Rare Metal Materials and Engineering
Volume 41, Suppl. 1, February. 2012
Cite this article as: Rare Metal Materials and Engineering, 2012, 41(S1): 0???-0???.

Critical Maximum Contact Pressure for Yielding in Hard Coating under Sliding Contact

Zhang Pengyu [1], **Diao Dongfeng** [1], **Wang Zhanjiang** [2]

[1] *Key Laboratory of Education Ministry for Modern Design and Rotor-Bearing System, School of Mechanical Engineering, Xi'an Jiaotong University, Xi'an 710049, China*
[2] *State Key Laboratory of Mechanical Transmission, Chongqing University, Chongqing 400030, China*

Abstract: In order to design the best combination of the coating and the substrate for preventing the yield, the semi-analytical method (SAM) was used to analyze the von Mises stress distributions in the contact system, and calculate the critical maximum contact pressure for yielding at the interface and surface in hard coating under both normal and tangential loads on three-dimensional elastic half space. From the results, it can be concluded that, to obtain a higher maximum critical contact pressure for the yield at the interface, increasing the substrate hardness and the ratio value of the coating thickness to the Hertzian contact radius(t/a_0) is more effective when t/a_0 is larger than 0.5 while increasing the substrate hardness or making the friction coefficient smaller is better when t/a_0 is very small; to prevent yielding at the surface, choosing a relatively lower friction system or increasing the yield strength ratio of the coating to the substrate(Y_f/Y_b) will be effective.

Key words: hard coating; sliding contact; contact pressure; three-dimension; yield; semi-analytical method

Coating technology is of great interest in industrial applications due to its enormous technological as well as economical importance. Depositing a hard coating on a soft substrate can significantly enhance the tribological properties of the surface, such as reducing the wear by preventing plough, reducing the contact area and, thus, reducing the friction in carrying the load. However, the trouble is fracture in the coating or delamination and spalling at the interface between coating and substrate under the critical contact condition [1-2]. Therefore, it is very important to know and control the critical contact condition for yielding to prevent the delamination and spalling.

Diao et al. [3-4] presented a two-dimensional finite element method (FEM) and used the critical condition for yielding in a hard coating to estimate the formation of a crack or a delamination. They showed that the yield appears at the coating-substrate interface under critical contact condition and for a special combination of coating and substrate. Holmberg [5] developed a three-dimensional FEM model to present a tribological analysis of deformations and stresses generated and their influence on crack generation and surface fracture in a layered surface loaded by a sliding sphere in dry conditions. Semi-analytical method (SAM) is more efficient than FEM because only the contact area of interest needs to be meshed and closed-form displacement solutions of surface tractions are used. Nogi and Kato [6] adopted O'Sullivan and King's [7] three-dimensional influence coefficients and applied the algorithm of the continuous convolution and fast Fourier transform (CC-FFT) to calculate the contact pressure, displacements, and the stress field on layered elastic solid. Polonsky et al. [8] constructed a discrete convolution and fast Fourier transform (DC-FFT) algorithm by adding a special correction procedure, which was based on the multi-level multi-summation (MLMS) technique and the conjugate gradient method (CGM), to the CC-FFT algorithm. The algorithm is a process of zero padding and wrap-around order and has been adopted in the studies of three-dimensional contact mechanisms for more recent years, owing to the fast solution of contact pressure by using the CGM and DC-FFT technique [9-12]. However, there is still dearth of analytical data for CGM and DC-FFT technique to calculate the critical maximum contact pressures for yielding at the interface or surface. Thus, further studies are still necessary for the efficient way to solve the problem.

In this paper, therefore, the CGM and DC-FFT technique were used to obtain the von Mises stress distributions under both normal and tangential loads on three-dimensional elastic half space, and then calculate the critical maximum contact pressure for yielding at the interface and surface. According to the calculated results, we can find some more effective ways

Received date: May. 25, 2011
Foundation item: NSFC(90923027 and 51050110137); The Fundamental Research Funds for Central Universities
Corresponding author: Diao Dongfeng, Ph.D., Professor, Key Laboratory of Education Ministry for Modern Design and Rotor-Bearing System, School of Mechanical Engineering, Xi'an Jiaotong University, Xi'an 710049, P. R. China, Tel: 0086-29-82669151, E-mail: dfdiao@mail.xjtu.edu.cn

to obtain a higher value of the maximum critical contact pressure for preventing the yield at the interface and surface.

