Replacement Scheme for Lubricating Oil Based on Bayesian Control Chart

Ying Du, Chaoqun Duan, and Tonghai Wu

Abstract—**Lubricating oil carries important health information of operating machines, and oil replacement scheme is crucial for ensuring machine health, reducing operation costs, and improving machine availability. However, few works have been done on the determination of reasonable replacement time for lubricating oil in the industry. Therefore, the main motivation of this article is to present a replacement scheme based on the Bayesian approach to detect and prevent the potential failures of lubricating oil. A three-state statistical model based on the hidden Markov chain is applied to characterize oil deterioration, which contains partially observable healthy and unhealthy states, and an observable failure state. A novel Bayesian control scheme for oil replacement based on the hidden stochastic process is proposed under the objective of long-term expected average availability maximization. A computational algorithm in a semi-Markov decision process is presented to estimate the optimal decision variable of the Bayesian control chart. The 2-D oil data based on wear debris collected at regular time epochs from a four-ball tester are adopted to validate the effectiveness of the proposed replacement approach. Given comparisons with the age-based scheme and the failure-based scheme, the proposed Bayesian replacement approach for lubricating oil is demonstrated to achieve better fault detection performance and the longer average availability.**

Index Terms—**Bayesian control chart, fault prognosis, hidden Markov model (HMM), oil deterioration modeling, oil replacement.**

I. Introduction

LUBRICATING oil is the "blood" and tribological information carrier of the machine, whose performance directly determines the safety, reliability, operating efficiency, and maintenance costs of the industrial production process [1]–[3]. Oil deterioration is a slow and complicated process and can result in insufficient lubrication, excessive wear, and even catastrophic machine failures under the condition of high temperature, high speed, and uncertain loads [3], [4]. It has been illustrated that the maintenance cost will be low when the machine is operating with deteriorated oil [5], as shown in Fig. 1. As the oil status degrades seriously, the maintenance cost is largely increased. Therefore, early fault detection and identification of lubricating oil are very crucial for machine operation and maintenance, and reasonable oil replacement is especially necessary for extending machine useful life, improving machine operational reliability, and saving energy consumption.

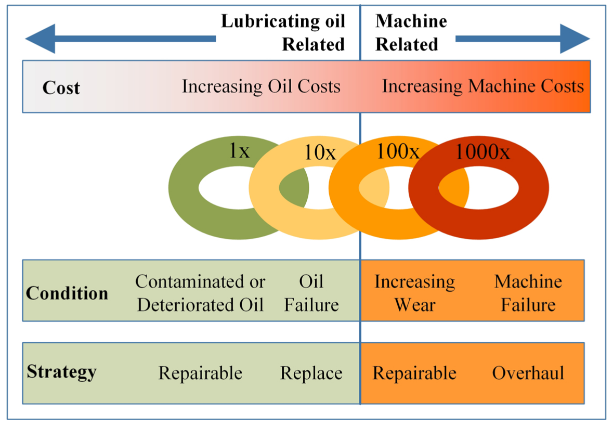


Fig. 1. Relationship between the failure condition and maintenance costs [5].

Condition-based monitoring has been widely applied in industry for better describing the machine's health condition, estimating its remaining useful life, and providing robust maintenance decision-making [6]–[9]. Compared with vibration-based monitoring and thermography, oil-based monitoring is more reliable on early detection of machine failures and is capable to obtain the wear progress [10], [11]. A wide range of research works have focused on oil condition monitoring based on wear debris in recent decades for the purpose of wear classification and machines' fault diagnosis, and up until now, a few articles studied oil performance assessment and replacement policy using statistical models [12], [13]. It is difficult to assess the actual health condition of the oil and determine the optimal failure replacement time.

Currently, it has been commonly depended on expert experiences and industrial demands to change the oil, and there is a lack of a standard for oil replacement. For example, for automobile engine oil, it is required to change the oil on the basis of driving kilometers (e.g., 5000 km) or driving

Manuscript received July 26, 2020; revised October 29, 2020; accepted November 19, 2020. Date of publication December 3, 2020; date of current version December 28, 2020. This work was supported in part by the National Science Foundation of China under Grant 51905330 and Grant 51975455, in part by the Shanghai Sail Plan for Talents Development under Grant 19YF1416000, in part by the International Collaborative Plan of Shaanxi Province under Grant 2017kw-034, and in part by the K. C. Wang Education Foundation. The Associate Editor coordinating the review process was Yuhua Cheng. *(Corresponding author: Chaoqun Duan.)*

Ying Du is with the School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, China (e-mail: yingdu@nwpu.edu.cn).

Chaoqun Duan is with the School of Mechatronic Engineering and Automation, Shanghai University, Shanghai 200444, China, and also with the Shanghai Institute of Intelligent Science and Technology, Tongji University, Shanghai 200092, China (e-mail: chaoqun.duan@hotmail.com).

Tonghai Wu is with the Key Laboratory of Education Ministry for Modern Design and Rotor-Bearing System, Xi'an Jiaotong University, Xi'an 710049, China (e-mail: wt-h@163.com).

Digital Object Identifier 10.1109/TIM.2020.3042231

time (e.g., half a year), and for aeroengine oil, it is mainly according to the flight mileage or operating time. However, the premature replacement will cause the waste of lubricating oil, and late replacement will cause oil deterioration and, thus, result in the failure of operating machines. In order to avoid this problem, it is essential to change the oil on the basis of its actual quality. Researchers have paid much more attention and efforts to effective oil replacement scheme in recent years. Puckace *et al.* [14] presented a method to optimize the oil drain interval of a vehicle. Wei *et al.* [15] proposed a novel method to determine the oil oxidation stability on-site and established a model based on oil oxidation stability to evaluate oil life and provide supports on oil replacement [16]. Raposo *et al.* [17] proposed an approach to evaluate oil deterioration and studied the oil degradation evolution to develop an oil replacement policy based on time-series analysis [18]. Hence, to advance above studies the main purpose of this article is to provide an effective scheme to estimate the optimal replacement time for lubricating oil when the oil performance cannot meet its requirements.

In this article, we present a Bayesian-based control scheme to obtain the optimal replacement time for lubricating oil. The concentration of wear debris in the oil system is considered as the deterioration feature to demonstrate oil degradation and evaluate machine health [3]. Then, 2-D oil data are collected from a four-ball tester at equidistant time epoch, and a vector autoregressive (VAR) model is fitted to the healthy oil monitoring data to compute residuals of the oil data. The hidden Markov process, which is considered as an efficient approach to model degrading systems [8], [19], is applied to model the oil deterioration behavior. The calculated oil residuals are regarded as the observation process for building the hidden Markov model (HMM) with partially observable healthy and unhealthy states and an observable failure state. The two types of unknown model parameters, state and observation parameters, can then be estimated by the expectation–maximization (EM) algorithm. Once parameter estimation is implemented, we develop a Bayesian oil replacement scheme based on the HMM for the optimal replacement decision-making [20], [21]. With two criteria to optimize the replacement scheme, which is the expected average availability maximization and cost minimization per unit time [13], [22], [23], we choose the average availability objective. A computational algorithm in a semi-Markov decision process (SMDP) is then developed to estimate the optimal control limit for oil replacement [24]. To the best of our knowledge, it is the first article to apply the HMM and Bayesian control chart to evaluate oil quality and provide an optimal oil replacement scheme in the field of tribology.

The replacement scheme for lubricating oil based on HMM and SMDP is illustrated in Fig. 2, and the main contributions are summarized as follows.

1) Multivariate HMM is presented for multidimensional oil deterioration modeling.
2) Development of a Bayesian control scheme for oil replacement.
3) SMDP approach to formulate the control problem and optimize the replacement scheme.

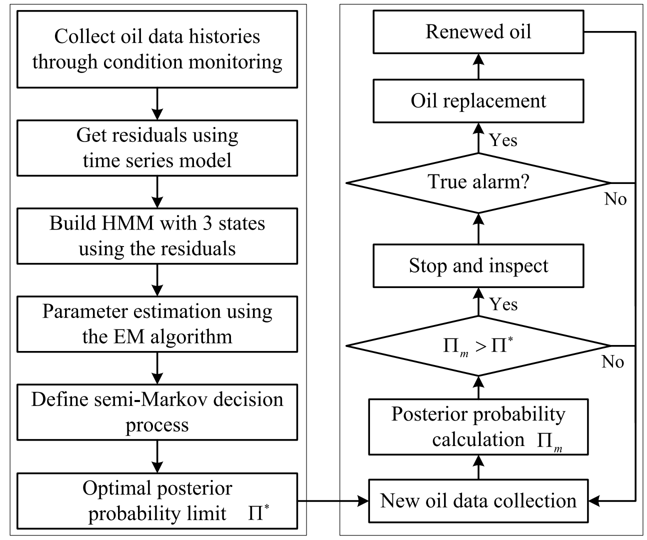


Fig. 2. Replacement scheme for lubricating oil based on HMM and SMDP.

4) Considerably better oil failure detection compared with published schemes.

The rest of this article is arranged afterward. In Section II, a four-ball tester is adopted to simulate the oil deterioration process, and a time-series model is applied to compute the residual observations of oil data. In Section III, the calculated residuals are used for the three-state hidden Markov modeling, and the EM algorithm is employed to obtain the unknown HMM parameters. In Section IV, the Bayesian replacement scheme for lubricating oil is proposed in an SMDP framework for the long-term expected average availability maximization. Ultimately, the conclusions and future works are summarized in Section V.

