

AN OPTIMIZATION ALGORITHM FOR BIORTHOGONAL WAVELET FILTER BANKS DESIGN

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A new approach for designing the Biorthogonal Wavelet Filter Bank (BWFB) for the purpose of image compression is presented in this paper. The approach is broken into two steps. First, an optimal filter bank is designed in the theoretical sense, based on Vaidyanathan's coding gain criterion in the SubBand Coding (SBC) system. Then, the above filter bank is optimized based on the criterion of Peak Signal-to-Noise Ratio (PSNR) in the JPEG2000 image compression system, resulting in a BWFB in practical application sense. With the approach, a series of BWFBs for a specific class of applications related to image compression, such as gray-level images, can be quickly designed. Here, new 7/5 BWFBs are presented based on the above approach for image compression applications. Experiments show that the 7/5 BWFBs not only have excellent compression performance, but also easy computation and are more suitable for VLSI hardware implementations. They perform equally well with respect to 7/5 filters in the JPEG2000 standard.

Keywords: Wavelet filter; optimization; coding gain; JPEG2000; BWFB.

AMS Subject Classification: 68U10, 94A12

1. Introduction

The Discrete Wavelet Transform (DWT) has been widely applied in the domain of image compressions.¹ For its excellent performance, DWT was selected as a core algorithm for the new standard, JPEG2000,^{2,3} where its implementation was accomplished based on the lifting scheme^{4–7} using the two-channel Biorthogonal Wavelet Filter Banks (BWFB).⁸ The BWFB can perform well in removing aliasing distortion, amplitude distortions, and phase distortions in the reconstructed image; thus, they are highly suitable for image compression applications. The evaluation criteria for a filter bank which tests its impulse response and step response in addition to regularity were also derived by Villasenor *et al.*⁹ In addition, based on

these criteria, several filters that belong to the BWFB were proposed by Villasenor *et al.* for implementing the DWT. Furthermore, based on the theoretical results, the Peak Signal-to-Noise Ratio (PSNR)¹⁰ of reconstructed images is then taken into account as a criterion for finding the optimal BWFB in the practical sense. Although numerical wavelet filters are available, there are still great difficulties in finding an optimal one for image compression applications. Many algorithmic issues aiming at these are not solved yet. The early studies in this field mostly focused on selecting a wavelet basis in the time domain.^{11–13} However, the wavelet filter design in the frequency domain (z transform) is simpler than that based on the time domain design,^{14–16} and the obtained wavelet filter can immediately be used to implement the DWT in an image compression system using the lifting scheme.

However, the majority of the referred design approaches for finding the optimal filter are from the view of theoretical investigation; they are based on one criterion or tradeoff among several criteria generally, i.e. they do not consider the PSNR criterion of the reconstructed image in the image compression system. A new design approach is proposed here which is based on the PSNR criterion to find the optimal BWFB for extending the JPEG2000 image compression system. In the approach, first, the one-dimensional functional relation between general BWFB and their lifting scheme is derived with respect to a free lifting parameter. Next, in the SBC system, an optimal value is drawn for the lifting parameter based on Vaidyanathan's coding gain criterion, which at the same time decides a series of filter banks with their lifting parameters. Then, Daubechies' regularity theorem is employed to determine the compact interval in which the free lifting parameter can be included. Finally, these filter banks are integrated into the Jasper1.701.0, which is a verification system provided by JPEG2000, resulting in an expandable multi-kernel system for image compression. The system integrates an independent quantization algorithm and EBCOT coding. At the same time, the lifting parameter is quantized with the interval of 0.01 for generating a series of filter banks, which are then used for the compression task. The compression results are then used for determining the best filter bank.

The paper is devoted to finding a suitable wavelet kernel for image compression with the requirement that its performance is at least equal to the 7/5 filters provided by JPEG2000. At the same time, the suitable wavelet kernel must be decided with all rational parameters, which results in a computational advantage and ease of VLSI implementation superior to that in the current JPEG2000 standard. The paper is organized as follows. In Sec. 2, the one-dimensional functional relation between general BWFB and their lifting scheme is derived with respect to a free lifting parameter. In Sec. 3, a theoretical approach for designing the BWFB is presented using the image model of first-order Markov process as input and based on the Vaidyanathan coding gain criterion in the SBC system. In Sec. 4, an optimization approach for the BWFB designed in Sec. 3 is presented using actual gray-level image samples and based on PSNR criterion in the expandable multi-kernel JPEG2000 image compression system, as well as the new 7/5 BWFB are proposed

using our approach that is applied to image compression. Finally, in Sec. 5, we conclude the paper.

