Improved hydrodynamic performance of liquid film seal by considering boundary slip and cavitation

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Abstract

Purpose – The purpose of this paper is to investigate the effect of boundary slip on hydrodynamic performance of liquid film seal considering cavitation.

Design/methodology/approach – A mathematical model of liquid film seal with slip surface was established based on the Navier slip model and Jakobsson–Floberg–Olsson (JFO) boundary condition. Liquid film governing equation was discretized by the finite difference method and solved by the SOR relaxation iterative algorithm and the hydrodynamic performance parameters of liquid film seal were obtained considering boundary slip and cavitation.

Findings – The results indicate that the values of performance parameters are affected significantly by the slip length under the condition of high speed and low differential pressure.

Originality/value – The performances of liquid film seal are investigated considering slip surface and cavitation. The results presented in the study are expected to provide a theoretical basis to improve the design method of liquid film seal.

Keywords Liquid film seal, Hydrodynamic performance, JFO boundary, Navier slip model

Paper type Research paper

1. Introduction

Mechanical face seals have found wide applications in industrial engineering, especially in petrochemical industry (Qiu and Khonsari, 2011a; Zhao et al., 2014). To improve the lubrication condition of surface, some types of grooves or dimples, which called surface texturing, are machined on the surface of the rotating or stationary ring (Etsion, 2005; Gachot et al., 2017; Rosenkrantz et al., 2019). There are various machining methods available for fabricating surface textures, including laser surface texturing (LST), pulsed air arc treatment, electro-polishing (Ta et al., 2015), and a novel approach of high-speed scratching (Zhang et al., 2015; Wang et al., 2018), etc.

The most typical type of groove is logarithmic spiral groove due to its effective hydrodynamic effect. Spiral groove liquid film seal (liquid film seal for short) mainly used in high-speed turbo-compressors (Gabriel, 1994; Wang et al., 2004; Hao et al., 2018a). The performances of liquid film seal are significantly affected by boundary conditions at the interfaces between the liquid and the solid surfaces. With the development of technology, the running conditions of sealing system become more harsh, which bring lubrication failure to liquid film seal. To tackle the problem, one potential attractive technique is by introducing a surface with boundary slip. Boundary slip states there is relative fluid-solid velocity at the fluid-solid interface (Granick et al., 2003; Wu et al., 2008) and the lubricant film easily appears boundary slip phenomenon on surface that used coated hydrophobic layer. Boundary slip will lead to the change of the velocity distribution and the characteristic parameters will also change.

Thus far, some researchers have paid attention to the utilization of the slip phenomenon in practical applications. Huang et al. (1999) pointed out that the no-slip boundary condition was no longer suitable to the visco-plastic fluid lubrication. Spikes (2003) analyzed the influence of boundary slip on sliding surface of stationary slider under light load condition based on finite difference method. Rao (2010) proposed a universal Reynolds equation based on Navier slip model and derived the pressure, shear stress with partial slip on the stationary surface. Aurelian et al. (2011) analyzed the load-carrying capacity and power loss of hydrodynamic fluid bearing...
and pointed out that well-chosen slip/no-slip surface pattern can considerably improve the bearing behavior. Zhao et al. (2019) proposed a generalized Reynolds equation considering boundary and heat transfer and investigated the coupled effects of boundary slip and heat on EHL contacts under large slide–roll ratio conditions. Song et al. (2017) showed that heterogeneous slip/no-slip surface enhances load-carrying capacity of tilting pad thrust bearing. The main flow of liquid film seal and dynamic pressure bearing is micro gap shear flow and the boundary slip in dynamic pressure bearing has reference significance for liquid film seal (Zhao et al., 2018). However, little attention has been paid to the effects of boundary slip on liquid film seal.