II. Experimental Setup and Data Preprocessing

A. Experimental Setup

A four-ball tester was employed to simulate and accelerate the deterioration process of lubricating oil. Lubricating oil monitoring data were collected by an online visual ferrograph (OLVF) sensor [25], which can determine the wear severity of tribo-pairs and indirectly reflect the performance and health condition of the oil. To avoid overparameterization, the 2-D oil data with the index of particle coverage area (IPCA) were adopted. IPCA is focusing on the wear debris concentration [25], which carries partial information about the hidden states and health condition of lubricating oil. The oil monitoring data obtained in each sampling epoch represent the generation of wear debris in the single sampling interval, which has set as $\Delta = 40$ min in this research.

The experimental system includes: 1) four-ball tribo-pairs; 2) lubrication oil system; and 3) online oil monitoring system with the OLVF sensor, as shown in Fig. 3. Fig. 3(a) shows the principle of the tribology system with both the oil circulation system and the sensing transmission system, and Fig. 3(b) shows the experimental four-ball tester with the OLVF sensor and the monitoring system.

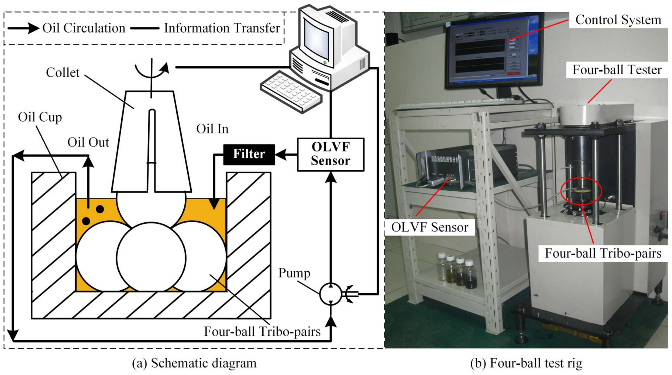


Fig. 3. Experimental system for lubricating oil deterioration.

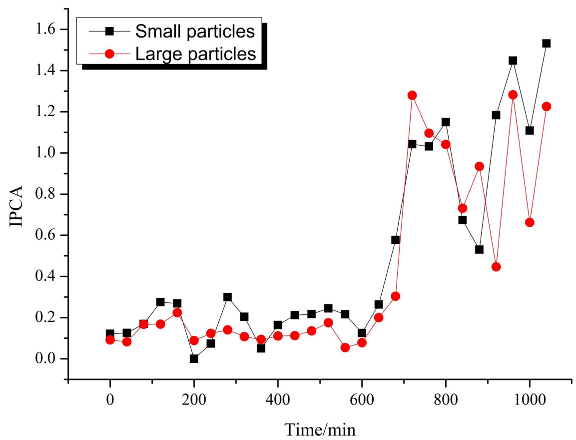


Fig. 4. Typical failure oil data history.

B. Oil Data Histories

The total sampling data histories of lubricating oil are $M+N=27$, where $M=16$ and $N=11$ represent the suspension data histories and failure histories (FHs), respectively. The suspension data represent that the data end when the oil is still working to provide sufficient lubrication. Failure data represent that the data end when the oil is in the failure state and cannot fulfill its functions anymore, i.e., the failure of lubricating oil has occurred and the oil replacement should be triggered. In industry, oil failure is often determined by some potential failure indicators, such as the element Fe (representing the wear condition) [18] and the physical and chemical indicators of lubricating oil. This article applied IPCA as the failure indicator to determine oil failure. One of the typical failure oil data histories is plotted in Fig. 4.

C. Data Preprocessing

Generally, oil properties obtained from condition monitoring are cross-correlated and autocorrelated, which will affect the results of oil analysis. Residual computation is a commonly used method to eliminate redundant and noise information of the monitoring data [13]. Therefore, the healthy portions of both failure and suspension oil data histories, Z_n, were first obtained by graphical examination and then fitted to a VAR model with the standard form by [3]

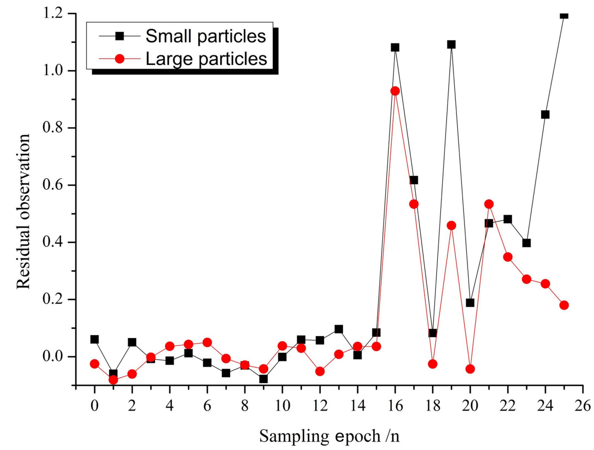


Fig. 5. Computed residual observations of a failure oil data history.

$$Z_n = \mu + \sum_{i=1}^{p} \Phi_r Z_{n-i} + \varepsilon_n, \quad n \in \mathbf{Z} \tag{1}$$

where $\mu = \mu_0 - \sum_{i=1}^{p} \Phi_i \mu_0$, ε_n are i.i.d. $N_2(\mathbf{0}, \mathbf{C})$, $p \in \mathbf{N}$ is the model order, $\mu_0 \in \mathbf{R}^2$ is the mean, $\mathbf{C} \in \mathbf{R}^{2\times 2}$ is the covariance, and $\Phi_i \in \mathbf{R}^{2\times 2}$ are the autocorrelation matrices.

The unknown parameters of the VAR model can be estimated (for explicit computation, see [3]), where the estimate of model order $\hat{p}$ is 4, and the estimates of parameters $\hat{\gamma} = (\hat{\mu}, \hat{\mathbf{C}}, \hat{\Phi}_1, \hat{\Phi}_2, \hat{\Phi}_3, \hat{\Phi}_4)$ are given as $\hat{\mu} = \begin{pmatrix} 0.0974 \\ 0.0236 \end{pmatrix}$, $\hat{\mathbf{C}} = \begin{pmatrix} 0.0034 & 0.0001 \\ 0.0001 & 0.0018 \end{pmatrix}$, $\hat{\Phi}_1 = \begin{pmatrix} 0.2492 & -0.0675 \\ 0.2318 & -0.1982 \end{pmatrix}$, $\hat{\Phi}_2 = \begin{pmatrix} 0.1887 & -0.1083 \\ 0.4168 & -0.0157 \end{pmatrix}$, $\hat{\Phi}_3 = \begin{pmatrix} 0.0011 & 0.0155 \\ -0.0045 & 0.0161 \end{pmatrix}$, $\hat{\Phi}_4 = \begin{pmatrix} 0.0143 & -0.0042 \\ 0.0129 & 0.0026 \end{pmatrix}$.

The residuals for oil data histories without cross-correlation and autocorrelation can be calculated with both healthy and unhealthy data by

$$Y_{n\Delta} = \begin{cases} Z_n - \left(\hat{\mu} + \sum_{r=1}^{\hat{p}} \Phi_r Z_{n-r} \right), & n > 4 \\ Z_n - E_{\hat{\gamma}}(Z_n | Z_1, Z_2, \ldots, Z_{n-1}), & n \leq 4. \end{cases} \tag{2}$$

The corresponding calculated residual observations of the oil data in Fig. 4 are plotted in Fig. 5, and the residuals have been tested to be independent and normal distributed (for details, see [3]). In Section III, the calculated residual observations will be considered as the observation process to develop an HMM framework, and the unknown model parameters with both state and observation processes will be estimated with the EM algorithm.

III. HMM Development and Parameter Estimation

The oil deterioration is assumed as a hidden stochastic process, $\{X_t : t \in \mathbf{R}^+\}$, with three states: $S = \{1, 2\} \cup \{3\}$. $S = \{1, 2\}$ are hidden states of lubricating oil that cannot

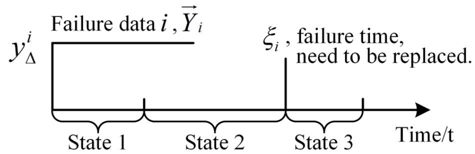


Fig. 6. Replacement time of the failure oil data.

be observed directly, where $S=\{1\}$ represents the healthy or good state, and $S=\{2\}$ represents the unhealthy or warning state. $S=\{3\}$ is an observable state of lubricating oil representing the failure state. We note that it can only convert from the lower state to the higher state, and it is irreversible, i.e., $\lambda_{ij}=0, j<i, i,j\in\{1,2,3\}$, and $\lambda_{33}=0$. Therefore, the state transition rate matrix of the lubricating oil can be written by

$$Q=\begin{pmatrix} -(\lambda_{12}+\lambda_{13}) & \lambda_{12} & \lambda_{13} \\ 0 & -\lambda_{23} & \lambda_{23} \\ 0 & 0 & 0 \end{pmatrix} \tag{3}$$

where $\lambda_{12}, \lambda_{13}, \lambda_{23} \in (0,+\infty)$ are the instantaneous transition rate of each state and need to be estimated. It is common that failure rate of lubricating oil in state 2 is higher than that in state 1, i.e., $\lambda_{23}>\lambda_{13}$.

We assume that the initial state of the oil is $S=\{1\}$, representing a new and healthy start, i.e., $X_0=1$, $P(X_0=1)=1$, and the sojourn time in each of the partially observable state follows exponential distributions. In addition, the state of lubricating oil can make transitions from $S=\{1\}$ to $S=\{2\}$ with probability p_{12} or from $S=\{1\}$ to $S=\{3\}$ with probability p_{13}. Let $\xi\geq 0$ represent the failure time, as shown in Fig. 6, and the oil should be replaced before the failure occurs.

