

An Optimization Approach of Compressive Sensing Recovery Using Split Quadratic Bregman Iteration with Smoothed ℓ_0 Norm

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Abstract—An optimization algorithm for image recovery is a core issue in the field of compressive sensing (CS). This paper deeply studied the CS reconstruction algorithm based on split Bregman iteration with ℓ_1 norm, which enables the ℓ_1 norm to approximate the original ℓ_0 norm during the optimization process. Consequently, we proposed another novel algorithm improving the precision and the convergence speed based on split quadratic Bregman iteration (SQBI) with ℓ_0 norm. Besides, we analyzed its convergence by proving two monotonically decreasing theorems. Inspired by previous researches, we applied smoothed ℓ_0 norm for the optimization problem to replace the traditional ℓ_0 norm in CS. The improvement is made by using a Gaussian function to approximate the ℓ_0 norm, transforming it into a convex optimization problem, and eventually achieved a convergent solution by the steepest descent method. The experimental results show that under the same conditions, compared with other state-of-the-art algorithms, the reconstruction accuracy of the CS reconstruction algorithm based on the SQBI with smoothed ℓ_0 norm is improved significantly, and its convergence rate is also accelerated as well.

Keywords—Compressive sensing, sparse representation, image recovery, split quadratic Bregman iteration, convex optimization

I. INTRODUCTION

Compressive sensing theory overcomes the drawback of traditional Shannon sampling theory such as large sample size, sampling time and data storage space wasted a lot of resources. Therefore, it has important theoretical value and widely future application [1]. Image reconstruction is an significant part of compressive sensing theory, and its key issue is how to recover the original high-dimensional data from compressed low-dimensional data as much as possible. According to the theory of compressive sensing, the sparsity of images plays a crucial role in image reconstruction: signals with higher sparsity can better be recovered. However, natural images tend to have lower sparseness in the pixel domain. Therefore, the main approach of natural image reconstruction is to find a suitable transform domain to sparsely represent it, and then implement inverse image transformation to achieve high-probability image recovery [2]. The sparsity of an image can reflect the essential characteristics of the image, and it is also an important prior knowledge for obtaining the original image in the process of compressive sensing reconstruction. In fact, sparsity is a very important basic theory in the field of image processing and computer vision, for example, approximation, estimation, compression, and dimensionality reduction are all

having a good performance based on sparsity [3]. In order to further improve sparsity, algorithms such as redundant dictionary, multi-dictionary, global sparse representations, and local sparse representation have been proposed by researchers [4,5]. Although these methods have kinds of limitations and need to be improved in theoretical aspects, they also made contributions to the development of compressed sensing theory.

At present, algorithms for compressed sensing reconstruction are mainly divided into two research directions based on making the ℓ_0 norm minimum or the ℓ_1 norm minimum. The minimum ℓ_0 norm based reconstruction algorithm does not require too much calculation and has a good performance. However, it is a non-convex NP-hard problem and cannot be solved directly. The general method to solving this problem is using the ℓ_1 norm gotten from the convex optimization solution instead of the ℓ_0 norm [1-4]. Compressive sensing theory shows that the ℓ_0 norm based optimization problem and the ℓ_1 norm based optimization problem are equivalent under certain conditions [5]. However, these conditions are not necessarily met in practice. Therefore, this paper focuses on the sparse reconstruction based on the minimum ℓ_0 norm, and the algorithms of compressive sensing recovery using the ℓ_0 norm commonly include Gradient Pursuit (GP), Subspace Pursuit (SP), Orthogonal Matching Pursuit (OMP), and Iteratively Reweighted Least Squares (IRLS) [6]. In order to overcome the defects of the ℓ_0 norm based compressive sensing reconstruction algorithm, we proposed an approach based on the split quadratic Bregman iteration (SQBI) which use the minimum smoothed ℓ_0 norm. The main idea is to divide the original optimization model into two sub-models (sub-model x and sub-model α) and to optimize them respectively to obtain a better approximation. A large number of experiments show that the split quadratic Bregman iterative algorithm optimized by the smoothed ℓ_0 norm has better reconstruction accuracy and higher convergence speed.