Over the past decades, cavitation phenomenon in the liquid film seal has attracted researchers’ attention. Etsion and Pascovici (1996, 1997) developed analytical models for two-phase mechanical face seal. Wang et al. (2014) investigated two-phase mechanical face seals with LST using a test rig and cavitation occurred in some of the dimples and annular vaporization regions attached to the dimples were observed. Dowson and Taylor (1979) summarized four kinds of cavitation boundary conditions, namely: Sommerfeld, half-Sommerfeld, Reynolds and Jakobsson–Floberg–Olsson (JFO) boundary condition. Qiu and Khonsari (2011b) analyzed thrust bearings with a dimpled surface texture-based on JFO boundary condition. Hao et al. (2018b) investigated the effect of cavitation with the means of both experiment and theoretical analysis. Lin et al. (2019) studied the evolution of cavitation bubbles of the water-lubricated spiral groove bearing under high speeds with the means of both experiment and theoretical analysis.

So far, some studies have mentioned the effect of boundary slip or cavitation on liquid film seal, respectively. However, few investigations analyze the influence of boundary slip with cavitation on liquid film seal. In this study, a numerical model of liquid film seal based on Navier slip model and JFO boundary is established. The numerical model is used to investigate the effect of boundary slip on the hydrodynamic performance. To improve the load-carrying capacity, the optimized slip length is obtained.

2. Theoretical model

2.1 Geometric model

The structure of sealing face is shown in Figure 1(a). In the operating condition, the liquid from the inner groove radius ports moves toward the groove outer radius by the rotation of the seal rings until it is impeded at the edges of the land and the dam regions and elevated pressure is generated during the process (Lee and Kim, 2011). The rotating and stationary surfaces are separated when the opening force formed by the liquid film pressure exceeds or equals to the closing force. Between the seal rings [Figure 1(b)], the groove and land produce hydrodynamic pressure while the seal dam produces hydrostatic pressure (Basu, 1992).

![Figure 1 Structural diagram of spiral groove liquid film seal](image)

**Notes:** (a) Structure of sealing face; (b) friction pairs

Where \( \theta_L \) and \( \theta_G \) represent the circumferential angles of the land and groove and \( \theta_s \) is the angle of single-period computational domain. \( r_i \) and \( r_o \) represent the inner and outer radii. \( p_i \) and \( p_o \) are the pressures at the inner radius and outer radius. \( \alpha \) is the spiral angle. \( h_g \) represents groove depth and \( h_o \) represents the film thickness on land. \( \omega \) represents the rotating angular velocity of rotating ring.

2.2 Modified Reynolds equation

The film thickness is small relative to other dimensions of length, to simplify the followed numerical analysis, basic assumptions for the model are made as follows (Lebeck, 1991):

- the lubricant is Newtonian and the flow is laminar;
- the film composed of a biphasic mixture is divided into full liquid film zones and cavitation zones and the pressure in the cavitation zones remains constant;
- thermal wedge of the fluid and thermal distortion of the friction pair are neglected;
- the angular misalignment of the friction pair and the fluid inertial are neglected; and
- the model is suitable for the isothermal and adiabatic conditions.

Based on the assumptions, the Navier–Stokes equations are simplified as follows:

\[
\begin{align*}
\frac{\partial p}{\partial x} &= \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right) \\
\frac{\partial p}{\partial y} &= \frac{\partial}{\partial z} \left( \mu \frac{\partial v}{\partial z} \right) \\
\frac{\partial p}{\partial z} &= 0
\end{align*}
\]

(1)

The Navier slip model is shown schematically in Figure 2. The velocity of the liquid at the wall-related to the slip length can be expressed as follows (Navier, 1823):

\[
u_{slip} = \frac{b}{\mu} \left. \frac{\partial u}{\partial z} \right|_{wall}
\]

(2)

where \( b \) is slip length, \( \mu \), m, \( b \) is defined as the ratio of the slip velocity to the absolute value of the velocity gradient in the normal direction of the wall; \( u_{slip} \) is the fluid velocity on the wall. The choice of Navier slip model is not only for its
Figure 2 Schematic diagram of boundary slip

simplicity but also it is the most common slip model that shows good correlation to experimental results (Neto et al., 2005).

