

Wide-spectrum reconstruction method for a birefringence interference imaging spectrometer

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We present a mathematical method used to determine the spectrum detected by a birefringence interference imaging spectrometer (BIIS). The reconstructed spectrum has good precision over a wide spectral range, 0.4–1.0 μm . This method considers the light intensity as a function of wavelength and avoids the fatal error caused by birefringence effect in the conventional Fourier transform method. The experimental interferogram of the BIIS is processed in this new way, and the interference data and reconstructed spectrum are in good agreement, proving this method to be very exact and useful. Application of this method will greatly improve the instrument performance. © 2010 Optical Society of America

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The birefringence interference imaging spectrometer (BIIS) has some great advantages, including no moving parts, high optical throughput, wide field of view, and compounded information carrier. It is becoming more and more valuable in remote sensing, resource investigation, environment observation, and object recognition [1,2]. In its application, to obtain useful information about the detected objects, spectrum reconstruction and interferogram processing is always necessary and important. The traditional reproduction technique is based on a scanning Michelson interferometer [3], which has a moving mirror to vary the optical path difference (OPD) at several moments. Under this condition the OPD is the same for each wavelength, but in practical applications just as the BIIS, the OPD varies owing to the birefringence effect. Thus different wavenumbers have different OPDs, and the conventional Fourier transform of interferogram is not applicable if one wants to obtain accurate information about the true spectrum. Therefore this Letter emphasizes the characteristics of BIIS' special interferogram imaging principle and presents a precise spectrum reconstruction algorithm.

Figure 1 shows the optical diagram of the BIIS [4,5]. It consists of a pretelescope system, a polarized interferometer (polarizer P1, Savart polariscope, and analyzer P2), an imaging lens, and a detector. The incident light is polarized by linear polarizer P1 and split into two polarized components by a Savart polariscope. The two components are made to interfere by the imaging lens, and the final interferogram and target's image are recorded by a 2D detector.

In traditional Fourier transform spectroscopy [6], the basic equation of the spectrum and interferogram is defined as

$$B(\sigma) = \int_{-\infty}^{+\infty} \left[I(\delta) - \frac{1}{2}I(0) \right] e^{-i2\pi\sigma\delta} d\delta, \quad (1)$$

where $B(\sigma)$ is known as the spectrum intensity and $I(\delta) - 1/2I(0)$ is the interferogram. So Eq. (1), at a

given wavenumber σ , states that if the flux versus optical path $I(\delta)$ is known as a function of OPD δ , the Fourier transform of $I(\delta) - 1/2I(0)$ yields $B(\sigma)$. The Fourier transform theory is perfect without doubt, but in BIIS, the OPD varies with the wavenumbers owing to the birefringence effect; thus the theory will be not so accurate. Taking calcite crystals, for example, as the material of the Savart plate in BIIS, according to the room-temperature dispersion formulas for calcite crystals, the refractive index of calcite can be expressed as [7]

$$n_o^2 - 1 = \frac{0.8559\lambda^2}{\lambda^2 - (0.0588)^2} + \frac{0.8391\lambda^2}{\lambda^2 - (0.141)^2} + \frac{0.0009\lambda^2}{\lambda^2 - (0.197)^2} + \frac{0.6845\lambda^2}{\lambda^2 - (7.005)^2}, \quad (2)$$

$$n_e^2 - 1 = \frac{1.0856\lambda^2}{\lambda^2 - (0.07897)^2} + \frac{0.0988\lambda^2}{\lambda^2 - (0.142)^2} + \frac{0.317\lambda^2}{\lambda^2 - (11.468)^2}. \quad (3)$$

In BIIS, the OPD expression is [8]

$$\Delta = t \left[\frac{a^2 - b^2}{a^2 + b^2} (\cos \omega + \sin \omega) \sin i + \frac{a^2(a^2 - b^2)}{\sqrt{2}(a^2 + b^2)^{3/2}} (\cos^2 \omega - \sin^2 \omega) \sin^2 i + \dots \right], \quad (4)$$

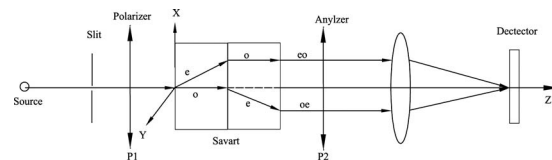


Fig. 1. Optical diagram of the BIIS.

where $a=1/n_e$, $b=1/n_o$, t is the thickness of the single Savart plate, and i is the incidence angle. Using the OPD definition expressed in Eq. (4), the different maximal OPD of each wave length is obtained as Fig. 2 shows. In Fig. 2, it is easy to see that the maximal OPD is decreased with the wavelength, and the biggest difference between these maximum OPDs is about $0.6 \mu\text{m}$, almost 15% of the maximum OPD at $0.862 \mu\text{m}$. This means that the OPD sampling step $d\delta$ in Eq. (1) for each wavenumber is quite different, and the traditional Fourier transform method will obviously not be so accurate for BIIS [9]. Therefore, we have to find a new method to rebuild the spectrum for BIIS.