The main organization of this paper is as follows: Section II describes the split quadratic Bregman iteration algorithm, Section III gives an application of split quadratic Bregman iterative algorithm to image reconstruction in Compressive Sensing, Section IV gives the experimental methods and the complete algorithm flow chart, Section V depicts the experiments results and analysis of image sparse representation, Section VI is the conclusion.

II. SPLIT QUADRATIC BREGMAN ITERATION

A. Bregman iteration

For the convenience of the following illustration, we first give a brief review of Bregman algorithm. Considering a constrained minimization problem:

$$\min_u J(u), \quad \text{s.t.} \quad H(u) = 0$$

$$H(u) = \frac{1}{2} \|Qu - q\|_2^2 \quad (1)$$

where $J(u)$ is convex functions with u belonging to $R^{n \times 1}$, and $H(u) = 0.5 \|Qu - q\|_2^2$ is a quadratic function with Q belonging to $R^{m \times n}$, q belonging to $R^{m \times 1}$, and then the problem can be solved by Bregman iteration [7] as follows:

$$u_{k+1} = \min_u D_J^{p_k}(u, u_k) + \lambda H(u)$$

$$p_{k+1} = p_k - \nabla H(u_{k+1}) \quad (2)$$

where λ is a positive number, p_k is the subgradient of J at u_k , and

$$D_J^{p_k}(u, u_k) = J(u) - J(u_k) - \langle p_k, u - u_k \rangle$$

$$= J(u) - f_{u_k}(u) \quad (3)$$

represents Bregman distance [8]. When u is a scalar, Bregman distance be shown in Fig. 1. The convergence of Bregman iteration has been amply analyzed in [7].

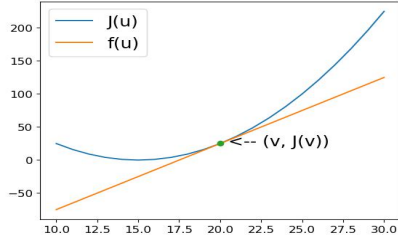


Fig.1 Bregman Distance

B. Quadratic Bregman iteration

It is obvious that Bregman iteration only works when $J(u)$ is convex, but it does not apply when $J(u)$ is non-convex. Motivated by it, we proposed a novel quadratic Bregman iteration (QBI) to approach the latter problem.

We begin with defining “quadratic Bregman distance”. The quadratic Bregman distance related with a non-convex and twice differentiable function J at the point v is:

$$D_{J,2}(u, v) = J(u) - f_{J,v,2}(u) \quad (4)$$

where $f_{J,v,2}$ is a quadratic function defined as:

$$f_{J,v,2}(u) = a_{J,v} u^T u + b_{J,v}^T u + c_{J,v} \quad (5)$$

the coefficients $a_{J,v} \in R$, $b_{J,v} \in R^{n \times 1}$ and $c_{J,v} \in R$ are uniquely decided by function J and the point v . For simplicity, during following discussion we will omit the subscript J and v of them without confusion.

1) $D_{J,2}(u, v)$ is convex for variable u ;

2) $D_{J,2}(v, v)$ is the minimum of quadratic Bregman distance;

3) The minimum of $D_{J,2}(u, v)$ is 0.

then we can get that:

- 1) $\nabla_u^2 D_{J,2}(u, v) = \nabla^2 J(u) - 2aI$ is positive semidefinite matrix;
- 2) $\nabla_u D_{J,2}(v, v) = \nabla J(v) - 2av - b = 0$;
- 3) $D_{J,2}(v, v) = J(v) - av^T v - b^T v - c = 0$.

Here, in order to obtain the value of coefficients a , b and c , quadratic Bregman distance is required to satisfy the following criterion (*):

So we can derive the value of coefficients a , b and c by:

- 1) a is the minimum value to ensure that $\nabla^2 J(u) - 2aI$ is positive semidefinite matrix (we can notice that $a < 0$, and for the same function J we can fix a unchanged a);
- 2) $b = \nabla J(v) - 2av$;
- 3) $c = J(v) - av^T v - b^T v$.

When u is a scalar, quadratic Bregman distance can be shown in Fig. 2.

