



On parameterized Lyapunov–Krasovskii functional techniques for investigating singular time-delay systems

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ABSTRACT

The problem of robust stability of a singular time-delay system is investigated. A novel Lyapunov–Krasovskii functional (LKF) is introduced which is a singular-type complete quadratic Lyapunov–Krasovskii functional with polynomial parameters. Stability conditions are derived in the form of linear matrix inequalities. Numerical examples are given to illustrate the effectiveness and lower conservatism of the new proposed stability criterion.

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1. Introduction

Over the past few decades, much attention has been focused on stability analysis and controller synthesis for singular time-delay systems, due to the fact that the singular system model is a natural representation of dynamic systems and it can describe a larger class of systems than regular ones such as large scale systems, power systems and constrained control systems. The study of singular systems is much more complicated than that of standard state-space time-delay systems, as the regularity and absence of impulses (for continuous systems) and causality (for discrete systems) must be considered simultaneously for singular systems [1,2].

A lot of attention has been dedicated to the Lyapunov–Krasovskii theory for the analysis of time-delay systems. In the existing literature, there are complete quadratic Lyapunov–Krasovskii functionals and simple Lyapunov–Krasovskii functionals for deriving the stability criteria for time-delay systems [3,4]. For singular systems with delays, several kinds of simple Lyapunov–Krasovskii functionals, i.e. functionals parameterized with constant matrices, have been proposed, which lead to different levels of conservatism due to the different model transformations and the bounding techniques for some cross-terms [5–11]. A tighter bounding for cross-terms can reduce the conservatism. However, there are no obvious ways to obtain less conservative results, even if one is willing to expend more computational effort on the problem, and to find a tighter bound for the cross-terms. This is the serious limitation for these criteria. To overcome this limitation, one has to find some more general LKF for handling the stability problem for singular systems. To the best of our knowledge, this problem has not been investigated for singular systems, which motivates the present study.

Note that there exist some singular systems which are stable with some nonzero delay, but are unstable without delay. For this class of singular systems, the stability cannot be obtained using simple Lyapunov–Krasovskii functionals, as the necessary condition for the application of the simple LKFs is asymptotic stability of the nondelayed system. The stability of such kinds of state-space systems was investigated using complete quadratic Lyapunov–Krasovskii functionals [12,13]. But to the best of our knowledge, there is no stability criterion available for singular systems from employing the complete

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quadratic Lyapunov–Krasovskii functional. Introducing a novel singular-type complete quadratic Lyapunov–Krasovskii functional is the other motivation of the present study.

In this work, a singular-type complete quadratic Lyapunov–Krasovskii functional is introduced in which the parameters are defined using polynomial functions inspired by [14]. A new delay-dependent stability criterion is derived via a linear matrix inequality formulation that can be easily solved by various convex optimization techniques. Numerical examples illustrate the efficiency of the new method.

2. Main result

Consider an uncertain singular time–delay system described by

$$\begin{cases} E\dot{x}(t) = (A_0 + \Delta A_0)x(t) + (A_1 + \Delta A_1)x(t - r), \\ x(t) = \phi(t), \quad t \in [-r, 0]. \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, r is a constant time delay and $\phi(t)$ is a compatible vector valued initial function. The matrix E may be singular and we shall assume that $\text{rank}E = q \leq n$. A_0 and A_1 are real constant matrices with appropriate dimensions, ΔA_0 and ΔA_1 are time-varying matrices representing norm-bounded parameter uncertainties and are assumed to be of the following form:

$$[\Delta A_0 \quad \Delta A_1] = MF(t) [N_0 \quad N_1], \quad (2)$$

where M , N_0 and N_1 are known real constant matrices with appropriate dimensions, and the uncertain matrix $F(t)$ satisfies

$$F^T(t)F(t) \leq I. \quad (3)$$

The nominal singular delay system of (1) can be written as

$$\begin{cases} E\dot{x}(t) = A_0x(t) + A_1x(t - r), \\ x(t) = \phi(t), \quad t \in [-r, 0]. \end{cases} \quad (4)$$

Definition 1 ([1]).

- (1) The pair (E, A_0) is said to be regular if $\det(sE - A_0)$ is not identically zero.
- (2) The pair (E, A_0) is said to be impulse free if $\deg(\det(sE - A_0)) = \text{rank}E$.

Definition 2 ([5]). The singular time–delay system (4) is said to be regular and impulse free if the pair (E, A_0) is regular and impulse free.