Based on Navier slip model, the surface boundary condition are as follows:

\[
\begin{align*}
  u(z = 0) &= b \frac{\partial u}{\partial z} |_{z=0} \\
  u(z = h) &= U - b \frac{\partial u}{\partial z} |_{z=h} \\
  \nu(z = 0) &= b \frac{\partial \nu}{\partial z} |_{z=0} \\
  \nu(z = h) &= -\nu \frac{\partial v}{\partial z} |_{z=h}
\end{align*}
\]  

Equation (1) can be readily integrated twice with respect to \( z \) as \( p \) is independent of \( z \) and substituting equation (3) into equation (1). Thus, the corresponding velocity equations can be expressed as follows:

\[
\begin{align*}
  u &= \frac{1}{2 \mu} \frac{\partial p}{\partial x} (x^2 - hz - bh) + U \frac{b + z}{h + 2b} \\
  \nu &= \frac{1}{2 \mu} \frac{\partial p}{\partial y} (x^2 - hz - bh)
\end{align*}
\]  

Combining continuity equation as follows:

\[
\frac{\partial p u}{\partial x} + \frac{\partial p v}{\partial y} + \frac{\partial p w}{\partial z} = 0
\]  

A modified Reynolds equation with boundary slip is obtained as following expression:

\[
\frac{\partial}{\partial x} \left( \frac{h^3 + 6bh^2}{12\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{h^3 + 6bh^2}{12\mu} \frac{\partial p}{\partial y} \right) = \frac{\partial}{\partial\theta} \left( \frac{U \partial h}{2} \right)
\]  

The equation can be written as follows in polar coordinate:

\[
\frac{1}{r} \frac{\partial}{\partial \theta} \left[ \frac{h^3 + 6bh^2}{12\mu} \frac{\partial p}{\partial \theta} \right] + \frac{\partial}{\partial r} \left[ \frac{r(h^3 + 6bh^2)}{12\mu} \frac{\partial p}{\partial r} \right] = \frac{\omega r \partial h}{2} 
\]  

Because of the groove structure, boundaries and the operating conditions, in the region where local pressure yielded by equation (7) may fall below the gas saturation pressure or evaporation pressure of lubricant at the operating temperature, resulting in the liquid film rupture and cavitation phenomena (Heshmat, 1991). \( \rho \) and \( \rho_c \) are the local and cavitation density of the fluid.

To automatically capture the locations of the liquid film rupture and reformation, a switch function \( g \) and a universal variable \( \phi \) are introduced to distinguish the cavitation zones from the computational domain and obtain the pressure or density in different zones (Elrod, 1981). The \( g \) and \( \phi \) are defined as follows:

\[
\phi = \frac{\rho}{\rho_c}
\]

\[
\begin{align*}
  g &= 1, \phi \geq 1, \text{(full film region)} \\
  g &= 0, \phi < 1, \text{(cavitation region)}
\end{align*}
\]  

The pressure of liquid film can be written as follows:

\[
p = p_c + g \beta \ln \phi
\]

where \( \beta \) is the bulk modulus of fluid, which is defined as \( \beta = \rho \frac{\partial \rho}{\partial p} \).

Through using the equations (8)-(10), the modified Reynolds equation considering boundary slip and cavitation can be written as follows:

\[
\frac{1}{r} \frac{\partial}{\partial \theta} \left[ g \beta \frac{h^3 + 6bh^2}{12\mu} \frac{\partial \phi}{\partial \theta} \right] + \frac{\partial}{\partial r} \left[ \frac{g \beta r(h^3 + 6bh^2)}{12\mu} \frac{\partial \phi}{\partial r} \right] = \frac{\omega r \partial h}{2}
\]  

3. Solution procedure

3.1 Discretization of the modified reynolds equation

The finite difference method is used to solve the modified Reynolds equation [equation (13)]. Figure 3 shows grid for the computation, \( p \) is the center node, while \( w, e, s \) and \( n \) denote the position relative \( p \). The parameter \( \Delta r \) is grid size in radial direction, while \( \Delta \theta \) is grid size in circumferential direction, as shown in Figure 3.