The residual observations $Y_{n\Delta}$ computed in Section II are regarded as the observation process to build the HMM framework. Given the oil state, i.e., $X_{n\Delta}=s$, $(s=1,2)$, $Y_{n\Delta}$ is assumed to follow a bivariate normal distribution $N_2(\mu_s,\Sigma_s)$, whose probability density function (pdf) is given by

$$f_{Y_{n\Delta}|X_{n\Delta}}(y|s)=\frac{1}{\sqrt{(2\pi)^2\det(\Sigma_s)}} \cdot\exp\left(-\frac{1}{2}(y-\mu_s)'\Sigma_s^{-1}(y-\mu_s)\right) \tag{4}$$

where $\mu_s\in\mathbf{R}^2$ and $\Sigma_s\in\mathbf{R}^{2\times 2}$, $(s=1,2)$ are the mean and variance of the observation process that need to be estimated. When the oil fails and needs to be replaced, we have $P(Y_{n\Delta}=\eta|X_{n\Delta}=3)=1$, where $\eta\notin\mathbf{R}^2$ is the observation for $X_{n\Delta}=3$.

We denote $O=\{\mathrm{FH},\mathrm{SH}\}$ as the oil data histories and $C=\{\bar{\mathrm{FH}},\bar{\mathrm{SH}}\}$ as the complete data set of lubricating oil, where $\mathrm{FH}=\{\mathrm{FH}_1,\ldots,\mathrm{FH}_{11}\}$ represents all the failure oil data histories, and $\mathrm{SH}=\{\mathrm{SH}_1,\ldots,\mathrm{SH}_{16}\}$ represents all the suspension histories (SHs). Let $L(\lambda,\varphi|O)$ represent the associated likelihood function, where $\lambda=(\lambda_{12},\lambda_{13},\lambda_{23})$ are the unknown parameters with state information, and $\varphi=(\mu_1,\mu_2,\Sigma_1,\Sigma_2)$ are the unknown parameters with observation information. The EM algorithm has been proven to be suitable

TABLE I

SPECIFIC STEPS OF EM ALGORITHM FOR PARAMETER ESTIMATION

Specific Steps:
• Input arbitrary values to unknown model parameters, (λ_0,φ_0)
• E-step: Calculate the expectation of the pseudo likelihood function: $Q\left(\lambda,\varphi\mid\widehat{\lambda},\widehat{\varphi}\right):=E_{\widehat{\lambda},\widehat{\varphi}}\left(\ln L\left(\lambda,\varphi\mid C\right)\mid O\right)$
• M-step: Calculate the estimates (λ^*,φ^*) of model parameters that maximize the likelihood function by $\lambda^*,\varphi^*\in\arg\max_{\lambda,\varphi} Q\left(\lambda,\varphi\mid\widehat{\lambda},\widehat{\varphi}\right)$
• Repeat E-step and M-step until satisfying the criteria: $\left\lvert(\lambda^*,\varphi^*)-\left(\widehat{\lambda},\widehat{\varphi}\right)\right\rvert<10^{-4}$
• Output the results, (λ^*,φ^*)

to solve the parameter estimation problem for hidden Markov modeling [13], [26], whose specific steps are shown in Table I.

The estimated state parameters $\lambda^*=(\lambda_{12}^*,\lambda_{13}^*,\lambda_{23}^*)$ can be computed by

$$\lambda_{12}^*=-\frac{\sum_{i=1}^N\widehat{b}_{12}^i+\sum_{j=1}^M\widehat{\gamma}_1^j+\frac{\sum_{j=1}^M\widehat{\gamma}_2^j\left(\sum_{i=1}^N\widehat{b}_{12}^i+\sum_{j=1}^M\widehat{\gamma}_1^j\right)}{\sum_{j=1}^M\widehat{\gamma}_1^j+\sum_{i=1}^N\widehat{b}_{12}^i+\sum_{i=1}^N\widehat{b}_{13}^i}}{\sum_{i=1}^N\widehat{a}_{12}^i+\sum_{j=1}^M\widehat{\alpha}_{12}^j}$$

$$\lambda_{13}^*=\lambda_{12}^*\frac{\sum_{i=1}^N\widehat{b}_{13}^i}{\sum_{i=1}^N\widehat{b}_{12}^i+\sum_{j=1}^M\widehat{\gamma}_1^j}$$

$$\lambda_{23}^*=-\frac{\sum_{i=1}^N\widehat{b}_{23}^i}{\sum_{i=1}^N\widehat{a}_{23}^i+\sum_{j=1}^M\widehat{\alpha}_{23}^j}.$$

The estimated observation parameters $\varphi^*=(\mu_1^*,\mu_2^*,\Sigma_1^*,\Sigma_2^*)$ can be calculated by

$$\mu_1^*=\frac{\sum_{i=1}^N\mathbf{n}_1^i\cdot\widehat{\mathbf{c}}^i+\sum_{j=1}^M\mathbf{n}_1^j\cdot\widehat{\beta}^j}{\sum_{i=1}^N\left\langle\widehat{\mathbf{c}}^i,\mathbf{d}_1^i\right\rangle+\sum_{j=1}^M\left\langle\widehat{\beta}^j,\mathbf{d}_1^j\right\rangle}$$

$$\Sigma_1^*=\frac{\sum_{i=1}^N\mathbf{n}_3^i\cdot\widehat{\mathbf{c}}^i+\sum_{j=1}^M\mathbf{n}_3^j\cdot\widehat{\beta}^j}{\sum_{i=1}^N\left\langle\widehat{\mathbf{c}}^i,\mathbf{d}_1^i\right\rangle+\sum_{j=1}^M\left\langle\widehat{\beta}^j,\mathbf{d}_1^j\right\rangle}$$

$$\mu_2^*=\frac{\sum_{i=1}^N\mathbf{n}_2^i\cdot\widehat{\mathbf{c}}^i+\sum_{j=1}^M\mathbf{n}_2^j\cdot\widehat{\beta}^j}{\sum_{i=1}^N\left\langle\widehat{\mathbf{c}}^i,\mathbf{d}_2^i\right\rangle+\sum_{j=1}^M\left\langle\widehat{\beta}^j,\mathbf{d}_2^j\right\rangle}$$

$$\Sigma_2^*=\frac{\sum_{i=1}^N\mathbf{n}_4^i\cdot\widehat{\mathbf{c}}^i+\sum_{j=1}^M\mathbf{n}_4^j\cdot\widehat{\beta}^j}{\sum_{i=1}^N\left\langle\widehat{\mathbf{c}}^i,\mathbf{d}_2^i\right\rangle+\sum_{j=1}^M\left\langle\widehat{\beta}^j,\mathbf{d}_2^j\right\rangle}$$

where vectors $\mathbf{d}_1^i=(0,1,\ldots,T_i)'$, $\mathbf{d}_2^i=(T_i,\ldots,1,0)'$, $\mathbf{n}_1^i=(0,\sum_{n\leq 1}y_n,\ldots,\sum_{n\leq T_i}y_n)$, $\mathbf{n}_2^i=(\sum_{n\geq 1}y_n,\ldots,y_{T_i},0)$, $\mathbf{n}_3^i=(0,\sum_{n\leq 1}(y_n-\mu_1^*)(y_n-\mu_1^*)',\ldots,\sum_{n\leq T_i}(y_n-\mu_1^*)(y_n-\mu_1^*)')$, and $\mathbf{n}_4^i=(\sum_{n\geq 1}(y_n-\mu_2^*)(y_n-\mu_2^*)',\ldots,(y_{T_i}-\mu_2^*)(y_{T_i}-\mu_2^*)',0)$.

Explicit formulas and specific computations for the estimation process of the unknown model parameters based on the EM algorithm can be found in [21] and [27] (see the Appendix). The model parameters for hidden Markov modeling are obtained and shown in Table II.

TABLE II
ESTIMATED MODEL PARAMETERS USING EM ALGORITHM

Parameters	Initial value	1^{st} iteration	2^{nd} iteration	Final iteration
λ_{12}	0.01	0.0118	0.0132	0.0192
λ_{13}	$0.5e-3$	$0.56e-3$	$0.59e-3$	$0.24e-4$
λ_{23}	0.5	0.4889	0.4585	0.1738
μ_1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -0.11e-2 \\ 0.43e-3 \end{pmatrix}$	$\begin{pmatrix} -0.96e-3 \\ 0.41e-3 \end{pmatrix}$	$\begin{pmatrix} -0.26e-2 \\ -0.13e-2 \end{pmatrix}$
μ_2	$\begin{pmatrix} 0.05 \\ 0.02 \end{pmatrix}$	$\begin{pmatrix} -0.39e-2 \\ -0.25e-2 \end{pmatrix}$	$\begin{pmatrix} -0.12e-1 \\ -0.72e-2 \end{pmatrix}$	$\begin{pmatrix} -0.52e-2 \\ -0.11e-1 \end{pmatrix}$
Σ_1	$\begin{pmatrix} 0.02 & 0.02 \\ 0.02 & 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.31e-2 & 0.26e-3 \\ 0.26e-3 & 0.18e-2 \end{pmatrix}$	$\begin{pmatrix} 0.31e-2 & 0.26e-3 \\ 0.26e-3 & 0.18e-2 \end{pmatrix}$	$\begin{pmatrix} 0.29e-2 & 0.25e-3 \\ 0.25e-3 & 0.18e-2 \end{pmatrix}$
Σ_2	$\begin{pmatrix} 0.03 & 0.02 \\ 0.02 & 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.42e-2 & 0.83e-3 \\ 0.83e-3 & 0.19e-2 \end{pmatrix}$	$\begin{pmatrix} 0.41e-2 & 0.12e-2 \\ 0.12e-2 & 0.21e-2 \end{pmatrix}$	$\begin{pmatrix} 0.99e-3 & 0.85e-3 \\ 0.85e-3 & 0.16e-2 \end{pmatrix}$

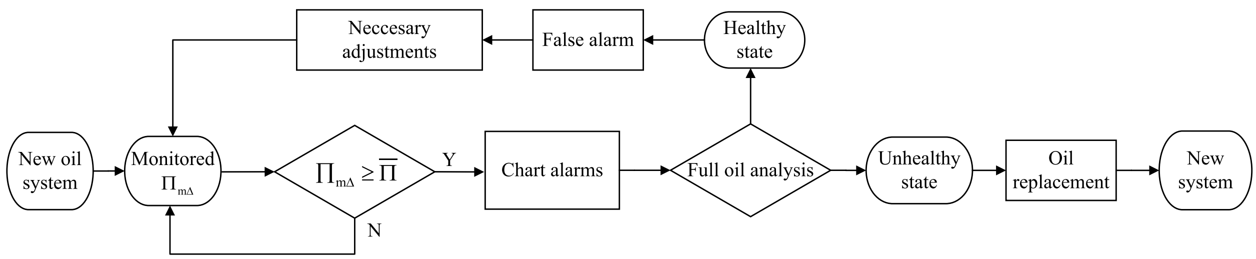


Fig. 7. Flowchart of the developed Bayesian oil replacement scheme.