Definition 3 ([7]). The system (4) is said to be stable if for any $\varepsilon > 0$, there exists a scalar $\delta(\varepsilon) > 0$ such that for any compatible initial function $\phi(t)$ satisfying $\sup_{-r \leq t \leq 0} \|\phi(t)\| \leq \delta(\varepsilon)$, the solutions $x(t)$ of system (4) satisfy $\|x(t)\| \leq \varepsilon$ for $t \geq 0$, and $x(t) \rightarrow 0$ as $t \rightarrow \infty$.

Definition 4. The uncertain singular delay system (1) is said to be robustly stable if the system (1) is regular, impulse free and stable for all admissible uncertainties ΔA_0 and ΔA_1 .

Without loss of generality, we can assume that the matrices in (1) have the forms

$$E = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}, \quad A_0 = \begin{bmatrix} A_{01} & A_{02} \\ A_{03} & A_{04} \end{bmatrix}, \quad A_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{13} & A_{14} \end{bmatrix},$$

and define $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ with $x_1(t) \in \mathbb{R}^q$, $x_2(t) \in \mathbb{R}^{n-q}$. Define the difference operator $\mathcal{D} : C_{n-q,r} \rightarrow \mathbb{R}^{n-q}$:

$$D(x_{2t}) = x_2(t) + A_{04}^{-1}A_{14}x_2(t - r).$$

Then we have the following lemma.

Lemma 1 ([5]). If the operator $D(x_{2t})$ is stable and there exist positive numbers α, β, γ and a continuous functional $V(x_t) : C_{n,r} \rightarrow \mathbb{R}$ such that

$$\begin{aligned} \beta \|x_1(t)\|^2 &\leq V(x_t) \leq \gamma \|x_t\|^2, \\ \dot{V}(x_t) &\leq -\alpha \|x_t\|^2, \end{aligned}$$

and the functional $V(x_t)$ is absolutely continuous for x_t satisfying (4), then (4) is asymptotically stable.

We introduce a singular-type complete quadratic Lyapunov–Krasovskii functional:

$$\begin{aligned}
 V(t, x_t) = & x_1^T(t)P_1x_1(t) + 2x_1^T(t) \int_{-r}^0 Q(\xi)x(t + \xi)d\xi + \int_{-r}^0 \int_{-r}^0 x^T(t + \xi)R(\xi, \eta)x(t + \eta)d\xi d\eta \\
 & + \int_{-r}^0 x^T(t + \xi)S(\xi)x(t + \xi)d\xi
 \end{aligned} \tag{5}$$

where $Q(\xi), S(\xi) = S^T(\xi)$ and $R(\xi, \eta) = R^T(\eta, \xi)$ are polynomial functions in the form of

$$\begin{aligned}
 Q(\xi) = & \sum_{i=1}^N \xi^{(i-1)}Q_i, S(\xi) = S + (r + \xi) \sum_{i=1}^N \sum_{j=1}^N \xi^{(i-1)}\xi^{(j-1)}T_{ij}, \\
 R(\xi, \eta) = & \sum_{i=1}^N \sum_{j=1}^N \xi^{(i-1)}\eta^{(j-1)}R_{ij},
 \end{aligned}$$

where Q_i, S_{ij} and R_{ij} for $i, j = 1, \dots, N$ are constant matrices. Introducing the partitioned matrix formation $W(\xi) = (I_n \ \xi I_n \ \dots \ \xi^{N-1}I_n)^T$, the expression for the polynomial parameters of LKF can be derived as

$$\begin{aligned}
 Q(\xi) = & \mathcal{Q}W(\xi), \quad S(\xi) = S + (r + \xi)W^T(\xi)\mathcal{T}W(\xi), \\
 R(\xi, \eta) = & W^T(\xi)\mathcal{R}W(\eta),
 \end{aligned} \tag{6}$$

where

$$\begin{aligned}
 \mathcal{Q} = & [Q_1 \quad Q_2 \quad \dots \quad Q_N], \\
 \mathcal{T} = & \begin{bmatrix} T_{11} & T_{12} & \dots & T_{1N} \\ * & T_{22} & \dots & T_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \dots & T_{NN} \end{bmatrix}, \\
 \mathcal{R} = & \begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ * & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \dots & R_{NN} \end{bmatrix}.
 \end{aligned}$$

The functions which defined the LKF are thus expressed in a simple way. A lemma for ensuring that the LKF is positive definite is formulated as follows.