According to the basic theory of Fourier transform spectroscopy, the key to solving the problem is to obtain interference intensities of each wavelength. For getting each wavenumber's interferogram, it is necessary to know the constitution of the obtained interferogram. According to the interferogram forming principle of the BIIS, each detected interferogram intensity I is the total interference intensity over the range of all detecting wavenumbers. Because the different light waves are incoherent light, the detected interferogram intensity I is the summation of total waves, just as Eq. (5) shows:

$$\sum_{n=1}^K I_{\lambda n} = I. \quad (5)$$

Here $I_{\lambda n}$ is each light wave's interference intensity and K is the number of wave bands. If each target's interferogram consists of N interference intensities, then Eq. (5) can be written as

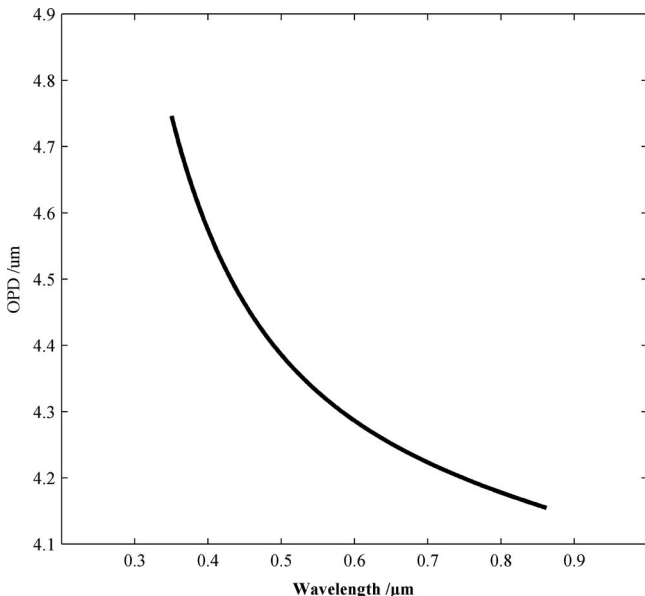


Fig. 2. Maximal OPD distribution of BIIS at the wavelength range of $0.350\text{--}0.862 \mu\text{m}$.

$$\begin{bmatrix} I_{\lambda 1}^0 \\ I_{\lambda 1}^1 \\ \cdot \\ \cdot \\ I_{\lambda 1}^N \end{bmatrix} + \begin{bmatrix} I_{\lambda 2}^0 \\ I_{\lambda 2}^1 \\ \cdot \\ \cdot \\ I_{\lambda 2}^N \end{bmatrix} + \begin{bmatrix} I_{\lambda 3}^0 \\ I_{\lambda 3}^1 \\ \cdot \\ \cdot \\ I_{\lambda 3}^N \end{bmatrix} + \dots + \begin{bmatrix} I_{\lambda K}^0 \\ I_{\lambda K}^1 \\ \cdot \\ \cdot \\ I_{\lambda K}^N \end{bmatrix} = \begin{bmatrix} I_0 \\ I_1 \\ \cdot \\ \cdot \\ I_N \end{bmatrix}, \quad (6)$$

where $I_{\lambda n}^0, I_{\lambda n}^1, \dots, I_{\lambda n}^N$ are the interference intensities of each light wave with different OPDs, I_0, I_1, \dots, I_N are a target's detected interferogram intensities and the discrete form of $I(\delta)$ in Eq. (1). In addition, because $I_{\lambda n}^0, I_{\lambda n}^1, \dots, I_{\lambda n}^N$ are the same light wave's interference intensities, their relationship is such that

$$I_{\lambda n}^m = I_{\lambda n}^0 (1 + \cos \Delta_{\lambda m}) / 2 \quad (n = 0, 1, 2, \dots, K; m = 0, 1, 2, \dots, N), \quad (7)$$

where $I_{\lambda n}^0$ is the intensity when the OPD is zero and equals each incident light wave's original intensity. $\Delta_{\lambda m}$ is the OPD related to each wavenumber's interference intensity, which is calculable and determined by design. Then Eq. (6) can be written as

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ C_1^1 & C_2^1 & \dots & C_K^1 \\ C_1^2 & C_2^2 & \dots & C_K^2 \\ \cdot \\ \cdot \\ C_1^N & C_2^N & \dots & C_K^N \end{bmatrix} \begin{bmatrix} I_{\lambda 1}^0 \\ I_{\lambda 2}^0 \\ \cdot \\ \cdot \\ I_{\lambda K}^0 \end{bmatrix} = \begin{bmatrix} I_0 \\ I_1 \\ \cdot \\ \cdot \\ I_N \end{bmatrix}, \quad (8)$$

where $C_n^0, C_n^1, \dots, C_n^K$ are constant coefficients, which are equal to $(1 + \cos \Delta_{\lambda m}) / 2$. By resolving Eq. (8), the intensity $I_{\lambda 1}^0, I_{\lambda 2}^0, \dots, I_{\lambda K}^0$ of each light wave could be obtained. As is well known, the spectrum referred to a plot of light intensity or power as a function of frequency or wavelength. So the spectrum that equals $(I_{\lambda 1}^0, I_{\lambda 2}^0, \dots, I_{\lambda K}^0)$ is finally obtained without Fourier transforms. We name this method birefringence interference transform (BIT).