1 Three-dimensional elastic contact model

Figure 1 shows a model of contact between a rigid ball and a smooth flat with a layer of coating. The coating has a uniform thickness t. Normal load W and tangential forces F_x are both applied on the rigid ball. Normal load is fixed and tangential load F_x is increased with the friction coefficient μ which changes from 0.25 to 0.70. The Young's modulus ratio of the coating to the substrate (E_f/E_b) is kept constant at two. Poisson's ratio is taken as 0.3 for both the coating and substrate. In the figure, a denotes the actual radius of the contact zone. In the following analyses, the stresses are normalized by the maximum Hertzian contact pressure p_0, and the coordinates are normalized by Hertzian contact radius a_0. The normalized thickness t/a_0 is changed from 0.125 to 4. In the coating layer, the stresses and displacements are taken as functions of x, y, and z_1, while in the substrate they are functions of x, y, and z_2. The applied loads consist of normal pressure p(x,y) and shear traction q_x(x,y). So the boundary conditions at the upper layer surface (z_1=0) are given by:

$$\sigma_{zz}^{(1)}(x,y,0)=-p(x,y),\ \sigma_{xz}^{(1)}(x,y,0)=q_x(x,y),\ \sigma_{yz}^{(1)}(x,y,0)=0. \quad (1)$$

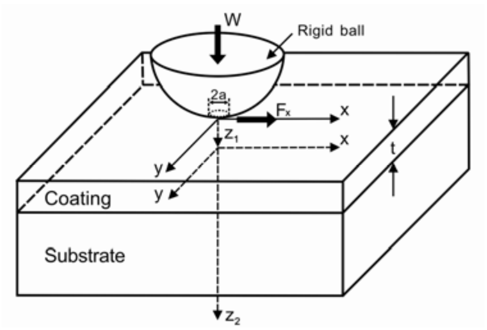

Fig.1 Contact model of a rigid ball on a layered substrate

The continuous conditions for the stresses and displacements at the interface (z_2=0) are as follows:

$$\sigma_{xz}^{(1)}(x,y,t)=\sigma_{xz}^{(2)}(x,y,0),\ \sigma_{yz}^{(1)}(x,y,t)=\sigma_{yz}^{(2)}(x,y,0),$$
$$\sigma_{zz}^{(1)}(x,y,t)=\sigma_{zz}^{(2)}(x,y,0),\ u_x^{(1)}(x,y,t)=u_x^{(2)}(x,y,0),$$
$$u_y^{(1)}(x,y,t)=u_y^{(2)}(x,y,0),\ u_z^{(1)}(x,y,t)=u_z^{(2)}(x,y,0). \quad (2)$$

In the substrate, the stresses and displacements should varnish at a large distance from the contact surface:

$$\sigma^{(2)}(x,y,\infty)=0,\quad u^{(2)}(x,y,\infty)=0. \quad (3)$$

For zero body force, the Papkovich-Neuber potentials φ and $\psi(\psi_1,\psi_2,\psi_3)$ are harmonic functions of x, y, z in space domain. So, the number of independent harmonic functions can be reduced to three by arbitrarily choosing one of ψ_1, ψ_2, ψ_3 to be zero. Here ψ_2 is assumed equal to zero. In frequency domain, the double Fourier transformed Papkovich-Neuber potentials are given by:

$$\tilde{\varphi}^{(k)}=A^{(k)}\exp(-\alpha z_k)+\bar{A}^{(k)}\exp(\alpha z_k),$$
$$\tilde{\psi}_1^{(k)}=B^{(k)}\exp(-\alpha z_k)+\bar{B}^{(k)}\exp(\alpha z_k),$$
$$\tilde{\psi}_3^{(k)}=C^{(k)}\exp(-\alpha z_k)+\bar{C}^{(k)}\exp(\alpha z_k). \quad (4)$$