Lemma 2. For a given delay $r > 0$, suppose that there exist positive definite matrices $S \in R^{n \times n}, \mathcal{T} \in R^{nN \times nN}$ and matrices $P_1 \in R^{q \times q}, \mathcal{Q} \in R^{q \times nN}, \mathcal{R} \in R^{nN \times nN}$ such that

$$\Phi = \begin{bmatrix} P_1 & \mathcal{Q} \\ * & \mathcal{R} + \mathcal{S}/r \end{bmatrix} > 0.$$

Then the functional V is positive definite, where

$$\mathcal{S} = \mathcal{L}^T S \mathcal{L}, \mathcal{L} = [I_n \quad 0 \quad \dots \quad 0].$$

Proof. Consider the functional (5) with the functions Q, R, S as in (6), and define the vector $\phi(t) = \int_{-r}^0 W(\xi)x(t + \xi)d\xi$; then

$$\begin{aligned}
 V(x_t) = & x_1^T(t)P_1x_1(t) + 2x_1^T(t)\mathcal{Q}\phi(t) + \int_{-r}^0 x^T(t + \xi)Sx(t + \xi)d\xi + \phi^T(t)\mathcal{R}\phi(t) \\
 & + \int_{-r}^0 (r + \xi)x^T(t + \xi)W^T(\xi)\mathcal{T}W(\xi)x(t + \xi)d\xi.
 \end{aligned}$$

As $S > 0$, Jensen’s inequality ensures that

$$\int_{-r}^0 x^T(t + \xi)Sx(t + \xi)d\xi \geq \left(\int_{-r}^0 x(t + \xi)d\xi \right)^T S / r \left(\int_{-r}^0 x(t + \xi)d\xi \right) = \phi^T(t)\mathcal{S}/r\phi(t),$$

and defining $\xi(t) = (x^T(t) \ \phi^T(t))^T$, the functional satisfies

$$V(x_t) \geq \xi^T(t)\Phi\xi(t) + \int_{-r}^0 (r + \xi)x^T(t + \xi)W^T(\xi)\mathcal{T}W(\xi)x(t + \xi)d\xi,$$

and then if $\Phi > 0$ and $\mathcal{T} > 0$, $V(x_t)$ is positive definite. \square

Note that differentiation of the partitioned matrix gives

$$\dot{W}(\xi) = \mathcal{D}W(\xi)$$

where $\mathcal{D} = D \otimes I_n$, $D = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & N-1 & 0 \end{bmatrix}$. On the basis of this, differentiation of the functions $Q(\xi)$, $S(\xi)$, $R(\xi, \eta)$

gives

$$\dot{Q}(\xi) = \mathcal{Q}\mathcal{D}W(\xi),$$

$$\dot{S}(\xi) = W^T(\xi)(\mathcal{D}^T\mathcal{T} + \mathcal{T}\mathcal{D})W(\xi),$$

$$\frac{\partial R(\xi, \eta)}{\partial \xi} = \frac{\partial R^T(\xi, \eta)}{\partial \eta} = W^T(\xi)\mathcal{D}^T\mathcal{R}W(\xi).$$

In this subsection, the stability analysis of the singular system (4) is provided. On the basis of the LKF of the form (5), easily computable LMI conditions are developed to ensure that the derivative of (6) is negative.

Theorem 1. For a given N and a constant delay $r > 0$, suppose that there exist matrices $P_1 \in R^{q \times q}$, $P_2 \in R^{q \times (n-q)}$, $P_3 \in R^{(n-q) \times (n-q)}$, $\mathcal{Q} \in R^{q \times nN}$, $\mathcal{R} \in R^{nN \times nN}$ and positive definite matrices $S \in R^{n \times n}$, $\mathcal{T} \in R^{nN \times nN}$ such that

$$\Phi > 0,$$

$$\Pi = \mathcal{T} + r(\mathcal{D}^T\mathcal{T} + \mathcal{T}\mathcal{D}) + \mathcal{T}\tilde{\mathcal{D}} + \tilde{\mathcal{D}}^T\mathcal{T} > 0,$$

$$\Psi = \begin{bmatrix} \Psi_{11} & PA_1 - \bar{\mathcal{Q}}W(-r) & A_0^T\bar{\mathcal{Q}} + W(0)^T\mathcal{R} - \bar{\mathcal{Q}}\mathcal{D} \\ * & -S & A_1^T\bar{\mathcal{Q}} - W^T(-r)\mathcal{R} \\ * & * & \Psi_{33} \end{bmatrix} < 0.$$