IV. BAYESIAN OIL REPLACEMENT SCHEME

In this section, we propose an oil replacement scheme based on the Bayesian control chart to monitor the quality of lubricating oil. The Bayesian control chart has been shown to be an effective method in decision-making for quality control [20]. For oil replacement decision-making, we formulate the Bayesian oil replacement scheme. When observations $Y_\Delta, \ldots, Y_{m\Delta}$ are collected up to the mth sampling epoch, the probability of the oil in the unhealthy or warning state ($X_{m\Delta} = 2$) can be computed by

$$\Pi_{m\Delta} = P(X_{m\Delta} = 2 | \xi > m\Delta, Y_\Delta, \ldots, Y_{m\Delta}, \Pi_0 = 0). \quad (5)$$

The Bayesian control chart monitors the posterior probability $\Pi_{m\Delta}$ at each decision epoch $m\Delta$. The flowchart of the developed Bayesian oil replacement scheme is illustrated in Fig. 7. When the monitored $\Pi_{m\Delta}$ exceeds a control limit $\overline{\Pi}$ at the mth decision epoch, the Bayesian control chart alarms, and full oil analysis is performed to examine the real quality of the lubricating oil. Upon the examination, if the actual status of oil is found to be in state 2 (unhealthy state), it is a true alarm, and an oil replacement is initiated so that the oil can be renewed as a new one. If the actual status of the oil is found to be in state 1 (healthy state), it is a false alarm, and the oil is left operating after necessary adjustments. The necessary adjustments usually involve some inspections, such as oil circuit inspection, oil leak inspection, lubrication condition inspection of the tribo-pairs, and sensor transmission system inspection. After full oil analysis, the oil quality is renewed to be its initial healthy state.

The objective of the Bayesian oil replacement scheme is to find out the optimal control limit $\overline{\Pi}^*$ under the average availability maximization. From the renewal theory, the availability maximization problem under the Bayesian control chart is to find out the optimal value of control limit for conducting the full oil analysis, which can be computed by

$$g(\overline{\Pi}^*) = \frac{E_{\overline{\Pi}^*}(\mathrm{UT})}{E_{\overline{\Pi}^*}(\mathrm{CL})} = \sup_{\overline{\Pi}\in(0,1)} \frac{E_{\overline{\Pi}}(\mathrm{UT})}{E_{\overline{\Pi}}(\mathrm{CL})} \quad (6)$$

where UT is the uptime in one cycle of the oil system, and CL is the cycle length of the oil system, which can be calculated, respectively, as

$$\begin{aligned} \mathrm{CL} &= I_{(\xi>\overline{T}, X_{\overline{T}}=1)}(\overline{T} + T_I) + I_{(\xi>\overline{T}, X_{\overline{T}}=2)}(\overline{T} + T_I + T_{\mathrm{PM}}) \\ \mathrm{UT} &= \overline{T} \end{aligned} \quad (7)$$

where T_I and T_{PM} are full oil analysis and replacement durations, respectively. $I_{(*,*)}$ is an indicator that, if the condition $(*, *)$ is satisfied, the value of $I_{(*,*)}$ is 1, otherwise, 0.

Using the Kolmogorov backward differential equations [28], given the state space $S = \{1, 2, 3\}$, the probability transition matrix of HMM can be obtained by

$$\mathbf{P}(t) = [P_{ij}(t)]_{i,j\in S} = \begin{pmatrix} P_{11}(t) & P_{12}(t) & P_{13}(t) \\ P_{21}(t) & P_{22}(t) & P_{23}(t) \\ P_{31}(t) & P_{32}(t) & P_{33}(t) \end{pmatrix} \quad (8)$$

where

$$P_{11}(t) = e^{-(\lambda_{12}+\lambda_{13})t}, \quad P_{21}(t) = P_{31}(t) = P_{32}(t) = 0$$

$$P_{12}(t) = \frac{\lambda_{12}(e^{-\lambda_{23}t} - e^{-(\lambda_{12}+\lambda_{13})t})}{\lambda_{12} + \lambda_{13} - \lambda_{23}}$$

$$P_{13}(t) = 1 - e^{-(\lambda_{12}+\lambda_{13})t} - \frac{\lambda_{12}(e^{-\lambda_{23}t} - e^{-(\lambda_{12}+\lambda_{13})t})}{\lambda_{12} + \lambda_{13} - \lambda_{23}}$$

$$P_{22}(t) = e^{-\lambda_{23}t}, \quad P_{23}(t) = 1 - e^{-\lambda_{23}t}, \quad P_{33}(t) = 1.$$

Then, the posterior probability $\Pi_{m\Delta}$ can be computed recursively after obtaining new multivariate data according to the Bayes theorem, as in (9) shown at the bottom of the page, where $ra = f_{Y_{m\Delta}|X_{m\Delta}}(y_{m\Delta}|1)/f_{Y_{m\Delta}|X_{m\Delta}}(y_{m\Delta}|2)$.

As we know that $(\mu_1, \Sigma_1) \neq (\mu_2, \Sigma_2)$, using the pdf formula, i.e., (4), we have

$$ra = h \exp\left[\frac{1}{2}(Y_{m\Delta} - B)^T A(Y_{m\Delta} - B) + C\right] \quad (10)$$

where $h = (|\Sigma_2| \cdot |\Sigma_1|^{-1})^{1/2}$, $A = \Sigma_2^{-1} - \Sigma_1^{-1}$, $B = (\Sigma_2^{-1} - \Sigma_1^{-1})^{-1}(\Sigma_2^{-1}\mu_2 - \Sigma_1^{-1}\mu_1)$, and $C = (\mu_2{}^T\Sigma_2^{-1}\mu_2 - \mu_1{}^T\Sigma_1^{-1}\mu_1) - B^T(\Sigma_2^{-1}\mu_2 - \Sigma_1^{-1}\mu_1)$.

Therefore, the posterior probability in (9) can be simplified to

$$\Pi_{m\Delta} = \frac{D^2_{\Pi_{(m-1)\Delta}}}{h \exp\left[\frac{1}{2}V_{m\Delta} + C\right] D^1_{\Pi_{(m-1)\Delta}} + D^2_{\Pi_{(m-1)\Delta}}} \quad (11)$$

where $V_{m\Delta} = (Y_{m\Delta} - B)^T A(Y_{m\Delta} - B)$, $D^1_{\Pi_{(m-1)\Delta}} = P_{11}(\Delta)(1 - \Pi_{(m-1)\Delta}) + P_{21}(\Delta)\Pi_{(m-1)\Delta}$, and $D^2_{\Pi_{(m-1)\Delta}} = P_{12}(\Delta)(1 - \Pi_{(m-1)\Delta}) + P_{22}(\Delta)\Pi_{(m-1)\Delta}$.

Now, an efficient computational algorithm is developed in the SMDP framework to determine the optimal control limit $\overline{\Pi}^* \in (0, 1)$. To formulate the SMDP state, we first need to discrete the posterior probability process. The interval [0, 1] is partitioned into L subintervals, and the control limit is defined as $\overline{\Pi} = \bar{k}/L$ for $0 < \bar{k} < L$. Suppose that, at sampling epoch $m\Delta$, $\Pi_{m\Delta}$ is updated as π, and the process is assumed to be in state $i \in \{1, \ldots, L\}$, where $\pi \in [(i-1)/L,\ i/L)$. If updated $\pi < \overline{\Pi}$, the full oil analysis is performed, and the SMDP is defined to be in (j, I) for $j \in \{\bar{k}, \ldots, L\}$. After analysis, if the actual status of lubricating oil is found to be in the healthy state, i.e., $X_{m\Delta} = 1$, only necessary adjustment is performed, and the oil continuously operates in the initial healthy state. If the actual status of lubricating oil is found to be in the unhealthy state, i.e., $X_{m\Delta} = 2$, oil replacement is triggered, and the SMDP process will be defined to be in state {PM}. Let $\mathbf{L}_1 = \{i : i \in \{1, \ldots, \bar{k} - 1\}\}$ and $\mathbf{I} = \{(j, I) : j \in \{\bar{k}, \ldots, L\}\}$, and then, the SMDP state space is defined by $\mathbf{L} = \mathbf{L}_1 \cup \mathbf{I} \cup \{\mathrm{PM}\}$.

The number of subintervals L is crucial to achieve sufficient precision of computation. An appropriate L to partition the continuous posterior probability is determined by a basic stopping rule: $g(\overline{\Pi}^* \mid L = 2^{U+1}) - g(\overline{\Pi}^* \mid L = 2^U) \leq \rho$, where U is a positive integer and ρ is the selected small number [29].