The discretized form is given by:

Figure 3 Grid for the computation
Further consolidation of equation (12), the algebraic iterative formulation of variable $\phi$ in the computational region is written as follows:

$$A_p \phi_p + A_w \phi_w + A_e \phi_e + A_n \phi_n + A_s \phi_s + F = 0$$  \hspace{1cm} (13)

where:

$$A_p = -\frac{1}{(\Delta r)^2} (a_n + a_s) g_p - \frac{1}{r_p (\Delta \theta)^2} (a_w + a_e) g_p - \frac{2}{2} \left( \frac{g_e + 2g_p + g_w}{2} \right) h_p$$  

$$A_w = \frac{1}{r_p (\Delta \theta)^2} a_w g_w + \frac{\lambda}{2(\Delta \theta)^2} \left( \frac{g_e + g_p}{2} \right) h_w,$$

$$A_e = \frac{1}{r_p (\Delta \theta)^2} a_e g_e - \frac{\lambda}{2(\Delta \theta)^2} \left( \frac{g_e + g_p}{2} \right) h_e,$$

$$A_n = \frac{1}{(\Delta r)^2} a_n g_n,$$

$$A_s = \frac{1}{(\Delta r)^2} a_s g_s,$$

$$\lambda = \frac{6 \mu \omega r}{2},$$

$$a_w = \frac{h_e + h_w}{2},$$

$$a_e = \frac{h_e + h_p}{2},$$

$$a_n = \frac{h_n + h_p}{2},$$

$$a_s = \frac{h_s + h_p}{2},$$

For the lubrication problem, iterative solvers are more attractive compared with direct solvers. Several numerical methods are available to solve the equation (13). In this study, the successive over relaxation scheme is adopted. For the above iterative solution, the convergence criterion is:

$$\varepsilon = \sum_{i=1}^{N} \sum_{j=1}^{M} \left| \frac{\phi_{new}^{ij} - \phi_{old}^{ij}}{\phi_{new}^{ij}} \right| < 10^{-6} \hspace{1cm} (15)$$

### 3.2 Performance parameters

Once the content parameter $\phi$ is obtained from the modified Reynolds equation in the whole computational domain. The hydrodynamic performances can be calculated by the following equations.

The load-carrying capacity $W$ is obtained by integrating pressure over the computational domain:

$$W = \int_{0}^{2\pi} \int_{r}^{r_{u}} \rho r \, dr \, d\theta$$  \hspace{1cm} (16)

**Friction torque:**

$$T = \int_{0}^{2\pi} \int_{r}^{r_{u}} \tau r^2 \, dr \, d\theta$$  \hspace{1cm} (17)

where

$$\tau = \frac{\mu \omega r}{h} - \frac{h_{w}}{2r} \frac{\partial \rho}{\partial \theta}, \quad (\text{full film region})$$

$$\tau = \frac{\phi \mu \omega r}{h}, \quad (\text{cavitation region})$$

**Radial leakage:**

$$Q = \int_{0}^{2\pi} \frac{(h^3 + 6b h^2) r \partial \rho}{12 \mu} \, dr \, d\theta$$  \hspace{1cm} (18)

**Cavitation ratio:**

$$\eta = 1 - \sum_{i=1}^{N_{\text{max}}} \sum_{j=1}^{M_{\text{max}}} \frac{g}{g_{i,j}}$$  \hspace{1cm} (19)

The flow chart of calculation procedure is shown in Figure 4.
4. Results and discussion

Table I presents the geometric and operating parameters used in this study.