Similarly, when function J is convex and differentiable, we can also rewrite Bregman distance as another form which also satisfies the criterion (*):

$$D_J^{p_k}(u, u_k) = D_{J,1}(u, u_k) = J(u) - f_{J,u_k,1}(u) \quad (6)$$

Where $f_{J,u_k,1}$ is a linear function decided by function J and the point u_k . Then, simulating Bregman iteration, we can derive QBI shown in Eq.(7) and Fig.2:

$$u_{k+1} = \min_u D_{J,2}(u, u_k) + \lambda H(u)$$

$$b_{k+1} = \nabla J(u_{k+1}) - 2au_{k+1} \quad (7)$$

$$c_{k+1} = J(u_{k+1}) - au_{k+1}^T u_{k+1} - b_{k+1}^T u_{k+1}$$

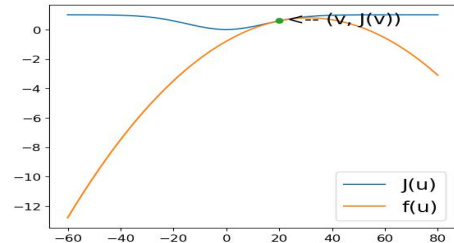


Fig.2 Quadratic Bregman Distance

C. Convergence analysis

Motivated by [9], we now study the convergence properties of the iteration. We show two monotonically properties of the constrained function and the quadratic Bregman distance.

THEOREM 1. Under the above assumptions, the sequence $H(u_k)$ obtained from QBI is monotonically nonincreasing:

$$H(u_{k+1}) \leq H(u_k) \quad (8)$$

Proof. Let $g_k(u) = D_{J,2}(u, u_k) + \lambda H(u)$. Then, from QBI we can notice that u_{k+1} is the minimizer of the convex function g_k , we can get that:

$$\begin{aligned} \lambda H(u_{k+1}) &\leq g_k(u_{k+1}) \leq g_k(u_k) \\ &= D_{J,2}(u_k, u_k) + \lambda H(u_k) = \lambda H(u_k) \end{aligned} \quad (9)$$

which implies theorem 1.

THEOREM 2. Under the same assumptions as THEOREM 1, let \hat{u} be the minimizer of the function H , then we can derive another monotonically nonincreasing property that:

$$D_{J,2}(\hat{u}, u_{k+1}) \leq D_{J,2}(\hat{u}, u_k) \quad (10)$$

Proof. According to the proof of THEOREM 1, we can derive that:

$$\begin{aligned} &\nabla_u g_k(u_{k+1}) \\ &= \nabla_u D_{J,2}(u_{k+1}, u_k) + \lambda \nabla H(u_{k+1}) \\ &= \nabla J(u_{k+1}) - 2au_{k+1} - b_k + \lambda \nabla H(u_{k+1}) \\ &= b_{k+1} - b_k + \lambda \nabla H(u_{k+1}) \\ &= 0 \end{aligned} \quad (11)$$

then we get that:

$$b_{k+1} = b_k - \lambda \nabla H(u_{k+1}) \quad (12)$$

then we can derive a further formula motivated by the identity of Bregman distance [10]:

$$\begin{aligned} &D_{J,2}(\hat{u}, u_{k+1}) + D_{J,2}(u_{k+1}, u_k) - D_{J,2}(\hat{u}, u_k) \\ &= (J(\hat{u}) - (a\hat{u}^T \hat{u} + b_{k+1} \hat{u} + c_{k+1})) + \lambda H(\hat{u}) \\ &+ (J(u_{k+1}) - (au_{k+1}^T u_{k+1} + b_k u_{k+1} + c_k)) + \lambda H(u_{k+1}) \\ &- (J(\hat{u}) - (a\hat{u}^T \hat{u} + b_k \hat{u} + c_k)) + \lambda H(\hat{u}) \\ &= (b_k - b_{k+1})(\hat{u} - u_{k+1}) \\ &= \lambda \nabla H(u_{k+1})^T (\hat{u} - u_{k+1}) \\ &\leq \lambda (H(\hat{u}) - H(u_{k+1})) \end{aligned} \quad (13)$$

then we get that:

$$\begin{aligned} &D_{J,2}(\hat{u}, u_{k+1}) \\ &\leq D_{J,2}(\hat{u}, u_{k+1}) + D_{J,2}(u_{k+1}, u_k) \\ &\leq D_{J,2}(\hat{u}, u_{k+1}) + D_{J,2}(u_{k+1}, u_k) + \lambda (H(u_{k+1}) - H(\hat{u})) \\ &\leq D_{J,2}(\hat{u}, u_k) \end{aligned} \quad (14)$$

which implies theorem 2.