Where, k=1 or 2 mark the layers, corresponding to the coating and substrate. $A^{(k)},\bar{A}^{(k)},B^{(k)},\bar{B}^{(k)},C^{(k)},\bar{C}^{(k)}$ denote twelve unknowns and the stresses and displacements at the large distance from the surface are assumed to be zero. The parameter $\alpha=\sqrt{m^2+n^2}$ is defined, where m, n are variables in frequency domain, correspond to x, y, respectively, in space domain. The expressions for stresses and displacements in frequency domain can be defined as:

$$\tilde{\sigma}_{ij}^{(k)}=FT[\phi_{,ij}^{(k)}-2\nu_k(\psi_{1,1}^{(k)}+\psi_{3,3}^{(k)})\delta_{ij}-(1-2\nu_k)(\psi_{i,j}^{(k)}+\psi_{j,i}^{(k)})$$
$$+x\psi_{1,ij}^{(k)}+z_k\psi_{3,ij}^{(k)}],$$
$$\tilde{u}_i^{(k)}=\frac{1}{2G_k}FT[\phi_{,i}^{(k)}+x\psi_{1,i}^{(k)}+z_k\psi_{3,i}^{(k)}-(3-4\nu_k)\psi_i^{(k)}]. \quad (5)$$

Where FT denotes the Fourier transform, G_k is shear modulus, and ν_k is Poisson's ratio. Once the influences coefficients are obtained, the elastic deformations of layered materials and the stresses can be calculated by using the CGM and DC-FFT algorithm[9-12].

2 Results and Discussion

2.1 Distributions of the von Mises stress

The position of yield in the coating and substrate is governed by the von Mises stress $\sqrt{J_2}/P_0$ 错误！未找到引用源。where J_2 is the second invariant of the stress deviator tensor [7]. The contours of the von Mises stress in the plane y=0 for different friction coefficients are plotted in Fig.2, where the coating and substrate elastic properties are the same. When μ=0.00, as shown in Fig.2a, the maximum von Mises stress is 0.362, which has the little different rate of 0.968% compared with the result in the O'Sullivan and King's investigation [7]. Similar little different rates can be obtained when μ=0.25 or μ=0.50. Despite some differences, our results are in good agreement with the reference [9]. Under the condition of

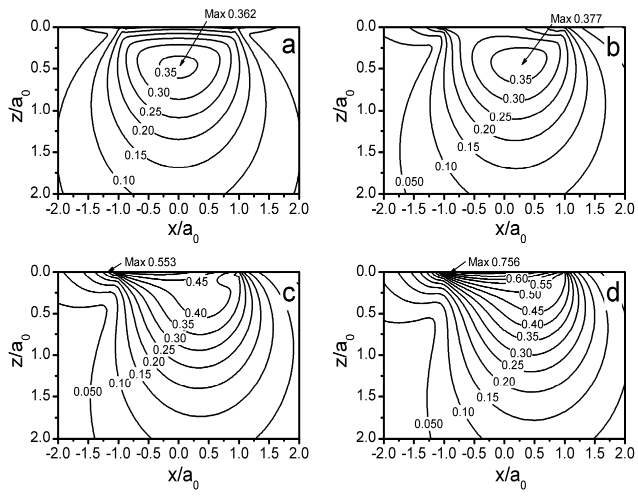

Fig.2 Contour plots of the von Mises stress in the y=0 plane for E_f/E_b=1, (a) μ=0.00; 错误！未找到引用源。(b) μ=0.25; (c) μ=0.50; (d) μ=0.70

non-layered status, the position of the maximum von Mises stress locates below the surface or contact center for relatively low friction. When the friction coefficient is increased (see Figs.2c-2d), the maximum von Mises stress appears at the positions on the surface.

Contour plots of the von Mises stress in the y=0 plane for μ=0.25 but with different coating thicknesses are presented in Figs.3a-3b. For t=0.5a_0, the maximum stress with a mild discontinuity occurs at the interface. And if the coating thickness is increased to t=a_0, we can see clearly that the position of the maximum stress moves upward in the coating. When the friction coefficient increases, as shown in Figs.3c-3f, the maximum von Mises stress is always at the surface.

