Abstracts

Schmidt 猜想、Schmidt 博弈和劣态逼近向量

Jinpeng AN (安金鹏 @ *Peking University*)

在联立丢番图逼近中, 关于二维加权劣态逼近向量的 Schmidt 猜想曾经是最基本的问题之一, 并被 Badziahin、Pollington 和 Velani 于 2011 年证明. 在本报告中, 我们以 Schmidt 博弈为工具来探讨与 Schmidt 猜想有关的若干结果.

Arithmetic properties of singular overpartitions

Shi-Chao CHEN (陈士超 @ *Henan University*)

Singular overpartitions was introduced by G. E. Andrews in 2015 and related many interesting combinatorial objects. Based on the theory of modular forms, we will talk arithmetic aspects of singular overpartitions, containing congruences and asymptotics.

On the largest prime factors of shifted primes

Yong-Gao CHEN (陈永高 @ *Nanjing Normal University*)

For any integer $n \geqslant 2$, let $P(n)$ be the largest prime factor of n. In this talk, we prove that the number of primes $p \leqslant x$ with $P(p-1) \geqslant p^c$ is more than $(1-c+o(1))\pi(x)$ for $0 < c \leqslant \frac{1}{2}$. This extends a recent result of F. Luca, R. Menares and A.P. Madariaga for $\frac{1}{4} \leqslant c \leqslant \frac{1}{2}$.

An additive problem on Piatetski-Shapiro primes

Yaming LU (陆亚明 @ *Xi'an Jiaotong University*)

In this talk, we will prove for γ sufficiently close to 1, that there are infinitely many primes p of the form $p = [n^{1/\gamma}]$ such that $p+2$ has at most 5 prime factors.

Estimates for coefficients of certain L-functions

Guangshi LÜ (吕广世 @ *Shandong University*)

It is an important problem to estimate various sums for coefficients of L-functions in number theory. In this talk, we shall introduce our recent work on the estimation of the coefficients of L-functions. In particular, we will talk about a general summation formula for the coefficients of a class of L-functions. As applications, we shall consider integral power moments of Fourier coefficients of Hecke-Maass cusp forms. Some techniques can be applied to study sign changes of Fourier coefficients of modular forms of half-integral weights.

Hyper-Kloosterman sums of different moduli and some applications

Xiumin REN (任秀敏 @ *Shandong University*)

Hyper-Kloosterman sums of different moduli appear naturally in Voronoi's summation formula for cusp forms for $GL_m(\mathbf{Z})$. In this talk, the square moment of the Kloosterman sum is evaluated in the case of consecutively dividing moduli. As an application, smooth sums of Fourier coefficients of a Maass form for $SL_m(\mathbf{Z})$ against an exponential functions are estimated.

On effective determination of $GL(3)$ cusp forms

Qingfeng SUN (孙庆峰 @ *Shandong University, Weihai*)

In this talk, we shall discuss the problem of effective determination of $GL(3)$ cusp forms by central values of L-functions. We settle this problem for the symmetric-square lifts from $GL(2)$-weight and $GL(2)$-level aspects, respectively.

Refining Lagrange's four-square theorem

Zhi-Wei SUN (孙智伟 @ *Nanjing University*)

Lagrange's four-square theorem asserts that any natural number can be written as $x^2+y^2+z^2+w^2$ with x,y,z,w integers. The speaker recently found that this can be refined in various ways. For example, we show that we may require additionally that $x+y+z$ (or $x+2y$, or $x+y+2z$) is a square (or a cube). Moreover, we have formulated lots of surprising conjectures on this topic; for example, we conjecture that any natural number can be written as $x^2+y^2+z^2+w^2$ with x,y,z,w nonnegative integers such that $x+3y+5z$ is a square. Another mysterious conjecture of the speaker asserts that any natural number can be written as $w^2+x^2+y^2+z^2+w^2$ with w,x,y,z nonnegative integers such that $(10w+5x)^2+(12y+36z)^2$ is a square. This reveals a surprising connection between Lagrange's theorem and Pythagorean triples. In this talk we will tell the story of such discoveries as well as related new results on partitions of integers motivated by our refinements of Lagrange's theorem.

On a generalization of a theorem of Sárközy and Sós

Min TANG (汤　敏 @ *Anhui Normal University*)

Let $\mathbf{N}_0$ be the set of all nonnegative integers and $\ell \geqslant 2$ be a fixed integer. For set $A \subseteq \mathbf{N}_0$ and $n \in \mathbf{N}_0$, let $r'_\ell(A,n)$ denote the number of solutions of $a_1+\cdots+a_\ell=n$ with $a_1,\ldots,a_\ell \in A$ and $a_1 \leqslant \cdots \leqslant a_\ell$. Let k be a fixed positive integer. In this talk, we prove that, for any given distinct positive integers u_i $(1 \leqslant i \leqslant k)$ and positive rational numbers α_i $(1 \leqslant i \leqslant k)$ with $\alpha_1+\cdots+\alpha_k=1$, there are infinitely many sets $A \subseteq \mathbf{N}_0$ such that $r'_\ell(A,n) \geqslant 1$ for all $n \geqslant 0$ and the set of n with $r'_\ell(A,n)=u_i$ has density α_i for all $1 \leqslant i \leqslant k$. This is a joint work with Y.-G. Chen.