These two theorems result to a general convergence conclusion. We can also see the convergence result with experiment in Section 4.

Based on the proof, we can see that since the coefficient c is a constant for every step of each iteration, we actually do not need to calculate its value. Besides, based on the coefficient b we derived above, we can rewrite QBI as the following form:

$$\begin{aligned} u_{k+1} &= \min_u D_{J,2}(u, u_k) + \lambda H(u) \\ b_{k+1} &= b_k - \lambda \nabla H(u_{k+1}) \end{aligned} \quad (15)$$

Noticing that $H(u) = \frac{1}{2} \|Qu - q\|_2^2$, according to [9], this complicated iteration is trickily equivalent to a more simplified form:

$$\begin{aligned} u_k &= \min_u J(u) + au^T u + \frac{\lambda}{2} \|Qu - q_k\|_2^2 \\ q_{k+1} &= q_k + q - Qu_k \end{aligned} \quad (16)$$

D. Split quadratic Bregman iteration

Based on the above illustrations and motivated by [7], we can briefly derive Split Quadratic Bregman iteration (SQBI) algorithm. Considering the optimized model with following constrained conditions:

$$\min_{q \in R^{N \times 1}, u \in R^{K \times 1}} g_1(q) + g_2(u), \quad s.t. \quad Qu = q \quad (17)$$

where $Q \in R^{N \times K}$, $g_1: R^{N \times 1} \rightarrow R$ is a convex function, and

$g_2: R^{K \times 1} \rightarrow R$ is non-convex, then SQBI procedure is

formulated as follows:

1) Set initial values:

$$n = 0, d_0 = 0, u_0 = 0, q_0 = 0, \eta > 0, \alpha;$$

$$2) \quad q_{n+1} = \arg \min_q g_1(q) + \frac{\eta}{2} \|q - Qu_n - d_n\|_2^2;$$

$$3) \quad u_{n+1} = \arg \min_u g_2(u) + au^T u + \frac{\eta}{2} \|q_{n+1} - Qu - d_n\|_2^2;$$

$$4) \quad d_{n+1} = d_n - (q_{n+1} - Gu_{n+1});$$

$$5) \quad n \leftarrow n + 1;$$

6) Repeat, if the condition is satisfied, then finish the iteration, if not return to step 2.

Experiment results show that SQBI converges quicker and takes up less memory when it is applied to solving large-scale optimization problems.

III. RECONSTRUCTION ALGORITHM BASED ON SPLIT QUADRATIC BREGMAN ITERATION (SQBI)

A. Optimized Model

According to the compressive sensing theory, signal intrinsic properties show sparsity in some transform domains. When a projection matrix is used to map a signal to a space which has lower dimension, the signal can be reconstructed with high probability. Suppose a signal $x \in R^{N \times 1}$, projection matrix $\Phi \in R^{M \times N}$ satisfying $M \ll N$, after linear projection we have $y = \Phi x$, where $y \in R^{M \times 1}$. For x has no sparsity in pixel domain, it means that most of the elements in x are not zero or don't approximate to zero, thus we are applying sparse

decomposition to x to find a proper space making $x = \Psi \alpha$, where $\alpha \in R^{K \times 1}$, $\Psi \in R^{N \times K}$, $N \leq K$, and most of the elements in α are zero or approximate to zero. Then we assume that x is sparse in transform space Ψ and the effect of sparse representation can be measured by the number of the elements which are zero or approximate to zero. The sparsity is higher and the reconstruction performance is better.

More generally, we have a model as follows:

$$\min_{\alpha} \frac{1}{2} \|y - \Phi x\|_2^2 + \lambda \|\alpha\|_0 \quad s.t. \quad \Psi \alpha = x \quad (18)$$

where $\lambda > 0$. The purpose of reconstruction algorithm is to recover the original signal x from a few observed signals y with a certain compression rate $S = M / N$.