For testing this method, we assumed that a uniform white light for which each light wave's intensity is equal to 80 in a normalized unit, and Fig. 3 is the simulated image according to the BIIS imaging principle. We processed Fig. 3 in this new way and got the final spectrum as Fig. 4 shows. It is obviously similar to the assumed spectrum; the maximum error is less than 0.2%. So we can say that through resolving each light wave's intensities at the target, BIT can successfully avoid the error caused by the birefringence crystal. The experimental results below will prove that this method is very suitable for BIIS in practice.

The experimental BIIS uses a $12 \text{ mm} \times 12 \text{ mm} \times (6+6) \text{ mm}$ Savart polariscope, a Mintron 512 \times 512 CCD camera, the size of each pixel is $10 \mu\text{m} \times 10 \mu\text{m}$, and the field of view is $i \approx 3^\circ$. Figure 5 shows a target's typical interferogram and image un-



Fig. 3. Simulated interferogram obtained by BIIS.

der white light obtained by BIIS. Through resolving each light wave's interferogram according to Eqs. (7) and (8), one reconstructed transmittance spectrum of the target is finally obtained as Fig. 6 shows. Its resolution is about 1 nm. The reconstructed spectrum is obviously better than the result of the traditional Fourier transform, which we presented in [9].

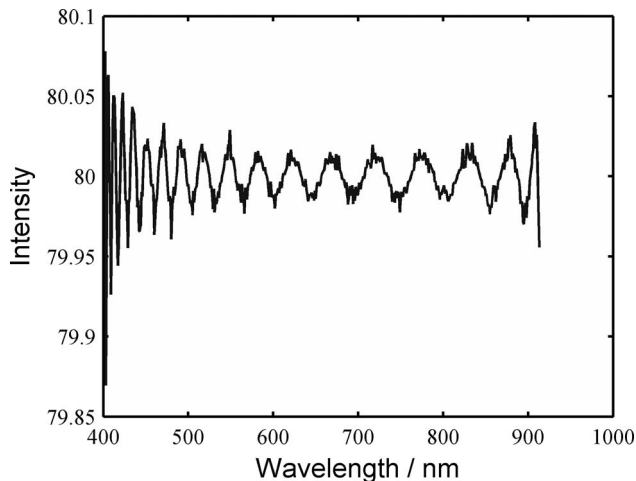


Fig. 4. Reconstructed spectrum, by BIT.

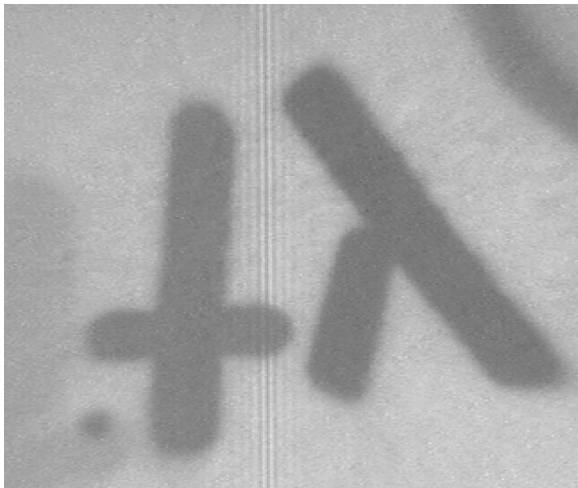


Fig. 5. Target plot's interferogram and image of white light, by BIIS.

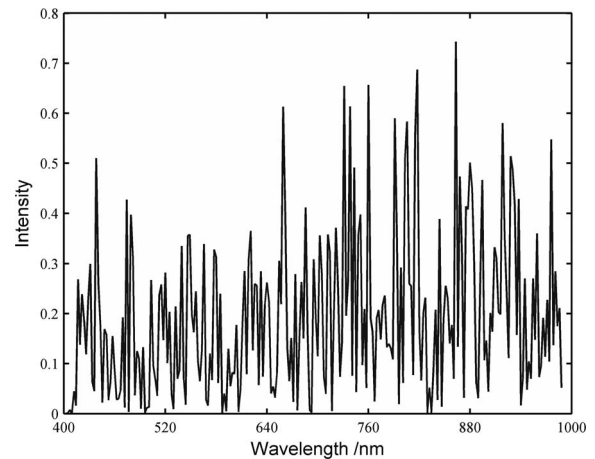


Fig. 6. Reconstructed spectrum of the polychromatic light.

In this Letter we have shown the possibility of accurately reconstructing the spectrum detected by BIIS over a wide spectrum range. In this method, each light wave's interferogram and intensity are calculated and reconstructed respectively. The final target spectrum is the combination of all light waves without Fourier transforms. The experimental results prove that this method is very efficient for processing the interference data of BIIS. There are many potential merits in BIT, such as high accuracy and high spectrum resolution, and it will facilitate spectrum calibration because the calculated intensity corresponds to its wavelength. In addition, the proposed method can also be used for other birefringence spectrometers. The only additional work to do is to count the OPD distribution.

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