2. Wavelet Lifting Scheme

The wavelet lifting scheme has been introduced for an efficient computation to the DWT because its computation speed is two times that of the convolution-based Mallat algorithm. Its main advantage with respect to the classical BWFB structure rests within its better computation efficiency, and in fact it enables us to use the new approach for designing the BWFB.

2.1. Lifting scheme of the BWFB

We consider a two-channel filter bank and suppose a symmetric FIR compactly supported BWFB $\{H_0(z), H_1(z), G_0(z), G_1(z)\}$. $H_0(z)$ and $G_0(z)$ denote low-pass filters. $H_1(z)$ and $G_1(z)$ denote high-pass filters for analysis and synthesis stages, respectively. They are as follows:

$$\begin{cases} H_0(z) = h_0 + \sum_{i=1}^n h_i(z^i + z^{-i}) \\ G_0(z) = g_0 + \sum_{j=1}^m g_j(z^j + z^{-j}). \end{cases} \quad (2.1)$$

The coefficients of low-pass filters $H_0(z)$ and $G_0(z)$ are denoted by h_i ($i = 1, 2, \dots, n$) and g_j ($j = 1, 2, \dots, m$), respectively. Let the BWFB satisfy the Perfect Reconstruction (PR) condition, so $H_1(z)$ and $G_1(z)$ denoting high-pass filters for analysis and synthesis stages are respectively given by

$$\begin{cases} H_1(z) = z^{-1}G_0(-z^{-1}) \\ G_1(z) = z^{-1}H_0(-z^{-1}). \end{cases} \quad (2.2)$$

The polyphase representations of both low-pass filters $H_0(z)$ and $G_0(z)$ are expressed as

$$\begin{cases} H_0(z) = H_{0e}(z^2) + z^{-1}H_{0o}(z^2) \\ G_0(z) = G_{0e}(z^2) + z^{-1}G_{0o}(z^2). \end{cases} \quad (2.3)$$

By using the Euclidian algorithm, the polyphase matrix of the BWFB can be represented as

$$\begin{aligned} \tilde{P}(z) &= \begin{bmatrix} H_{0e}(z) & G_{0e}(z) \\ H_{0o}(z) & G_{0o}(z) \end{bmatrix} = \prod_{i=1}^q \begin{bmatrix} 1 & S_i(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ T_i(z) & 1 \end{bmatrix} \begin{bmatrix} K & 0 \\ 0 & \frac{1}{K} \end{bmatrix} \\ &= \prod_{i=1}^{r/2} \begin{bmatrix} 1 & \alpha_{2i-1}(1+z^{-1}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \alpha_{2i}(1+z) & 1 \end{bmatrix} \begin{bmatrix} K & 0 \\ 0 & \frac{1}{K} \end{bmatrix} \end{aligned} \quad (2.4)$$

where r is the smallest number for which $H_{0o}(z) = 0$ while calculating the Greatest Common Divisor (GCD) to $H_{0e}(z)$ and $H_{0o}(z)$, and $q = r/2 + 1$, where q represents the total lifting steps required. Moreover, K is a normalization constant, $S_i(z)$ and $T_i(z)$ denote primary and dual lifting polynomials, and α_{2i-1} and α_{2i} denote primary and dual lifting parameters, respectively. Generally, a lifting scheme starts with low-pass filtering, but in the case of n of Eq. (2.1) being an odd number, α_1 is zero, and then it starts with high-pass filtering. According to Eq. (2.4), we can establish one functional relation between the coefficients of the BWFB and the lifting parameters of the lifting scheme. We assemble them as

$$\begin{cases} h_i = f_{h_i}(\alpha_1, \alpha_2, \dots, \alpha_r) \\ g_j = f_{g_j}(\alpha_1, \alpha_2, \dots, \alpha_r). \end{cases} \quad (2.5)$$

In addition, from wavelet properties and their normalization conditions we can get

$$\begin{cases} h_0 + 2 \sum_{k=1}^n h_k = 1 \\ g_0 + 2 \sum_{k=1}^m g_k = 1, \end{cases} \quad (2.6)$$

$$\begin{cases} h_0 + 2 \sum_{k=1}^n (-1)^k h_k = 0 \\ g_0 + 2 \sum_{k=1}^m (-1)^k g_k = 0. \end{cases} \quad (2.7)$$