Then system (4) is regular, impulse free and stable. where

$$\Psi_{11} = PA_0 + A_0^TP^T + \mathcal{Q}W(0) + W^T(0)\mathcal{Q} + S + rW^T(0)\mathcal{T}W(0),$$

$$\Psi_{33} = -\Pi/r - \mathcal{D}^T\mathcal{R} - \mathcal{R}\mathcal{D},$$

$$\tilde{\mathcal{D}} = \text{diag}\{0, 1, \dots, N-1\} \otimes I_n,$$

$$\bar{\mathcal{Q}} = \begin{bmatrix} \mathcal{Q} \\ 0 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & P_2 \\ 0 & P_3 \end{bmatrix}.$$

Proof. We assume that the matrices \bar{Q}_1 , T_{11} and S_0 have the admissible partitions

$$\bar{Q}_1 = \begin{bmatrix} Q_{11} & Q_{12} \\ 0 & 0 \end{bmatrix}, \quad S = \begin{bmatrix} S_1 & S_2 \\ * & S_3 \end{bmatrix}, \quad T_{11} = \begin{bmatrix} T_{111} & T_{112} \\ * & T_{113} \end{bmatrix},$$

and then $\Psi < 0$ implies that

$$\begin{bmatrix} \Psi_{11} & PA_1 - \bar{\mathcal{Q}}W(-r) \\ * & -S \end{bmatrix} = \begin{bmatrix} \bar{\Delta}_{11} & \bar{\Delta}_{12} & \bar{\Delta}_{13} & \bar{\Delta}_{14} \\ * & P_3A_{04} + A_{04}^TP_3 + S_3 + rT_{113} & \bar{\Delta}_{23} & P_3A_{14} \\ * & * & \bar{\Delta}_{33} & \bar{\Delta}_{34} \\ * & * & * & -S_3 \end{bmatrix} < 0.$$

Obviously, $P_3A_{04} + A_{04}^TP_3 + S_3 + rT_{113} < 0$ implies that A_{04} is nonsingular, and then system (4) is regular and impulse free.

Like in [7], it follows from

$$\begin{bmatrix} P_3A_{04} + A_{04}^TP_3 + S_3 + rT_{113} & P_3A_{14} \\ * & -S_3 \end{bmatrix} < 0,$$

that we can conclude that $\rho(A_{04}^{-1}A_{14}) < 1$. The differentiation of V leads to

$$\begin{aligned} \dot{V}(t, x_t) = & x^T(t)[PA_0 + A_0^T P^T + \bar{Q}_1 + \bar{Q}_1^T + S + rW(0)^T \mathcal{T} W(0)]x(t) + 2x^T(t)[PA_1 - \bar{Q}W(-r)]x(t-r) \\ & - x^T(t-r)Sx(t-r) + 2x^T(t)[A_0^T \bar{Q} - W^T(0)\mathcal{R} - \bar{Q}\mathcal{D}]\phi(t) + 2x^T(t-r)[A_1^T \bar{Q} - W^T(-r)\mathcal{R}]\phi(t) \\ & - \phi^T(t)(\mathcal{D}^T \mathcal{R} + \mathcal{R}\mathcal{D})\phi(t) - \int_{-r}^0 x^T(t+\xi)W^T(\xi)\Pi W(\xi)x(t+\xi)d\xi. \end{aligned}$$

Since $\Pi > 0$, Jensen’s inequality ensures that the last term of the previous expression is bounded by $-\phi^T(t)\Pi/r\phi(t)$. Introducing the vector $\zeta(t) = (x^T(t) \quad x^T(t-r) \quad \phi^T(t))^T$, one has $\dot{V}(t, x_t) \leq \zeta^T(t)\Psi\zeta(t)$. Then provided that $\Psi < 0$, the derivative of the LKF is negative definite. Combining this with Lemma 2, we derive that the system (4) is regular, impulse free and stable by Lemma 1. \square

In this subsection, the stability criterion is derived for uncertain singular time-delay systems. Before the main theorem, a lemma which is extensively used in uncertain system research is formulated.

Lemma 3 ([15]). For appropriate dimension matrices Γ, \mathcal{E} , symmetric matrix Ω , and all the $F(t)$ satisfying $F^T(t)F(t) \leq I$,

$$\Omega + \Gamma F(t)\mathcal{E} + \mathcal{E}^T F^T(t)\Gamma^T < 0$$

if and only if there exists a constant $\varepsilon > 0$ such that

$$\Omega + \varepsilon \Gamma \Gamma^T + \varepsilon^{-1} \mathcal{E}^T \mathcal{E} < 0.$$

Now, extending Theorem 1 to uncertain singular time-delay system (1) yields the following theorem.