4.1 Influence of slip length

Figure 5 illustrates how the hydrodynamic performances are influenced by the slip length. As shown in Figure 5(a), the load-carrying capacity increases and then decreases with the increase of slip length. Load-carrying capacity can get a maximum value at \( b = 1.0 \) \( \mu \)m and the load-carrying capacity raises by about 29.01 per cent compared with \( b = 0 \) \( \mu \)m. The cavitation ratio becomes nil when the slip length is greater than \( b = 4.5 \) \( \mu \)m. The existence of boundary slip has obvious effect on restraining cavitation and changing the load-carrying capacity. In relation to cavitation, the greater the slip length is, the smaller the cavitation ratio becomes. When the slip length \( b < 1.0 \) \( \mu \)m, the existence of boundary slip decreases the cavitation ratio significantly, so that the load-carrying capacity increases. However, cavitation ratio is small when the slip length \( b > 1.0 \) \( \mu \)m and the load-carrying capacity decreases with the increase of slip length.

As shown in Figure 5(b), with the increase of slip length, leakage increases approximately linearly, while friction torque decreases at first and then increases. Friction torque can get a minimum value at slip length \( b = 1.0 \) \( \mu \)m and the friction torque decreases by about 0.35 per cent compared with \( b = 0 \) \( \mu \)m. In general, the slip length has no obvious effect on the friction torque, for example, friction torque with \( b = 10 \) \( \mu \)m increases about 0.92 per cent compared to \( b = 0 \) \( \mu \)m.

4.2 Influence of rotating speed

Figure 6(a) shows the effect of rotating speed on load-carrying capacity with different slip lengths. The load-carrying capacity increases and then decreases slightly with the increase of rotating speed under no-slip condition (\( b = 0 \) \( \mu \)m). When slip length \( b = 0 \) \( \mu \)m, the cavitation ratio is significantly affected by the rotating speed, which leads to the decrease of the load-carrying capacity of liquid film with the increase of rotating speed. However, when boundary slip occur (\( b \neq 0 \) \( \mu \)m), the cavitation ratio decreases significantly and the load-carrying capacity increases with the increase of rotating speed. The slip length increases to a certain value, for example, when the value is \( b = 10 \) \( \mu \)m, the cavitation eventually disappears and the load-carrying capacity increases with the increase of rotating speed, but the increase is limited. The cavitation ratio increases and then decreases to some extent with the increase of rotating speed and cavitation ratio has similar trend with the increase of rotating speed for different slip lengths. The reason is that the hydrodynamic effect is stronger with higher rotating speed, so that the negative pressure effect at the boundary of the groove is stronger and reach the cavitation pressure, which leads to occurrence of cavitation.

Combined Figure 6(a) with Figure 6(b), the results indicate that the cavitation ratio decreases with the increase of slip length. As shown in Figure 6(b), the maximum pressure of liquid film increases then decreases with the increase of slip length. With the increase of slip length, the position of film rupture is not affected, but the position of film reformation boundary is closer to the position of film rupture while the cavitation ratio decreases and the results are consistent with Figure 6. When the slip length increases to a certain extent, the minimum pressure is greater than the cavitation pressure, the

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**Table I**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>inner radius, ( r_i/\text{mm} )</td>
<td>45.75</td>
</tr>
<tr>
<td>outer radius, ( r_o/\text{mm} )</td>
<td>54.25</td>
</tr>
<tr>
<td>spiral groove inner radius, ( r_{gi}/\text{mm} )</td>
<td>47.75</td>
</tr>
<tr>
<td>spiral groove outer radius, ( r_{go}/\text{mm} )</td>
<td>52.75</td>
</tr>
<tr>
<td>spiral groove depth, ( h_d/\mu\text{m} )</td>
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<tr>
<td>film thickness, ( h_f/\mu\text{m} )</td>
<td>12</td>
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<tr>
<td>spiral groove angle, ( \alpha/^{\circ} )</td>
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<tr>
<td>spiral groove number, ( N_g )</td>
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<tr>
<td>lubricant temperature/°C</td>
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<tr>
<td>lubricant viscosity, ( \mu/\text{Pa·s} )</td>
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</tr>
<tr>
<td>lubricant density, ( \rho/\text{kg·m}^{-3} )</td>
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</tr>
<tr>
<td>rotating speed, ( \omega/\text{r/min} )</td>
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<tr>
<td>inner pressure, ( p_i/\text{MPa} )</td>
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<tr>
<td>outer pressure, ( p_o/\text{MPa} )</td>
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</tr>
<tr>
<td>cavitation pressure, ( p_c/\text{MPa} )</td>
<td>0.03</td>
</tr>
</tbody>
</table>
cavitation eventually disappears at the end face of the liquid film seal.