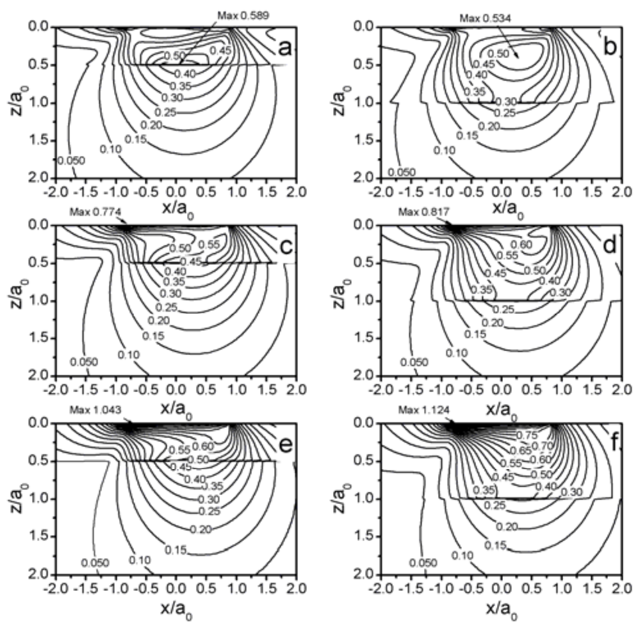

Fig.3 Contour plots of the von Mises stress in the y=0 plane for E_f/E_b=2, (a)μ=0.25, t=0.5a_0; (b)μ=0.25, t=a_0; (c)μ=0.5, t=0.5a_0; (d)μ=0.5, t=a_0; (e)μ=0.7, t=0.5a_0; (f)μ=0.7, t=a_0

2.2 Critical maximum contact pressure for yielding at the interface and surface

In previous analyses [13-16], the von Mises stress for yield initiation became a good standard for estimating the formation of a crack or a delamination. Based on the above results, the position of the maximum von Mises stress most appears at the interface or surface. So it is necessary to evaluate the critical maximum contact pressure $P_{max,c}$ for the yield at the interface and surface. It should be emphasized that the hardness of substrate (H_b) and coating (H_f) is easier to be measured than yield strength and Y_f/Y_b=H_f/H_b is widely accepted in practical application. Therefore, we use the hardness of substrate or coating to normalize $P_{max,c}$ and select H_b=3Y_b, H_f=3Y_f as a representative combination for the hard coating. It has been mentioned in previous studies of Diao et al. [3-4] that the critical maximum contact pressure $P_{max,c}$ for von Mises yielding obtained from

$$P_{max,c} = Min[Y(x,0,z_1)/\psi_Y(x,0,z_1)]. \quad (6)$$

Here, $\psi_Y(x,0,z_1)$ is the ratio of maximum von Mises stress to maximum Hertzian contact pressure $\sigma_{vm,max}/P_0$, and $Y(x,0,z_1)$ is the distribution of the yield strength in the y=0 plane. In this paper, $Y(x,0,z_1)$ only depends on the value of z_1, when z_1<t, $Y(x,0,z_1)$=Y_f, and when z_1>t, $Y(x,0,z_1)$=Y_b, Y_f and Y_b are the yield strength of the coating and substrate materials. Only Y_f>Y_b is included by considering the hard coating in this paper, so the yield condition at the interface can be given by $Y(x,0,z_1)$=Y_b.

Figure 4 shows the relationship between the normalized critical maximum contact pressure for the yield at the interface $P_{max,c}/H_b$ and Y_f/Y_b for different friction coefficients. It can be seen that, when μ is increased from 0.25 to 0.70, no effect of the yield strength ratio on $P_{max,c}/H_b$ can be seen. So the effect of Y_f/Y_b on the critical maximum contact pressure for the yield at the interface can be ignored. The relationship between the substrate hardness normalized critical maximum contact pressure for the yield at the interface $P_{max,c}/H_b$ and t/a_0 is shown in Fig.5. From Figs.4-5 and our previous studies [3-4], we find a more effective way to obtain a higher critical maximum contact pressure for the yield at the interface, that is: increasing the substrate

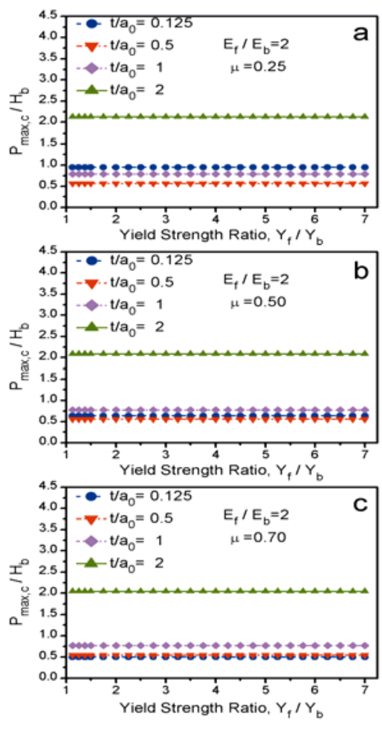

Fig.4 Critical maximum contact pressure for yielding at the interface, (a) μ=0.25; (b)μ=0.50; (c) μ=0.70