Noticing that it is difficult to handle ℓ_0 norm directly, motivated by [11,12], we use smoothed ℓ_0 norm to replace it. Here we use Gaussian function as a smoothed function.

Let $\|\alpha\|_0 = \sum_{i=1}^K h(\alpha_i)$, $\alpha_i \in R$ is the i th value of α , and then

we have:

$$h(\alpha_i) = \begin{cases} 0 & \alpha_i = 0 \\ 1 & \alpha_i \neq 0 \end{cases} \quad (19)$$

To obtain an approximation of this function by using Gaussian function, we have

$$h_{\xi}(\alpha_i) \approx 1 - e^{-\frac{\alpha_i^2}{2\xi^2}} \quad (20)$$

where $\xi > 0$. Thus we have:

$$\lim_{\xi \rightarrow 0} h_{\xi}(\alpha_i) = \begin{cases} 0 & \alpha_i = 0 \\ 1 & \alpha_i \neq 0 \end{cases} \quad (21)$$

Approximately, we have

$$h(\alpha_i) \approx h_{\xi}(\alpha_i) \quad (22)$$

hence

$$\|\alpha\|_0 = \sum_{i=1}^K h(\alpha_i) \approx \sum_{i=1}^K h_{\xi}(\alpha_i) \quad (23)$$

Then the reconstruction model (18) can be rewritten as:

$$\min_{\alpha} \frac{1}{2} \|y - \Phi x\|_2^2 + \eta \tilde{h}(\alpha) \quad s.t. \quad \Psi \alpha = x \quad (24)$$

where $\tilde{h}(\alpha) = \sum_{i=1}^K h_{\xi}(\alpha_i)$, η is the correction parameter.

B. Reconstruction Algorithm Based on Quadratic Split Bregman Iteration

If we want to reconstruct the original signal using Split Bregman iteration, a transformation for Eq. (18) is necessary:

$$\min_{\alpha, \hat{x}} \frac{1}{2} \|y - \Phi x\|_2^2 + \lambda \tilde{h}(\alpha), \quad s.t. \quad \hat{x} = \Psi \alpha \quad (25)$$

Let $f(x) = \frac{1}{2} \|y - \Phi x\|_2^2$, $g(\alpha) = \lambda \|\alpha\|_0$, then begin Split Bregman iteration. The 2nd step above is transformed as

$$\hat{x}_{n+1} = \arg \min_{\hat{x}} f(x) + \frac{\eta}{2} \|\hat{x} - \Psi \alpha_n - d_n\|_2^2 \quad (26)$$

The 3rd step transformed as

$$\alpha_{n+1} = \arg \min_{\alpha} g(\alpha) + a \alpha^T \alpha + \frac{\eta}{2} \|x_{n+1} - \Psi \alpha - d_n\|_2^2 \quad (27)$$

Therefore, the initial model is transformed into two sub-models by Split Bregman iteration. In the next discussion, subscript n will be neglected to make our states more simple.

C. Sub-model x

With Eq. (26), x model can be expressed as

$$\min_{\hat{x}} Q_1(\hat{x}) = \min_{\hat{x}} \frac{1}{2} \|y - \Phi x\|_2^2 + \frac{\eta}{2} \|x - \Psi \alpha - d\|_2^2 \quad (28)$$

It is a strict convex function. The derivation of it is

$$\frac{\partial Q_1(\hat{x})}{\partial \hat{x}} = (\Phi^T \Phi + \eta I)x - [\Phi^T y + \lambda(\Psi \alpha + d)] \quad (29)$$

where I is unit matrix. Eq. (29) can obtain a precise solution by a few of iterations using gradient descent method.

