of system (1).

Remark 1. Theorem 2 presents general sufficient conditions for the local and global exponential stability of the time-varying delayed neural networks (1), which are independent of the delays. New criteria can directly be deduced from Theorem 2 for the exponential stability of those models included by (1), which extend and improve the existing results in [2] for BAMD, [7–11] for CNND and [1,6] for HNND.

Example 1. Consider the following neural networks model:

$$\begin{cases}\dfrac{\mathrm{d}u_1(t)}{\mathrm{d}t}=-2u_1(t)+\dfrac{1}{2}f_1(u_1(t))+\dfrac{1}{4}f_2(u_2(t-\tau_{12}(t)))+I_1,\\[2mm] \dfrac{\mathrm{d}u_2(t)}{\mathrm{d}t}=-2u_2(t)+\dfrac{2}{9}f_2(u_2(t))+\dfrac{1}{4}f_1(u_1(t-\tau_{21}(t)))+I_2,\end{cases} \tag{11}$$

where $f_1(r)=-r-\sin r$ and $f_2(r)=-2r-\sin 2r$, $r\in R$.

Since f_i $(i=1,2)$ is neither bounded nor monotonically nondecreasing, none of the existing criteria in [8–11] for CNND can be applicable. However, it is easy to verify that conditions (H_1) and (H_2) are satisfied in this example, with $f_i^\tau=f_i$, $v_i=v_i^\tau=1$, $\lambda_1=\lambda_2=2$, $m_1=m_1^\tau=2$, $m_2=m_2^\tau=4$ for $\Omega=R^2$. We compute

$$\max_{1\leqslant i\leqslant 2}\frac{1}{\lambda_i}\left\{m_i\sum_{j=1}^2|w_{ji}|+m_i^\tau\sum_{j=1}^2|w_{ji}^\tau|\right\}=\max\{\tfrac{3}{4},\tfrac{17}{18}\}=\tfrac{17}{18}<1,$$

i.e., condition (8) holds on R^2 with $d_1=d_2=1$. Therefore, by Theorem 2, we deduce that for any fixed external inputs I_1,I_2, system (11) has a unique equilibrium $u^*=(u_1^*,u_2^*)^{\mathrm{T}}$ which is globally exponentially stable and the exponential decay estimate satisfies (9) where σ is the unique positive solution of $\frac{9}{10}\sigma-1+\frac{9}{10}\mathrm{e}^{\tau\sigma}=0$, i.e., the exponential decay estimate obeys to $|u_1(t)-u_1^*|+|u_2(t)-u_2^*|\leqslant\mathrm{e}^{-\sigma(t-t_0)}\sup_{s\in[t_0-\tau,t_0]}\|\phi(s)-u^*\|_1$, where $u(t)=(u_1(t),u_2(t))^{\mathrm{T}}$ is any solution of (11) initiated from $\phi(s)\in R^2$, $s\in[t_0-\tau,t_0]$.

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