Some topics on the derivative of the Riemann zeta function

Yoshio TANIGAWA (谷川好男 @ *Nagoya University, Japan*)

I will report two of my recent results with Makoto Minamide and Jun Furuya on the derivative of the Riemann zeta function.

It is well known that the error term $\Delta(x)$ in the Dirichlet divisor problem has a representation as $\Delta(x)=-2\sum_{n\leqslant\sqrt{x}}\psi(x/n)+O(1)$ called Chowla and Walum formula. The first topic is to give a generalization of this formula to the error term obtained from the summatory function of the coefficients of $(-1)^{k+l}\zeta^{(k)}(s)\zeta^{(l)}(s)$. We also give its upper bound by using the exponent pairs.

The second topic is the approximate functional equation for $\zeta'(s)^2$. Approximate functional equation for $\zeta'(s)^2$ was given by Hall in 1999, whose error term contains the factor $((x+y)/|t|)^{1/4}$. Hall suggested to remove this factor as in the classical case $\zeta(s)^2$. In fact we can remove this factor by using Titchmarsh's method. I am going to explain the outline of proof.

Almost prime points on smooth cubic surfaces

Yuchao WANG (王玉超 @ *Shanghai University*)

Sarnak and his collaborators initiated a program to investigate the distribution of points whose coordinates have few prime factors on varieties equipped with a group structure. We settle this problem for several new families of varieties having no group structure. During the talk we shall concentrate on the case of smooth cubic surfaces defined by $F = 0$, where F is an integral smooth cubic form in 4 variables. We prove that there exists an integer r such that rational points for which the product of the coordinates has at most r prime factors form a Zariski dense subset, provided that the cubic surface has one rational point. Moreover, we are able to get a rather small r under the assumption that the cubic surface contains two skew rational lines. Our approach is based on weighted sieve arguments combined with birational geometry and conic bundle structure. This is joint work with E. Sofos (Leiden University).

On sums of powers of almost equal primes

Bin WEI (魏　斌 @ *Tianjin University*)

In 1938, Hua established that whenever $s > 2^k$, and n is a sufficiently large natural number satisfying the necessary congruence conditions, then the equation

$$n = p_1^k + p_2^k + \ldots + p_s^k$$

is soluble in prime numbers p_j. An intensively studied refinement of Hua's theorem is that in which the variables are constrained to be almost equal. In this talk, I will review the history of the "almost equal" problems. Some new techniques are introduced in recent progresses, which leads to uniform results for degree k.

Spectral decomposition for GL_2

Han WU (吴　涵 @ *ETH Zürich, Switzerland*)

Since the publication of my thesis article, many people especially Ph.D students have asked me about the preliminary part on Fourier inversion for GL_2 automorphic representations. In the first part of the talk, I will survey various versions of spectral decomposition. In the second part, I will talk about an extension of the theory followed from Zagier's work on regularized integrals. The later is an important ingredient of my recent work on Burgess-like subconvexity for Hecke characters. I will present within the scope of $\mathrm{PGL}_2(\mathbf{Z}) \setminus \mathrm{PGL}_2(\mathbf{R})$, avoiding the general setting over adeles and the second part will be attacked only if time permits.

Quadratic polynomials at prime arguments

Jie WU (吴　杰 @ *CNRS* & *Université de Lorraine, France*)

It is a fundamental and challenging problem to determine in general whether a given irreducible polynomial in $\mathbf{Z}[X]$ can capture infinitely many prime values. This is known in the linear case in view of Dirichlet's theorem on primes in arithmetic progressions, but no answer is valid for any non-linear cases. A much more ambitious conjecture asserts that the above infinitude also holds if one is restricted to prime variables and there are no fixed prime factors; however, even the linear case seems beyond the current approach as predicted by the twin prime conjecture. Nevertheless, we are nowadays much heartened since $p+h$ can present infinitely many primes for certain h with $1 < |h| \leqslant 7 \times 10^7$, thanks to Zhang's breakthrough on prime gaps.

In this talk, we are interested in the case of quadratic polynomials at prime arguments. It is of course beyond the current approach to prove the infinitude of primes captured by such polynomial, and alternatively, we consider the greatest prime factors and almost prime values as two approximations. This is joint work with Ping Xi.

On the fourth power mean of the general Kloosterman sums

Wenpeng ZHANG (张文鹏 @ *Northwest University*)

The main purpose of this paper is using the analytic methods and the properties of Gauss sums to study the computational problem of the fourth power mean of the general Kloosterman sums for any primitive character $\chi \pmod q$, and give an exact computational formula for it.

Quadratic forms in dense sets

Lilu ZHAO (赵立璐 @ *Shandong University*)

We consider the nontrivial zeros of quadratic forms in dense sets. Let $f(x_1, \ldots, x_n)$ be a translation invariant quadratic form with $n \geqslant 9$. We establish that $f(x_1, \ldots, x_n) = 0$ has nontrivial zeros in $A \subseteq \mathbf{Z}$ provided that A is a dense set.