After defining the state space of the SMDP process, we introduce a computational algorithm in the SMDP framework to find out the optimal control limit for full oil analysis. First, we define SMDP quantities to formulate the SMDP algorithm. Given the current state of lubricating oil, $i \in \mathbf{L}$, the three SMDP quantities are presented as follows.

1) τ_i is the expected sojourn time until the next decision epoch.
2) C_i is the expected cost incurred until the next decision epoch.
3) P_{ij} is the probability that the oil system will be in state $j \in \mathbf{L}$ at the next decision epoch.

Second, with the SMDP quantities, the control limit $\overline{\Pi}^*$ that maximizes the average availability can be computed by solving the following equations [30]:

$$\begin{aligned} \upsilon_i &= g(\overline{\Pi}^*)\tau_i - C_i + \sum_{j \in \mathbf{L}} P_{i,j} \cdot \upsilon_j, \quad i \in \mathbf{L} \\ \upsilon_s &= 0, \quad s \in \mathbf{L} \end{aligned} \quad (12)$$

where the quantities υ_i are related to the so-called relative values for the control limit policy (parameterized by $\overline{\Pi}$) when starting in state i. The so-called relative values indicate that the transient effect of the starting states on the total expected costs under the given policy. In our case, the quantities $\upsilon_i - \upsilon_j$ measures the difference in total expected costs between the system whose initial state is i and the system whose initial state is j (see [30] for more details). Both $g(\overline{\Pi}^*)$ and υ_i satisfy the abovementioned simultaneous system of linear equations.

Using (12), we can obtain $\overline{\Pi}^*$ with corresponding availability $g(\overline{\Pi}^*)$. However, the abovementioned quantities are unknown and need to be estimated. Then, the remainder of the mathematical analysis in this section is the derivation of SMDP quantities for $P_{i,j}$, τ_i, and C_i, where $i, j \in \mathbf{L}$.

The transition probability from state $i \in \mathbf{L}_1$ to state $j \in \mathbf{I}$ in the SMDP framework can be obtained as

$$\begin{aligned} P_{i,j} &= P\left(\frac{j-1}{L} \leq \Pi_{m\Delta} < \frac{j}{L}, \xi > m\Delta | \xi > (m-1)\Delta, \Pi_{m\Delta}\right) \\ &= P\left(\frac{j-1}{L} \leq \Pi_{m\Delta} < \frac{j}{L} | \xi > m\Delta, \Pi_{m\Delta}\right) R(\Delta|\Pi_{m\Delta}) \end{aligned} \quad (13)$$

where the conditional reliability function (CRF) $R(t|\Pi_{m\Delta})$ can be derived by

$$\begin{aligned} R(t|\Pi_{m\Delta}) &= P(\xi > m\Delta + t | \xi > m\Delta, Y_\Delta, \ldots, Y_{m\Delta}, \Pi_{m\Delta}) \\ &= P(X_{m\Delta+t} \neq 3 | \xi > m\Delta, Y_\Delta, \ldots, Y_{m\Delta}, \Pi_{m\Delta}) \\ &= (1 - \Pi_{m\Delta})(1 - P_{13}(t)) + \Pi_{m\Delta}(1 - P_{23}(t)). \end{aligned} \quad (14)$$

The left formula $P(((j-1/L) \leq \Pi_{m\Delta} < (j/L)|\xi > m\Delta, \Pi_{m\Delta})$ in (13) can be computed by (15), where $a_1 = 2\ln[(((1-(j/L))D^2_{\Pi_{(m-1)\Delta}})/((j/L)D^1_{\Pi_{(m-1)\Delta}}))h] - C$, $a_2 = 2\ln[(((1-((j-1)/L))D^2_{\Pi_{t(m-1)}})/(((j-1)/L)D^1_{\Pi_{t(m-1)}}))h] - C$, and $V_{m\Delta}$ is defined in (11).

Reference [31, Th. 3.1] proves that the pdf of $V_{t(m)}|X_{t(m)}$ obeys normal distribution, which can be expressed as

$$\Pi_{m\Delta} = \frac{P_{12}(\Delta)(1 - \Pi_{(m-1)\Delta}) + P_{22}(\Delta)\Pi_{(m-1)\Delta}}{ra \cdot (P_{11}(\Delta)(1 - \Pi_{(m-1)\Delta}) + P_{21}(\Delta)\Pi_{(m-1)\Delta}) + P_{12}(\Delta)(1 - \Pi_{(m-1)\Delta}) + P_{22}(\Delta)\Pi_{(m-1)\Delta}} \quad (9)$$

$V_{m\Delta}|X_{m\Delta}=1 \sim N_{X=1}(\mu_1 - B, \Sigma_1)$ and $V_{m\Delta}|X_{m\Delta}=2 \sim N_{X=2}(\mu_2 - B, \Sigma_2)$. Therefore

$$\begin{aligned} P&\left(\frac{j-1}{L} \le \Pi_{m\Delta} < \frac{j}{L} | \xi > m\Delta, \Pi_{(m-1)\Delta}\right) \\ &= P(a_1 \le V_{m\Delta} < a_2 | X_{m\Delta}=1)\left[\frac{D^1_{\Pi_{(m-1)\Delta}}}{D^2_{\Pi_{(m-1)\Delta}} + D^1_{\Pi_{(m-1)\Delta}}}\right] \\ &\quad + P(a_1 \le V_{m\Delta} < a_2 | X_{m\Delta}=2)\left[\frac{D^2_{\Pi_{(m-1)\Delta}}}{D^2_{\Pi_{(m-1)\Delta}} + D^1_{\Pi_{(m-1)\Delta}}}\right] \end{aligned} \tag{15}$$

can be simplified as

$$\begin{aligned} P&\left(\frac{j-1}{L} \le \Pi_{m\Delta} < \frac{j}{L} | \xi > m\Delta, \Pi_{(m-1)\Delta}\right) \\ &= [\Phi_1(a_2) - \Phi_1(a_1)]\left[\frac{D^1_{\Pi_{(m-1)\Delta}}}{D^2_{\Pi_{(m-1)\Delta}} + D^1_{\Pi_{(m-1)\Delta}}}\right] \\ &\quad + [\Phi_2(a_2) - \Phi_2(a_1)]\left[\frac{D^2_{\Pi_{(m-1)\Delta}}}{D^2_{\Pi_{(m-1)\Delta}} + D^1_{\Pi_{(m-1)\Delta}}}\right] \end{aligned} \tag{16}$$

where Φ_1 and Φ_2 represent the cumulative distribution function of $V_{m\Delta}|X_{m\Delta}=1$ and $V_{m\Delta}|X_{m\Delta}=2$.

Then, the SMDP state transition probability in (13) from state $i \in \mathbf{L}_1$ to state $j \in \mathbf{I}$ can be rewritten by

$$\begin{aligned} P_{i,j} &= P(\overline{\Pi} \le \Pi_{m\Delta} < 1 | \xi > m\Delta, \Pi_{(m-1)\Delta}) R(\Delta | \Pi_{(m-1)\Delta}) \\ &= [1 - \Phi_1(a_{\overline{\Pi}})]\left[\frac{D^1_{\Pi_{t(m-1)}}}{D^2_{\Pi_{(m-1)\Delta}} + D^1_{\Pi_{(m-1)\Delta}}}\right] \\ &\quad + [1 - \Phi_2(a_{\overline{\Pi}})]\left[\frac{D^2_{\Pi_{(m-1)\Delta}}}{D^2_{\Pi_{(m-1)\Delta}} + D^1_{\Pi_{(m-1)\Delta}}}\right] \end{aligned} \tag{17}$$

where $a_{\overline{\Pi}} = 2\ln[(((1-\overline{\Pi}) \cdot D^2_{\Pi_{(m-1)\Delta}})/(\overline{\Pi} \cdot D^1_{\Pi_{t(m-1)}}))h] - C$.

In addition, the SMDP state transition probability from state $(j, I) \in \mathbf{I}$ to state {PM} or to the initial healthy state $i \in \mathbf{L}_1$ with a false alarm can be computed, respectively, by

$$P_{(j,I),\mathrm{PM}} = \Pi_{m\Delta}, \; P_{(j,I),1} = 1 - \Pi_{m\Delta}. \tag{18}$$

The SMDP state transition probability from {PM} state to initial healthy state can be computed by

$$P_{\mathrm{PM},1} = 1. \tag{19}$$

The following step is devoted to calculating SMDP mean costs and mean sojourn times. Using the CRF defined in (14), for state $i \in \mathbf{L}_1$, the mean sojourn time can be computed by

$$\tau_i = \int_0^\Delta R(t|\Pi_{m\Delta})dt \tag{20}$$

and the remaining mean sojourn times are defined as

$$\begin{aligned} \tau_{\mathbf{I}} &= T_I \\ \tau_{\mathrm{PM}} &= T_{\mathrm{PM}} \end{aligned} \tag{21}$$

where T_I and T_{PM} are the inspection time and replacement time, respectively.

