当数论遇上组合

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高维东: Long sequences having no two nonempty zero-sum subsequences of distinct lengths

Let G be an additive finite abelian group. We call a sequence of elements (repetition allowed) from G a ***zero-sum sequence*** if the sum of all terms of S equals to 0 (the identity element of G). In 2012, Girard posed a problem of determining the smallest positive integer t, denoted by $\mathrm{disc}(G)$, such that every sequence S over G of length $|S| \geq t$ has two nonempty zero-sum subsequences of distinct lengths. The study on problems related to $\mathrm{disc}(G)$ can track back to 1970's, Graham conjectured that if p is a prime and S is a sequence of length $|S| = p$ over C_p such that all (nontrivial) zero-sum subsequences have the same length, then S must contain at most two distinct terms. In 1976, Erdős and Szemerédi confirmed this conjecture for sufficiently large primes p. We will present some new results and open problems on $\mathrm{disc}(G)$.

李吉有: Enumeration of polynomials over finite fields

The distribution of factorization patterns in polynomials over finite fields is an important problem in algebra and number theory. In this talk we will introduce certain distribution problems for polynomials with prescribed leading coefficients, which naturally arise from the list sizes of Reed Solomon codes and the spectrum of Wegner graphs. This is joint work with Daqing Wan.

孙智伟: Permutations of $\{1, \ldots, n\}$ and related permanents

In this talk we introduce recent results on permutations of $\{1, \ldots, n\}$. For example, we show that for any positive integer n there is a unique permutation $\pi \in S_n$ such that all the numbers $k + \pi(k)$ $(k = 1, \ldots, n)$ are powers of two.

We also mention some divisibility properties of the permanent

$$\mathrm{per}[i^{j-1}]_{1\leqslant i,j\leqslant n} = \sum_{\sigma\in S_n}\prod_{i=1}^{n} i^{\sigma(i)-1},$$

as well as related applications to groups.

We also introduce some open conjectures of the speaker, one of which states that for any integer $n > 6$ there is a permutation $\pi \in S_n$ such that

$$\sum_{k=1}^{n-1}\frac{1}{\pi(k)+\pi(k+1)} = 1.$$

周进鑫: Symmetry of bi-Cayley graphs

A graph X admitting a group H of automorphisms acting semi-regularly on the vertices with exactly two orbits is called a ***bi-Cayley graph over H.*** This is a natural generalization of Cayley graph. The bi-Cayley graphs play an important role in the study of symmetry of graphs.

This class of graphs can be used to construct some highly symmetrical graphs, such as, vertex-transitive non-Cayley graphs, semisymmetric graphs, half-arc-transitive graphs and so on. In this talk, I shall introduce some of our recent work regarding the symmetry of bi-Cayley graphs.