D. Sub-model α

With Eq. (27), α model can be represented as

$$\min_{\alpha} Q_2(\alpha) = \min_{\alpha} \frac{1}{2} \|\Psi \alpha - (x - d)\|_2^2 + \frac{\lambda}{\eta} (\tilde{h}(\alpha) + a \alpha^T \alpha) \quad (30)$$

Solving such kind of problems directly is impossible, thus we assume: $\delta = \Psi \alpha - (x - b)$, for all image blocks, δ obeys individual Gaussian distribution with average 0, variance σ^2 . This assumption is reasonable in some way according to the simulation experiments. With this assumption, we consider $x - b$ as a coarse estimation of signal x . Therefore we can transform this model into a simpler one.

$$\begin{aligned} \min_{\alpha} Q_2(\alpha) &= \min_{\alpha} \tilde{h}(\alpha) + a \alpha^T \alpha \\ s.t. \quad &\frac{1}{2} \|\Psi \alpha - (x - d)\|_2^2 \leq \varepsilon \end{aligned} \quad (31)$$

where $\varepsilon > 0$. This model can be solved by the gradient descent method effectively.

IV. COMPLETE ALGORITHM PROCEDURE

Completed algorithm procedure are as follows:

1) Let

$$n=0, x_0=0, \\ \alpha_0=0, d_0=0, \Psi_0=O,$$

be initiating every parameter;

2) Calculate x_{n+1} using gradient descent method:

$$x_{n+1} = x_n - s_1[(\Phi^T \Phi + \eta I)x_n - \Phi^T y - \lambda(\Psi_n \alpha_n + d_n)]$$

3) $r_{n+1} = u_{n+1} - d_n$

4) Calculate Ψ_{n+1} using K-SVD algorithm;

5) Calculate α_{n+1} using gradient descent method ;

6) $b_{n+1} = b_n - (x_{n+1} - \Psi_{n+1} \alpha_{n+1})$;

7) $n \leftarrow n+1$;

8) Repeat, if the condition is satisfied or the iterations run out, then finish the iteration, if not return to step 2.

Note that there adopted K-SVD method because it allows to solve all the ℓ_0 norm problem using MP or OMP.

V. EXPERIMENTAL RESULTS AND ANALYSIS

To illustrate the performance of improved algorithm, a large number of experiments have been carried out on MATLAB R2014a, the experiment results demonstrated that split quadratic Bregman iteration compressive sensing reconstruction algorithm using the minimum smoothed ℓ_0 norm is superior to the other state-of-the-art algorithms. The algorithm we proposed is compared with two representative Compressive Sensing recovery methods in literature, multi-hypothesis (MH) method [13] and collaborative sparsity (CoS) method [14], which deal with image signals in the random projection residual domain, and the hybrid space-transform domain. It is worthy to recognize that both of MH and CoS methods are known as the current state-of-the-art algorithms for image CS recovery.

To evaluate the quality of the image after reconstruction, in addition to Peak Signal to Noise Ratio (PSNR, unit: dB), which is used to evaluate the objective image quality, a recently proposed powerful perceptual quality metric Feature SIMilarity (FSIM) [15] is calculated to evaluate the visual quality. The higher FSIM value of a image always means the better visual quality.

Compare the reconstruction performance. Apply all the algorithms to test image “zelda”, “barbara” and “camera” with size 512*512*8 BMP format and the results are shown below. Visual performance shows that improved algorithm has better performance, as follows.

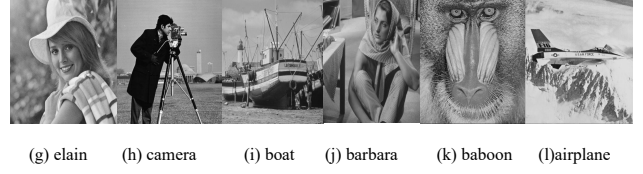
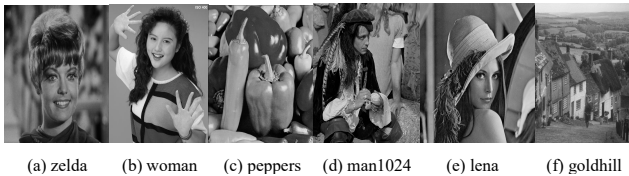