Finally, we derive all the coefficients of the BWFB and the lifting parameters of the lifting scheme according to Eqs. (2.5)–(2.7), which are the functions with respect to one parameter α_1 as below

$$\begin{cases} \alpha_i = f_{\alpha_i}(\alpha_1) \\ h_j = f_{h_j}(f_{\alpha_i}(\alpha_1)) \\ g_j = f_{g_j}(f_{\alpha_i}(\alpha_1)). \end{cases} \quad (2.8)$$

Therefore, the design and optimization procedure of the BWFB are greatly simplified by using Eq. (2.8). In particular, Eq. (2.8) is very convenient for analyzing the coding gain of the SBC system, the PSNR of the reconstructed images, and the computational complexity and suitability of VLSI hardware implementations.

2.2. Regularity property of the BWFB

To find the BWFB $\{H_0(z), H_1(z), G_0(z), G_1(z)\}$ for image compression, the regularity condition of the BWFB must be satisfied. Let N_1 and N_2 denote the numbers of the vanishing moments of low-pass filters on analysis and synthesis stages,

respectively. We employ Daubechies' theorem to determine the interval that the free lifting parameter α_1 can be included, and then we have

$$\begin{cases} H_0(z) = [(1 + z^{-1})/2]^{N_1} F(z) \\ G_0(z) = [(1 + z^{-1})/2]^{N_2} Q(z) \end{cases} \quad (2.9)$$

where $F(z)$ and $Q(z)$ are both trigonometric polynomials for free lifting parameter α_1 . By using Daubechies' theorem, we have the following inequalities

$$\begin{cases} \sup_{t \in \mathbf{R}, |z|=1} |F(z)F(z^2) \cdots F(z^{k_1-1})|^{1/k_1} < 2^{N_1-1/2} \\ \sup_{t \in \mathbf{R}, |z|=1} |Q(z)Q(z^2) \cdots Q(z^{k_2-1})|^{1/k_2} < 2^{N_2-1/2} \end{cases} \quad (2.10)$$

where k_1 and k_2 are integers. Thus we can determine the interval with respect to the free lifting parameter α_1 to design and optimize the BWFB respectively for image compression applications. In general, Eq. (2.10) also indicates restriction conditions in the design of the BWFBs for image compression.

3. Design of the BWFB

This section will design the BWFB based on the coding gain criterion in the SBC system. We adopt two-channel filter banks to implement the subband coding with an input image of model, which is the first-order Markov process with an input signal of Gaussian white noise. From Ref. 17, the Power Spectral Density (PSD) function of the first-order Markov process is expressed as

$$S_{xx}(e^{j\omega}) = \frac{1}{1 + \rho^2 - 2\rho \cos \omega}. \quad (3.1)$$

The SBC system is shown in Fig. 1. The subband noises are uncorrelated and remain after passing through the synthesis filter, then from Ref. 18 the output noise variance on the SBC system can be written as

$$\sigma_{\text{SBC}}^2 = \frac{C}{M} \sum_{k=1}^M 2^{-2b_k} \frac{1}{2\pi} \int_{-\pi}^{\pi} |G_k(e^{j\omega})|^2 d\omega \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{xx}(e^{j\omega}) |H_k(e^{j\omega})|^2 d\omega \quad (3.2)$$

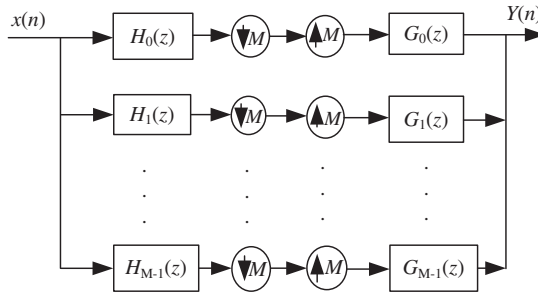


Fig. 1. M -channel SBC system for implementing the DWT.