Theorem 2. For a given N and a constant delay $r > 0$, suppose that there exist matrices $P_1 \in R^{q \times q}, P_2 \in R^{q \times (n-q)}, P_3 \in R^{(n-q) \times (n-q)}, \bar{Q} \in R^{q \times nN}, \mathcal{R} \in R^{nN \times nN}$ and positive definite matrices $S \in R^{n \times n}, \mathcal{T} \in R^{nN \times nN}$, and a scalar $\varepsilon > 0$ such that

$$\Phi > 0,$$

$$\Pi = \mathcal{T} + r(\mathcal{D}^T \mathcal{T} + \mathcal{T} \mathcal{D}) + \mathcal{T} \tilde{\mathcal{D}} + \tilde{\mathcal{D}}^T \mathcal{T} > 0,$$

$$\bar{\Psi} = \begin{bmatrix} \bar{\Psi}_{11} & PA_1 - \bar{Q}W(-r) + \varepsilon N_0^T N_1 & A_0^T \bar{Q} + W(0)^T \mathcal{R} - \bar{Q}\mathcal{D} & PM \\ * & -S + \varepsilon N_1^T N_1 & A_1^T \bar{Q} - W^T(-r)\mathcal{R} & 0 \\ * & * & \Psi_{33} & \bar{Q}^T M \\ * & * & * & -\varepsilon I \end{bmatrix} < 0.$$

Then system (1) is robustly stable. Here $\bar{\Psi}_{11} = \Psi_{11} + \varepsilon N_0^T N_0$.

Proof. Replace A_0 and A_1 in Ψ with $A_0 + MF(t)N_0$ and $A_1 + MF(t)N_1$ respectively. Then $\Psi < 0$ for system (1) is equivalent to the following condition:

$$\Psi + \mathcal{M}F\mathcal{N} + \mathcal{N}^T F^T \mathcal{M}^T < 0 \tag{7}$$

where Ψ has the same expression as in Theorem 1 and

$$\mathcal{M}^T = [M^T P^T \quad 0 \quad M^T \bar{Q}], \quad \mathcal{N} = [N_0 \quad N_1 \quad 0].$$

By Lemma 3, (7) holds for any $F(t)$ satisfying (3) if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Psi + \varepsilon^{-1} \mathcal{M} \mathcal{M}^T + \varepsilon \mathcal{N}^T \mathcal{N} < 0,$$

which is equivalent to $\bar{\Psi} < 0$ according to the Schur complement. \square

3. Numerical example

Example 1. Consider a singular time-delay system described by

$$\begin{bmatrix} \dot{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} -0.3012 & 0.1257 \\ 0.2351 & -1.0998 \end{bmatrix} x(t) + \begin{bmatrix} -0.5c & 0 \\ 0 & -0.1c \end{bmatrix} x(t-r)$$

where c is a scalar. Table 1 shows the results for the upper bounded of delay for different values of c . It can be seen that Theorem 1 in this work provides larger delay bounds than the previous results given in Table 1 when N is increasing.

Table 1

Comparison of upper bounds of the delay.

c	1	1.2	1.4	1.6	2
Fridman and Shaked [6]	2.0362	1.7691	1.5619	1.3977	1.1548
Xu et al. [11]	2.2750	1.9635	1.7282	1.5438	1.2729
Theorem 1 ($N = 2$)	2.1660	1.8760	1.6470	1.4730	1.2160
Theorem 1 ($N = 3$)	4.3770	3.9190	3.5430	3.2280	2.7230

Table 2

Comparison of stability conditions for Example 2.

	Fridman [6]	Su [8]	Zhu [9]	Feng [11]	Theorem 1 ($N = 2$)	Theorem 1 ($N = 3$)
r	1.150	1.1547	1.1547	1.2052	1.1547	2.3810

Example 2. Consider the following singular time-delay system:

$$\begin{bmatrix} \dot{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 1 \\ 0 & 0.5 \end{bmatrix} x(t-r).$$

This system was studied in [5]. Table 2 gives the maximum upper bound of r obtained using different methods, and shows that the proposed stability criterion for this system improves the stability when $N = 3$.

4. Conclusion

In this work, the stability of a singular time-delay system is studied. A novel Lyapunov–Krasovskii functional is introduced which is a singular-type complete quadratic LKF with polynomial parameters. The stability condition is expressed in terms of easily computable LMIs. Applying the delay-dependent robust stability criterion proposed here to solve the robust control problem for singular time-delay systems, such as H_∞ control, guaranteed cost control, variable structure control and so on, will be of interest for further research.

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