Fig.3 Test Images



Fig.4 zelda with different algorithms



Fig.5 barbara with different algorithms

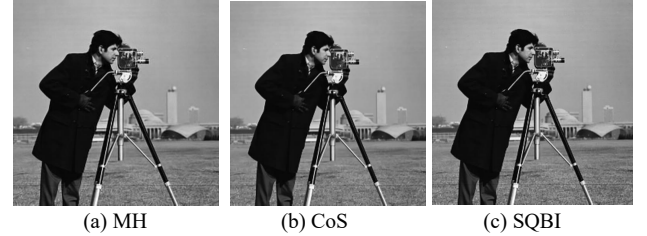


Fig.6 camera with different algorithms

TABLE I. PSNR COMPARISON AMONG THREE ALGORITHMS

PSNR/dB	zelda	woman	peppers	man1024	lena	goldhill
MH	38.1273	32.3356	34.5889	32.1322	33.3328	31.0359
CoS	37.8470	32.9203	35.2212	32.1835	33.3268	31.4048
SQBI	38.7904	33.2174	35.3380	32.8340	33.7968	31.4865

PSNR/dB	elain	camera	boat	barbara	baboon	airplane
MH	34.0252	36.8456	32.5623	34.1854	23.9912	34.0277
CoS	32.8793	36.4567	33.8180	28.7188	23.5590	34.5178
SQBI	34.5868	40.3767	34.9115	35.2767	23.7920	35.5760

TABLE II. FSIM Comparison among Three Algorithms

FSIM	zelda	woman	peppers	man1024	lena	goldhill
MH	0.9905	0.9721	0.9834	0.9917	0.9817	0.9665
CoS	0.9885	0.9797	0.9857	0.9913	0.9798	0.9678
SQBI	0.9910	0.9808	0.9862	0.9933	0.9828	0.9708

FSIM	elain	camera	boat	barbara	baboon	airplane
MH	0.9800	0.9853	0.9737	0.9841	0.9282	0.9780
CoS	0.9757	0.9838	0.9817	0.9657	0.9195	0.9781
SQBI	0.9820	0.9930	0.9861	0.9877	0.9301	0.9841

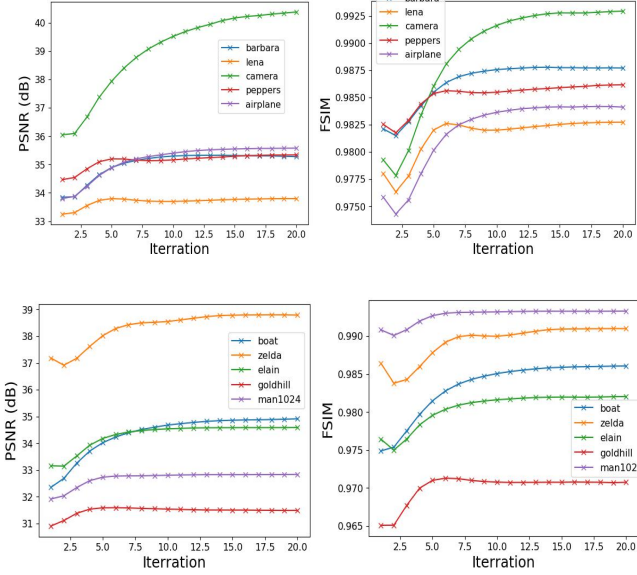


Fig.7 Evolution of PSNR and FSIM, Convergence Result

There are 11 different images with size 512*512 and 1 image with size 1024*1024, 8 bit depth, BMP format, grayscale images listed in TABLE I and TABLE II. We apply the different algorithms to all the images, and accomplish the objective evaluation using both the Peak-Signal to Noise Ratio (PSNR) and Feature Similarity (FSIM) Index for Image.

A visual comparison of test image “zelda”, “barbara” and “camera” in Fig. 3, we can see that the performance of reconstruction image algorithm proposed in this paper is superior to other state-of-the-art algorithms.

An objective comparison of test images respectively and results are shown in TABLE I and TABLE II. We can see that PSNR and FSIM are both improved. From Fig. 4-6, we can see that SQBI is superior to the CoS and MH algorithm.

Compare the convergence rate. Apply the algorithms mentioned above to all the images respectively and results are shown in Fig. 7. We can see that our algorithm has a general converges result.