TABLE III
VALUES OF $\overline{\Pi}^*$ AND $g(\overline{\Pi}^*)$ UNDER A DIFFERENT DISCRETIZATION LEVEL L

L	32	64	128	256
$\overline{\Pi}^*$	0.2223	0.2233	0.2235	0.2235
$g(\overline{\Pi}^*)$	0.9362	0.9373	0.9378	0.9378

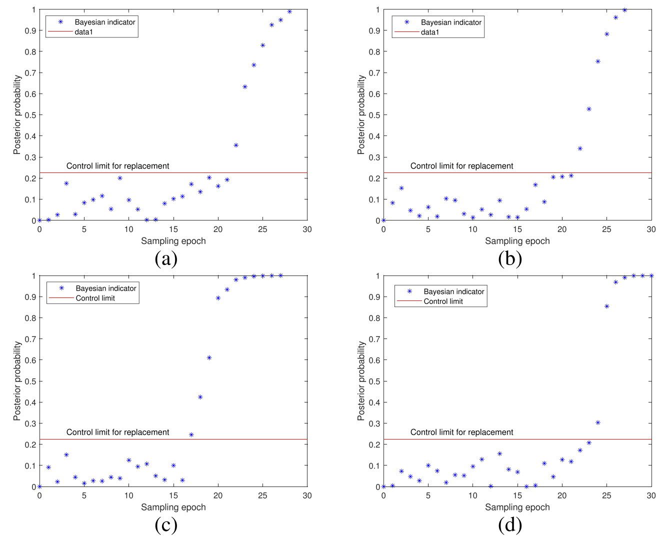


Fig. 8. Oil replacement process illustrated in the Bayesian control chart for different FHs. (a) FH #1. (b) FH #2. (c) FH #3. (d) FH #4.

For the problem of average availability maximization, the mean "costs" of the SMDP process are, in fact, the uptimes for each corresponding SMDP state, which are given by

$$\begin{aligned} C_i &= \tau_i, \quad i \in \mathbf{L}_1 \\ C_{(j,I)} &= C_{\mathrm{PM}} = 0. \end{aligned} \tag{22}$$

After computing the derivations of SMDP quantities by substituting the abovementioned quantities into (13), the optimal control limits $\overline{\Pi}^*$ for oil replacement scheme with the corresponding average availability $g(\overline{\Pi}^*)$ can be obtained.

By coding (13)–(22), the optimal control limit for oil replacement scheme is obtained as $\overline{\Pi}^* = 0.2235$ with corresponding $g(\overline{\Pi}^*) = 0.938$. The computation results under different level of discretization L are illustrated in Table III. We have found that, when $L \ge 32$, the partition leads to a sufficient degree of precision. We, therefore, choose 64 as the appropriate value of L to guarantee the high computation accuracy.

Examples of the Bayesian oil replacement scheme are given in the following with different failure oil data histories (i.e., FHs #1–#4), as shown in Fig. 8. Once the Bayesian indicator exceeds the control limit, $\overline{\Pi}^* = 0.2235$, the full oil analysis is initiated to examine the real quality of the lubricating oil. For all the FHs, the Bayesian indicator alarms before a physic failure occurrence. This is especially true when we check the Bayesian control chart in the following four cases. In Fig. 8, we can observe that the Bayesian indicator gives an alarm when the real state of lubricating oil is in severe

TABLE IV
COMPARISON OF DIFFERENT CONTROL SCHEMES

Control scheme	Control limit $\overline{\Pi}^*$	Control limit $\overline{T}^*$	Rate N_1	Rate N_2	Replacement time	Average availability
Proposed scheme	0.2235	-	3.7%	7.4%	21.3	0.938
Age-based scheme	-	18	14.8%	37%	18	0.792
Failure-based scheme	-	-	100%	0	22.9	0.611

deterioration. For FH #3, we can notice that the risks of the potential failures increase rapidly, and the oil fails quickly thereafter.

From the four illustrated cases, we find that the Bayesian control chart can predict the incoming risks and give the optimal replacement time to prevent sudden failures of oil systems. Besides, we also have compared our Bayesian oil replacement scheme with the typical age-based oil replacement scheme [32]. Consider that the oil replacement is initiated at time $n\Delta$. From the renewal theory, the expected availability for the scheme is given by

$$g(\overline{T}^*, \overline{T}^* = n\Delta) = \frac{\int_0^{n\Delta} \bar{F}(t)dt}{T_{\text{PM}}\bar{F}(n\Delta) + C_F F(n\Delta)} \quad (23)$$

where $F(n\Delta) = p_{13}(n\Delta)$ is the distribution function of ξ, and $\bar{F}(n\Delta) = 1 - F(n\Delta)$. Barlow and Hunter [32] showed that, under the age-based oil replacement scheme, the optimal replacement time $\overline{T}^*$ satisfies

$$h(\overline{T}^*)\int_0^{\overline{T}^*} R(t|0)dt - (1-R(\overline{T}^*|0)) = \frac{T_{\text{PM}}}{T_F - T_{\text{PM}}} \quad (24)$$

where the CRF is given by

$$\begin{aligned} R(t|0) &= 1 - p_{13}(t) \\ &= e^{-(\lambda_{12}+\lambda_{13})t} + \frac{\lambda_{12}(e^{-\lambda_{23}t} - e^{-(\lambda_{12}+\lambda_{13})t})}{\lambda_{12}+\lambda_{13}-\lambda_{23}} \end{aligned} \quad (25)$$

and the hazard rate function is given by (26), as shown at the bottom of the page.

Using the same replacement parameters in Section III and solving (24), the optimal oil replacement time is $\overline{T}^* = 18$, and the maximum availability is 0.792. The results are rather low, and the scheme is less effective than the proposed Bayesian oil replacement. The results of the comparison for the same failure oil histories are presented in Fig. 9. For FHs #1, #2, and #4, the age-based scheme stops the oil system and replaces the oil at least 3 sampling intervals earlier than the Bayesian control scheme, while, for FH #3, the age-based scheme stops the oil system much later when the risk is already high. We have examined all the oil data histories and presented the average replacement time in Table IV. Then, we have found that the proposed Bayesian replacement scheme for lubricating oil gives approximately 3.3 longer survival time than the age-based scheme. In addition, the age-based scheme gives a fixed replacement time, which results in much conservative or delay replacement control results. This is because the age-based oil replacement scheme does not use the updated oil information in decision-making, which leads to suboptimal results.

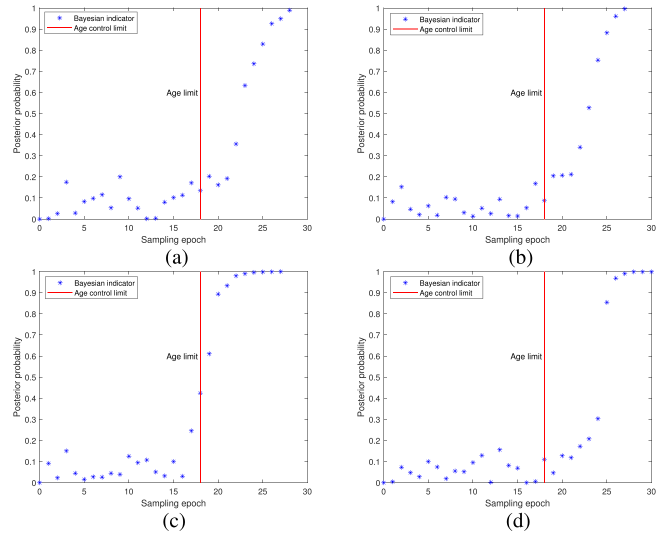


Fig. 9. Comparison with the age-based oil replacement scheme for different FHs. (a) FH #1. (b) FH #2. (c) FH #3. (d) FH #4.

We have further examined the misalarm rate and false-alarm rate of failure prediction for the age-based scheme, the failure-based scheme, and our proposed scheme, as shown in Table IV. Let N_1 and N_2 denote the misalarm rate and false-alarm rate of the replacement scheme, respectively. We have found that our proposed model achieves the lowest misalarm rate of 3.7% and the false-alarm rate of 7.4% in the failure prediction. The age-based scheme gives as high as 37% of false-alarm rate, while the failure-based scheme replaces the oil upon failures and misses alarming all the incoming failures. These two conventional schemes cannot give a satisfactory performance with high degrees of true-alarm rate, which may not be effective in the real oil replacement applications.

$$\begin{aligned} h(t) &= \frac{1}{R(t|0)} \cdot \left(-\frac{dR(t|0)}{dt}\right) \\ &= \frac{(\lambda_{12}+\lambda_{13}-\lambda_{23})(\lambda_{12}+\lambda_{13})e^{-(\lambda_{12}+\lambda_{13})t} - \lambda_{12}((\lambda_{12}+\lambda_{13})e^{-(\lambda_{12}+\lambda_{13})t} - \lambda_{23}e^{-\lambda_{23}t})}{(\lambda_{12}+\lambda_{13}-\lambda_{23})e^{-(\lambda_{12}+\lambda_{13})t} + \lambda_{12}(e^{-\lambda_{23}t} - e^{-(\lambda_{12}+\lambda_{13})t})} \end{aligned} \quad (26)$$

Through the abovementioned comparisons, we can conclude that the proposed Bayesian oil replacement scheme gives considerably better prediction and prevention of imminent failures.

V. Conclusion

The oil replacement scheme plays an important role in reducing unexpected random failures caused by oil deterioration, improving machine availability, and extending its service life. In this article, an oil replacement scheme based on the Bayesian approach has been proposed for the optimal replacement time determination for lubricating oil. Oil deterioration was simulated and implemented by a four-ball tester, and the 2-D vector oil data histories were regularly collected by an OLVF sensor. The lubricating oil deterioration process was assumed to be modeled as a stochastic Markov chain with three states, and the sojourn time in each hidden state was assumed to be exponentially distributed. A VAR model was adopted to fit the healthy parts of the oil monitoring data set for obtaining the residuals, which were regarded as the observation process for hidden Markov modeling. The EM algorithm was employed, and the unknown HMM parameters were estimated. A Bayesian control chart was presented to formulate the oil replacement scheme in an SMDP framework to find out the optimal control limit $\overline{\Pi}^*$ by maximizing the long-term expected average availability. Ultimately, comparisons with the age-based control scheme and the failure-based scheme were given, which demonstrated that our approach achieved better failure detection performance and the longer average availability. In addition, it is very new to apply the Bayesian control chart to provide a replacement scheme for lubricating oil.