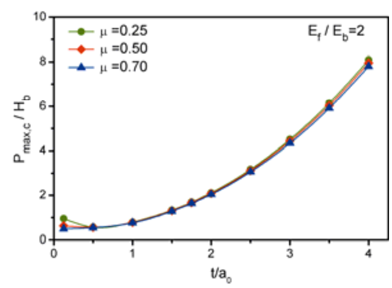

Fig.5 Relationship between the normalized critical maximum contact pressure for the yield at the interface $P_{max,c}/H_b$ and t/a_0

hardness H_b and t/a_0 rather than μ when t/a_0 is larger than 0.5. Furthermore, when t/a_0 is smaller than 0.5, we can increase H_b for the effective prevention of yielding at the interface.

Figure 6 shows the relationship between critical maximum contact pressure for yielding at the surface and the ratio of yield strength. We see the value of $P_{max,c}/H_b$ increases when increasing Y_f/Y_b with a certain slope and the slope changes with changing t/a_0.It proves that, to prevent the yield at the surface, a rise in the Y_f/Y_b will be effective. Figure 7 shows the relationship between the coating hardness normalized critical maximum contact pressure for yielding at the surface and t/a_0 for different friction coefficients μ when E_f/E_b=2 is kept constant at two. We also find when the friction coefficient is kept constant, the effect of t/a_0 on $P_{max,c}$ gradually decreases. If we keep the value of t/a_0 constant, we will find $P_{max,c}$ decreases with the increase of the friction coefficient. So we can conclude a more effective way from the Figs.6-7 to obtain a higher value of the maximum critical maximum contact pressure at the surface to prevent yielding, which is to choose a relatively lower friction system or increasing the yield strength ratio.

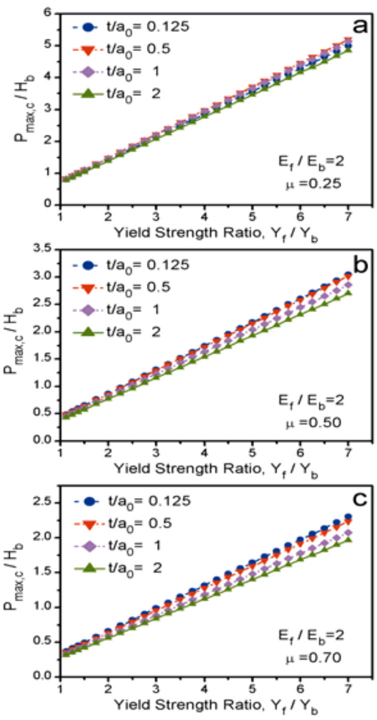

Fig.6 Critical maximum contact pressure for yielding at the surface, (a) μ =0.25; (b) μ =0.50; (c) μ =0.70

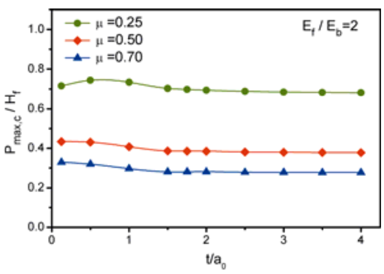

Fig.7 Relationship between the normalized critical maximum contact pressure for yielding at the surface $P_{max,c}/H_f$ and t/a_0

Moreover, Fig.5 and Fig.7 can be used for definition of the critical maximum contact pressure for yielding at the interface and the surface. According to the two figures and Eq. (6), we can control the occurrence of the yield at the interface and the surface. At the same time, we believe that if we are able to control the yield occurrence at the interface or surface, then the crack initiation there should be prevented.

3 Conclusions

1) In order to obtain a higher value of the critical maximum contact pressure for yielding at the interface, increasing the substrate hardness or t/a_0 is more effective than μ when t/a_0 is larger than 0.5. When t/a_0 is very small, we increase substrate hardness for effective prevention of the yield at the interface.

2) For the sake of obtaining a higher value of the critical maximum contact pressure to prevent yielding at the surface, choosing a relatively lower friction system or increasing the yield strength ratio will be a more effective way.

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