To study the effect of rotating speed on leakage and friction torque, plots for different values of slip length under different rotating speeds are presented in Figure 6(c). It can be seen that the leakage increases linearly with the increase of rotating speed when boundary slip occurs, and $Q_s$ (leakage caused by static pressure) is affected slightly by rotating speed. The reason is that the increase of rotating speed leads to the increase of hydrodynamic effect, which results in the increase of pumping capacity, so that leakage increases. Leakage has similar trend with the increase of rotating speed under different slip lengths conditions. Besides, the results show that the leakage increases with the increase of slip length at the same rotating speed. The figure shows that the friction torque increases linearly with the increase of rotating speed and the friction torques under different slip lengths have the same change rule and the overall trends are consistent.

4.3 Influence of differential pressure

Figure 7(a) illustrates how the load-carrying capacity and cavitation ratio are influenced by differential pressure. As can...
Figure 7 Influence of differential pressure on hydrodynamic performance ($\omega = 1,500$ rpm)

Notes: (a) Load-carrying capacity and cavitation ratio; (b) leakage and friction torque

be seen, the bigger the differential pressure is, the stronger the load-carrying capacity is and the load-carrying capacity under different slip lengths has the similar change rule. The reason is that the greater differential pressure lift the static pressure, so that the load-carrying capacity can be improved. Besides, the influence of slip length on load-carrying capacity is more significantly when differential pressure is lower than 0.5 MPa. The results indicate that the cavitation is serious when $\Delta p < 0.5$ MPa and the existence of boundary slip suppresses cavitation and improves the load-carrying capacity to some extent. On the contrary, when $\Delta p > 0.5$ MPa, the slip length has little effect on the load-carrying capacity with the same differential pressure, so that the load-carrying capacity increases linearly with the increase of differential pressure.

Figure 7(b) shows the effects of differential pressure on leakage and friction torque. With the increase of differential pressure, the leakage increases approximately linearly and the variation law of the leakage is similar with different slip lengths. In addition $Q_d$ (leakage caused by dynamic pressure) is affected slightly by differential pressure. The reason is that the leakage is affected by the radial pressure gradient and the increase of differential pressure result in the increase of the radial pressure gradient, which leads to the increase of leakage. The friction torque increases with the increase of differential pressure, while the change of amplitude is very small, which can be neglected. For example, when the slip length $b = 10 \mu m$, friction torque with $\Delta p = 1.1$ MPa only increases by about 0.23 per cent compared to $\Delta p = 0.1$ MPa.

5. Conclusions

In this paper, the influence of slip surface on the hydrodynamic performances of liquid film seal are investigated and both the slip length and cavitation are taken account. The main conclusions are as follows:

- the paper proposes a mathematical model of liquid film seal considering boundary slip and cavitation. The hydrodynamic performance parameters take on a similar trend as the slip length increase under different working conditions;
- the influence of slip length on the hydrodynamic performance parameters is more significantly under the condition of high speed and low differential pressure; and
- the existence of boundary slip reduces the cavitation ratio, which can improve the load-carrying capacity of liquid film seal. Under the calculation conditions, the optimum slip length is $b = 1.0 \mu m$, which increases the load-carrying capacity of liquid membrane by 29.01 per cent.

References


Improved hydrodynamic performance

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