The experiments demonstrated above that the split quadratic Bregman iteration with a minimum smoothed ℓ_0 norm has effectively improved the reconstruction performance and the convergent rate in compressive sensing recovery than other state-of-the-art algorithms. Simultaneously, earlier researches stated briefly that the reconstruction algorithm based on Bregman iteration is superior to algorithms such as GP, SP, OMP and IRLS. Therefore, our approach proposed in this paper is superior to a large part of the other algorithm.

VI. CONCLUSIONS

Benefited from the deep studies of the reconstruction algorithm of compressive sensing based on the split quadratic Bregman iteration, this paper proposed an approach which uses a minimum smoothed ℓ_0 norm to further solve the optimization problem for the lack of solving precision and the parameters’ setting. Moreover, the reconstruction accuracy and convergence speed are improved effectively compared with other state-of-the-art algorithms. Experiments have shown the our approach is superior to other state-of-the-art reconstruction algorithms for image recovery using the compressive sensing such as MH and CoS.

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REFERENCES

- [1] D. Donoho, “Compressed sensing,” *IEEE Trans. Inform. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.
- [2] E. Candès, J. Romberg, and T. Tao, “Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information,” *IEEE Trans. Inform. Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.
- [3] Ran He, Weishi Zheng, Tieniu Tan, Zhenan Sun, “Half-Quadratic-Based Iterative Minimization for Robust Sparse Representation,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol.36, no.2, 2014, pp.261-275.
- [4] R. Rubinstein, M. Zibulevsky, and M. Elad, “Double Sparsity: Learning Sparse Dictionaries for Sparse Signal Approximation,” *IEEE Transactions on Signal Processing*, vol. 58, no. 3, 2010, pp.1553-1564.
- [5] Yair Carmon, John C Duchi, Oliver Hinder, Aaron Sidford, “Accelerated Methods for NonConvex Optimization,” *SIAM Journal on Optimization*, vol.28, no.2, 2018, pp.1751-1772.
- [6] E. J. Candès and M. B. Wakin, “An Introduction to Compressive Sampling,” *IEEE Signal Process. Mag.*, vol.25, no. 2, Mar. 2008, pp. 21–30.
- [7] Tom Goldstein, Stanley Osher, “The Split Bregman Method for L1-Regularized Problems,” *SIAM Journal on Imaging Sciences*, 2009, 2(2):323-343.
- [8] Xinwu Liu, Lihong Huang, “Split Bregman iteration algorithm for total bounded variation regularization based image deblurring,” *Journal of Mathematical Analysis and Applications*, vol.372, no.2, 2010, pp.486-495.
- [9] Stanley Osher, Martin Burger, Donald Goldfarb, Jinjun Xu, Wotao Yin, “An iterative regularization method for total variation-based image restoration,” *Multiscale modeling and simulation*, 2005, 4(2):460-489.
- [10] Gong Chen, Mare Teboulle, “Convergence Analysis of a Proximal-Like Minimization Algorithm Using Bregman Functions,” *SIAM Journal on Optimization*, 1993, 3(3):538-543.
- [11] G. Hosen Mohimani, Massoud Babaiezadeh, Christian Jutten, “Fast Sparse Representation Based on Smoothed ℓ_0 Norm,” 2007 International conference on independent component analysis and signal separation, LNCS 4666, pp.389-396, Springer London, UK.
- [12] G. Hosen Mohimani, Massoud Babaiezadeh, Christian Jutten, “A Fast Approach for Overcomplete Sparse Decomposition Based on Smoothed ℓ_0 Norm,” *IEEE Transactions on Signal Processing*, 2009, 57(1):289-301.
- [13] Chen Chen, Eric W. Tramel, James E. Fowler, “Compressed-sensing recovery of images and video using multi-hypothesis predictions,” *The 45th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, 2011, pp. 1193–1198.
- [14] Jian Zhang, Debin Zhao, Chen Zhao, Ruiqin Xiong, Siwei Ma, Wen Gao, “Compressed sensing recovery via collaborative sparsity,” *IEEE International Conference on Data Compression*, Snowbird, UT, USA, 2012, pp. 287–296.
- [15] Lin Zhang, Lei Zhang, Xuanqin Mou, David Zhang, “FSIM: a Feature SIMilarity index for image quality assessment,” *IEEE Trans. Image Process*, 2012, 20(8) : 2378–2386.