In future research works, a more efficient hidden semi-Markov model for early detection of a degradation oil system can be considered to model the deterioration process. A three-state HMM can also be extended to an N-state model to accurately describe the deterioration process of lubricating oil. However, it will be more difficult to realize the modeling and analysis process. Another possible topic for future works will be to apply a two-level Bayesian control chart to formulate an oil replacement scheme and consider the average cost minimization as the optimization objective. In addition, an adaptive sampling strategy for online oil monitoring based on the Bayesian control chart will be a worthwhile research work, which can obtain appropriate sampling intervals in each deterioration stage according to the actual status of lubricating oil.

Appendix

The pseudo-log-likelihood function is defined as $Q(\lambda,\varphi|\widehat{\lambda},\widehat{\varphi}) = \sum_{i=1}^{N} Q_{\mathrm{FH_i}}(\lambda,\varphi|\widehat{\lambda},\widehat{\varphi}) + \sum_{j=1}^{M} Q_{\mathrm{SH_j}}(\lambda,\varphi|\widehat{\lambda},\widehat{\varphi})$, and for each single oil history, the function can be decomposed as $Q_\kappa(\lambda,\varphi|\widehat{\lambda},\widehat{\varphi}) = Q_\kappa{}^{\mathrm{State}}(\lambda|\widehat{\lambda},\widehat{\varphi}) + Q_\kappa{}^{\mathrm{Obs}}(\varphi|\widehat{\lambda},\widehat{\varphi})$.

For FHs, $Q_{\mathrm{FH}}{}^{State}(\lambda|\widehat{\lambda},\widehat{\varphi}) = \langle\widehat{\mathbf{A}},\lambda\rangle + \langle\widehat{\mathbf{B}},\ln\lambda\rangle$ and $Q_{\mathrm{FH}}{}^{Obs}(\varphi|\widehat{\lambda},\widehat{\varphi}) = \langle\widehat{\mathbf{C}},\ln\mathbf{G}\rangle$, where $\mathbf{G} = (g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,\Delta),\ldots, g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,T\Delta), g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,t))'$, which is the vector of the conditional density function $g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,k\Delta)$, $\overrightarrow{\mathbf{Y}} = (Y_1,\ldots,Y_T)$ is the residual observations, $\xi = t$ is the failure time with $T\Delta < t \le (T+1)\Delta$, $\tau_1 \in ((k-1)\Delta, k\Delta] \le t$, $k = 1,\ldots,T$, and the vectors are defined as $\widehat{\mathbf{A}} = (\widehat{a}_{12},\widehat{a}_{13},\widehat{a}_{23})'$, $\widehat{\mathbf{B}} = (\widehat{b}_{12},\widehat{b}_{13},\widehat{b}_{23})'$, and $\widehat{\mathbf{C}} = (\widehat{c}_1,\ldots,\widehat{c}_T,\widehat{c}_t)'$. For SHs, $Q_{\mathrm{SH}}{}^{State}(\lambda|\widehat{\lambda},\widehat{\varphi}) = \langle\widehat{\alpha},\lambda\rangle + \widehat{\gamma}_1\ln(\lambda_{12}) + \widehat{\gamma}_2\ln(\lambda_{12}+\lambda_{13})$ and $Q_{\mathrm{SH}}{}^{Obs}(\varphi|\widehat{\lambda},\widehat{\varphi}) = \langle\widehat{\beta},\ln\mathbf{G}\rangle$, where $\widehat{\alpha} = (\widehat{\alpha}_{12},\widehat{\alpha}_{13},\widehat{\alpha}_{23})'$ and $\widehat{\beta} = (\widehat{\beta}_1,\ldots,\widehat{\beta}_T,\widehat{\beta}_t)'$.

The explicit formulas for estimating state parameters and observation parameters mentioned earlier are given by

$$
\begin{aligned}
\widehat{a}_{12} &= -\frac{\widehat{p}_{12}\widehat{\lambda}_{23}e^{-\widehat{\lambda}_{23}t}}{\widehat{d}}\langle\widehat{\delta}_2,\widehat{\mathbf{G}}\rangle - \frac{t\,\widehat{p}_{13}e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13})t}}{\widehat{d}}g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,t)\\
\widehat{a}_{12} &= \widehat{a}_{13}\\
\widehat{a}_{23} &= \frac{\widehat{p}_{12}\widehat{\lambda}_{23}e^{-\widehat{\lambda}_{23}t}}{\widehat{d}}(\langle\widehat{\delta}_2,\widehat{\mathbf{G}}\rangle - t\langle\widehat{\delta}_1,\widehat{\mathbf{G}}\rangle)\\
\widehat{b}_{12} &= \widehat{b}_{23} = \frac{\widehat{p}_{12}\widehat{\lambda}_{23}e^{-\widehat{\lambda}_{23}t}}{\widehat{d}}\langle\widehat{\delta}_1,\widehat{\mathbf{G}}\rangle\\
\widehat{b}_{13} &= \frac{\widehat{p}_{13}e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13})t}}{\widehat{d}}g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,t)\\
\widehat{c}_k &= \frac{\widehat{p}_{12}\widehat{\lambda}_{23}e^{-\widehat{\lambda}_{23}t}\widehat{\delta}_1^k}{\widehat{d}}g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,k\Delta),\quad k=1,\ldots,T\\
\widehat{c}_t &= \left(\frac{\widehat{p}_{12}\widehat{\lambda}_{23}e^{-\widehat{\lambda}_{23}t}\widehat{\delta}_1^t + \widehat{p}_{13}e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13})t}}{\widehat{d}}\right)g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,t)\\
\widehat{d} &= \widehat{p}_{12}\widehat{\lambda}_{23}e^{-\widehat{\lambda}_{23}t}\langle\widehat{\delta}_1,\widehat{\mathbf{G}}\rangle + \widehat{p}_{13}e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13})t}g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,t)\\
\widehat{\alpha}_{12} &= -\frac{\widehat{\lambda}_{12}e^{-\widehat{\lambda}_{23}t}}{\widehat{\nu}}\langle\widehat{\delta}_2,\widehat{\mathbf{G}}\rangle - \frac{(t+(\widehat{\lambda}_{12}+\widehat{\lambda}_{13})^{-1})e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13})t}}{\widehat{\nu}}\\
&\quad\times g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,t)\\
\widehat{\alpha}_{12} &= \widehat{\alpha}_{13}\\
\widehat{\alpha}_{23} &= \frac{\widehat{\lambda}_{12}e^{-\widehat{\lambda}_{23}t}}{\widehat{\nu}}(\langle\widehat{\delta}_2,\widehat{\mathbf{g}}\rangle - t\langle\widehat{\delta}_1,\widehat{\mathbf{g}}\rangle)\\
\widehat{\beta}_k &= \frac{\widehat{\lambda}_{12}e^{-\widehat{\lambda}_{23}t}\widehat{\delta}_1^k}{\widehat{\nu}}g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,k\Delta),\quad k=1,\ldots,T\\
\widehat{\beta}_t &= \left(\frac{\widehat{\lambda}_{12}e^{-\widehat{\lambda}_{23}t}\widehat{\delta}_1^t + e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13})t}}{\widehat{\nu}}\right)g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,t)\\
\widehat{\gamma}_1 &= \frac{\widehat{\lambda}_{12}e^{-\widehat{\lambda}_{23}t}}{\widehat{\nu}}\langle\widehat{\delta}_1,\widehat{\mathbf{G}}\rangle\\
\widehat{\gamma}_2 &= \frac{e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13})t}}{\widehat{\nu}}g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,t)\\
\widehat{\nu} &= \widehat{\lambda}_{12}e^{-\widehat{\lambda}_{23}t}\langle\widehat{\delta}_1,\widehat{\mathbf{G}}\rangle + e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13})t}g_{\overrightarrow{\mathbf{Y}}|\xi,\tau_1}(\overrightarrow{y}|t,t).
\end{aligned}
$$

The vectors $\widehat{\delta}_1 = (\widehat{\delta}_1^1,\ldots,\widehat{\delta}_1^T,\widehat{\delta}_1^t)$ and $\widehat{\delta}_2 = (\widehat{\delta}_2^1,\ldots,\widehat{\delta}_2^T,\widehat{\delta}_2^t)$ can be computed by

$$
\begin{aligned}
\widehat{\delta}_1^k\cdot(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23}) &= e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23})(k-1)\Delta} - e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23})k\Delta}\\
\widehat{\delta}_1^t\cdot(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23}) &= e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23})T\Delta} - e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23})t}\\
\widehat{\delta}_2^k\cdot(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23}) &= \widehat{\delta}_1^k - k\Delta e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23})k\Delta}\\
&\quad + (k-1)\Delta e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23})(k-1)\Delta}\\
\widehat{\delta}_2^t\cdot(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23}) &= \widehat{\delta}_1^t - te^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23})t}\\
&\quad + T\Delta e^{-(\widehat{\lambda}_{12}+\widehat{\lambda}_{13}-\widehat{\lambda}_{23})T\Delta}.
\end{aligned}
$$

To obtain the maximization of the pseudo-log-likelihood function, we set

$$\frac{\partial Q(\lambda,\varphi|\widehat{\lambda},\widehat{\varphi})}{\partial\lambda_{12}}=\frac{\partial Q(\lambda,\varphi|\widehat{\lambda},\widehat{\varphi})}{\partial\lambda_{13}}=\frac{\partial Q(\lambda,\varphi|\widehat{\lambda},\widehat{\varphi})}{\partial\lambda_{23}}=0$$
$$\frac{\partial Q(\lambda,\varphi|\hat{\lambda},\hat{\varphi})}{\partial\mu_{1}}=\frac{\partial Q(\lambda,\varphi|\hat{\lambda},\hat{\varphi})}{\partial\mu_{2}}=0$$
$$\frac{\partial Q(\lambda,\varphi|\hat{\lambda},\hat{\varphi})}{\partial{\Sigma_1}^{-1}}=\frac{\partial Q(\lambda,\varphi|\hat{\lambda},\hat{\varphi})}{\partial{\Sigma_2}^{-1}}=0.$$

Then, we can obtain the state parameters $\lambda^* = (\lambda_{12}^*, \lambda_{13}^*, \lambda_{23}^*)$ and the observation parameters $\varphi^* = (\mu_1^*, \mu_2^*, \Sigma_1^*, \Sigma_2^*)$, respectively.

References

[1] Y. Li, J. Wu, and Q. Guo, "Electromagnetic sensor for detecting wear debris in lubricating oil," *IEEE Trans. Instrum. Meas.*, vol. 69, no. 5, pp. 2533–2541, May 2020.

[2] J. M. Wakiru, L. Pintelon, P. N. Muchiri, and P. K. Chemweno, "A review on lubricant condition monitoring information analysis for maintenance decision support," *Mech. Syst. Signal Process.*, vol. 118, pp. 108–132, Mar. 2019.

[3] Y. Du, T. Wu, and V. Makis, "Parameter estimation and remaining useful life prediction of lubricating oil with HMM," *Wear*, vols. 376–377, pp. 1227–1233, Apr. 2017.

[4] J. Zhu, J. M. Yoon, D. He, and E. Bechhoefer, "Online particle-contaminated lubrication oil condition monitoring and remaining useful life prediction for wind turbines," *Wind Energy*, vol. 18, no. 6, pp. 1131–1149, Jun. 2015.

[5] R. M. Mortier, M. F. Fox, and S. T. Orszulik, *Chemistry and Technology of Lubricants*, 3rd ed. Dordrecht, The Netherlands: Springer, 2013.

[6] L. Cui, X. Wang, H. Wang, and J. Ma, "Research on remaining useful life prediction of rolling element bearings based on time-varying Kalman filter," *IEEE Trans. Instrum. Meas.*, vol. 69, no. 6, pp. 2858–2867, Jun. 2020.

[7] C. Duan and C. Deng, "Prognostics of health measures for machines with aging and dynamic cumulative damage," *IEEE/ASME Trans. Mechatronics*, vol. 25, no. 5, pp. 2264–2275, Oct. 2020.

[8] M. Kordestani, M. Saif, M. E. Orchard, R. Razavi-Far, and K. Khorasani, "Failure prognosis and applications—A survey of recent literature," *IEEE Trans. Rel.*, early access, Sep. 17, 2019, doi: 10.1109/TR.2019.2930195.

[9] Y. Yang, Q. Wu, Y. Wang, W. Qin, and K. Lu, "Dynamic characteristics of cracked uncertain hollow-shaft," *Mech. Syst. Signal Process.*, vol. 124, pp. 36–48, Jun. 2019.

[10] X. Zhu, C. Zhong, and J. Zhe, "Lubricating oil conditioning sensors for online machine health monitoring–A review," *Tribol. Int.*, vol. 109, pp. 473–484, May 2017.

[11] J. Sun, L. Wang, J. Li, F. Li, J. Li, and H. Lu, "Online oil debris monitoring of rotating machinery: A detailed review of more than three decades," *Mech. Syst. Signal Process.*, vol. 149, Feb. 2021, Art. no. 107341.

[12] S. Yan, B. Ma, C. Zheng, and J. Chen, "An optimal lubrication oil replacement method based on selected oil field data," *IEEE Access*, vol. 7, pp. 92110–92118, 2019.

[13] M. J. Kim, R. Jiang, V. Makis, and C.-G. Lee, "Optimal Bayesian fault prediction scheme for a partially observable system subject to random failure," *Eur. J. Oper. Res.*, vol. 214, no. 2, pp. 331–339, 2011.

[14] J. S. Puckace, J. J. Redmon, H. Fitzgerald, and O. K. Ismail, "Method of determining optimal change interval for a functional fluid," U.S. Patent 9 195 964, Nov. 24, 2015.

[15] L. Wei, H. Duan, S. Chen, D. Jia, X. Qian, B. Cheng, J. Liu, and J. Li, "A field test method to quantitatively determine oxidation stability of gasoline engine oil," *Lubrication Sci.*, vol. 30, no. 2, pp. 57–64, Mar. 2018.

[16] L. Wei, H. Duan, Y. Jin, D. Jia, S. Chen, J. Liu, and J. Li, "Oil change interval evaluation of civilian cars in urban traffic condition," *Proc. Inst. Mech. Eng., J: J. Eng. Tribol.*, vol. 233, no. 6, pp. 956–964, Jun. 2019.

[17] H. Raposo, J. T. Farinha, L. Ferreira, and D. Galar, "Dimensioning reserve bus fleet using life cycle cost models and condition based/predictive maintenance: A case study," *Public Transp.*, vol. 10, no. 1, pp. 169–190, May 2018.

[18] H. Raposo, J. T. Farinha, I. Fonseca, and D. Galar, "Predicting condition based on oil analysis–A case study," *Tribol. Int.*, vol. 135, pp. 65–74, Jul. 2019.

[19] J. Yu, "Health condition monitoring of machines based on hidden Markov model and contribution analysis," *IEEE Trans. Instrum. Meas.*, vol. 61, no. 8, pp. 2200–2211, Aug. 2012.

[20] V. Makis, "Multivariate Bayesian control chart," *Oper. Res.*, vol. 56, no. 2, pp. 487–496, Apr. 2008.

[21] C. Lin and V. Makis, "Optimal Bayesian maintenance policy and early fault detection for a gearbox operating under varying load," *J. Vib. Control*, vol. 22, no. 15, pp. 3312–3325, Aug. 2016.

[22] H.-C. Yan, J.-H. Zhou, and C. K. Pang, "Machinery degradation inspection and maintenance using a cost-optimal non-fixed periodic strategy," *IEEE Trans. Instrum. Meas.*, vol. 65, no. 9, pp. 2067–2077, Sep. 2016.

[23] R. Jiang, M. J. Kim, and V. Makis, "Availability maximization under partial observations," *OR Spectr.*, vol. 35, no. 3, pp. 691–710, Jul. 2013.

[24] C. Duan, V. Makis, and C. Deng, "A two-level Bayesian early fault detection for mechanical equipment subject to dependent failure modes," *Rel. Eng. Syst. Saf.*, vol. 193, Jan. 2020, Art. no. 106676.

[25] T. Wu, J. Wang, J. Wu, Y. Xie, and J. Mao, "Wear characterization by an on-line ferrograph image," *Proc. Inst. Mech. Eng., J: J. Eng. Tribol.*, vol. 225, no. 1, pp. 23–34, Jan. 2011.

[26] R. Jiang, J. Yu, and V. Makis, "Optimal Bayesian estimation and control scheme for gear shaft fault detection," *Comput. Ind. Eng.*, vol. 63, no. 4, pp. 754–762, Dec. 2012.

[27] M. J. Kim, V. Makis, and R. Jiang, "Parameter estimation for partially observable systems subject to random failure," *Appl. Stochastic Models Bus. Ind.*, vol. 29, no. 3, pp. 279–294, May 2013.

[28] S. M. Ross, *Introduction to Probability Models*, 8th ed. San Diego, CA, USA: Academic, 2003.

[29] C. Duan, Z. Li, and F. Liu, "Condition-based maintenance for ship pumps subject to competing risks under stochastic maintenance quality," *Ocean Eng.*, vol. 218, Dec. 2020, Art. no. 108180.

[30] H. M. Tijms, *Stochastic Models–An Algorithmic Approach*. Hoboken, NJ, USA: Wiley, 1994.

[31] S. B. Provost and E. M. Rudiuk, "The exact distribution of indefinite quadratic forms in noncentral normal vectors," *Ann. Inst. Stat. Math.*, vol. 48, no. 2, pp. 381–394, Jun. 1996.

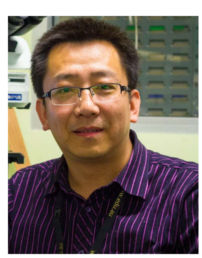
[32] R. Barlow and L. Hunter, "Optimum preventive maintenance policies," *Oper. Res.*, vol. 8, no. 1, pp. 90–100, Feb. 1960.

Ying Du received the Ph.D. degree from the School of Mechanical Engineering, Xi'an Jiaotong University, Xi'an, China, in 2018.

From 2015 to 2017, she was a Joint Visiting Ph.D. Student with the Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, ON, Canada. She is currently a Post-Doctoral Researcher with the School of Aeronautics, Northwestern Polytechnical University, Xi'an. Her research interests include fault diagnostics and prognostics, condition-based maintenance, and image processing.

Chaoqun Duan received the Ph.D. degree from the School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan, China, in 2018.

From 2015 to 2017, he was a Joint Visiting Ph.D. Student with the Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, ON, Canada. He is currently an Assistant Professor with the School of Mechatronic Engineering and Automation, Shanghai University, Shanghai, China. His research interests include fault diagnostics and prognostics, reliability modeling and estimation, and condition-based maintenance.



Tonghai Wu received the Ph.D. degree from the School of Mechanical Engineering, Xi'an Jiaotong University, Xi'an, China, in 2006.

From 2013 to 2014, he was a Visiting Scholar with the University of New South Wales, Sydney, NSW, Australia. He is currently a Professor with the School of Mechanical Engineering, Xi'an Jiaotong University. His current research interests include machinery diagnosis and prognosis, machine vision on wear particles, and surface analysis.