where C is some constant which depends on the statistics of each bank signal, M is the band number of filter banks and equals 2, and $H_k(z)$, $k = 0, 1$ denotes both $H_0(z)$ and $H_1(z)$, which are low-pass and high-pass filters on the analysis stage. Similarly $G_k(z)$, $k = 0, 1$ denotes both $G_0(z)$ and $G_1(z)$, which are low-pass and high-pass filters on synthesis stages, respectively. $S_{xx}(e^{j\omega})$ denotes the PSD function given by Eq. (3.1). b_k are the numbers of bits allocated to the k th channels, to which we quantize the input signal. Without any subband decomposition, that is, with just Pulse Coding Modulation (PCM), the noise variance can be written as

$$\sigma_{\text{PCM}}^2 = C2^{-2b}\sigma_x^2 = C2^{-2b}\frac{1}{2\pi}\int_{-\pi}^{\pi} S_{xx}(e^{j\omega}) d\omega \quad (3.3)$$

where b is the average bit rate. Therefore we can obtain the coding gain which was defined as the ratio of the above variances as follows:

$$\begin{aligned} G &= \frac{\sigma_{\text{PCM}}^2}{\sigma_{\text{SBC}}^2} \\ &= \left[2^{-2b} \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{xx}(e^{j\omega}) d\omega \right] \\ &\quad \div \left[\frac{1}{M} \sum_{k=1}^M 2^{-2b_k} \frac{1}{2\pi} \int_{-\pi}^{\pi} |G_k(e^{j\omega})|^2 d\omega \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{xx}(e^{j\omega}) |H_k(e^{j\omega})|^2 d\omega \right]. \end{aligned} \quad (3.4)$$

Because one of the optimization steps is an optimal bit allocation, we can use this step to minimize the denominator. The optimal bit allocation turns the sum in the denominator into a product. So the following expression for the coding gain under optimal bit allocation is derived as follows:

$$\begin{aligned} G_{\text{opt}} &= \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} S_{xx}(e^{j\omega}) d\omega \right] \\ &\quad \div \left[\left(\prod_{k=1}^M \frac{1}{2\pi} \int_{-\pi}^{\pi} |G_k(e^{j\omega})|^2 d\omega \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{xx}(e^{j\omega}) |H_k(e^{j\omega})|^2 d\omega \right)^{1/M} \right]. \end{aligned} \quad (3.5)$$

In general, the relations of adjacent samples of the image model in horizontal and vertical directions are supposed to be the same, and $\rho_h = \rho_v = \rho = 0.95$. When we combine Eqs. (2.1), (2.2), (3.1) and (3.5), the one-dimensional function G_{opt} in optimal coding gain with respect to the free lifting parameter α_1 can be derived. Moreover, according to $dG_{\text{opt}}(\alpha_1)/d\alpha_1 = 0$, we obtain the optimal BWFB in terms of the theory defined by α_1 that the optimal coding gain reach the maximum value in SBC system, thus all the coefficients and lifting parameters of the BWFB which are calculated based on Vaidyanathan optimal coding gain criterion can be determined by Eq. (2.8).

4. Results and Discussion

In general, the optimal BWFB in the theoretical sense designed in Sec. 3 cannot achieve the best performance in practical image compression applications. Therefore, the sequential step must be taken to further optimize the above BWFB using the PSNR criterion for practical image compression applications. For this purpose, we establish an image compression verification system, which is based on verification software Jasper 1.701.0 with quantization and EBCOT coding algorithm in the JPEG2000 standard.^{19,20} It supports multi-kernel BWFB for image compression applications, and all the filter coefficients and their lifting parameters only depend on the free lifting parameter α_1 . Thus we can select the interval according to the result calculated in Sec. 3 and Eq. (2.10) to determine the finite number of free lifting parameter α_1 quantized by step size 0.01, and each α_1 value in this interval defines a BWFB. Finally, we take these BWFB with respect to a set of free lifting parameters α_1 to realize the image compression automatically and very continuously in our verification system, and then obtain the optimal BWFB in practical image compression application based on PSNR criterion. This section will provide a new 7/5 BWFB defined by our approach applied to images, which is a popular gray-level test image, Lena.bmp (512×512 pixels \times 8 bits).

4.1. Design of 7/5 BWFB

In Eq. (2.1), if $n = 3$, $m = 2$, then both low-pass filters on analysis and synthesis stages of the 7/5 BWFB are given by

$$\begin{cases} H_0(z) = h_0 + \sum_{i=1}^3 h_i(z^i + z^{-i}) \\ G_0(z) = g_0 + \sum_{j=1}^2 g_j(z^j + z^{-j}). \end{cases} \quad (4.1)$$

According to Eq. (2.3), the polyphase representation of the 7/5 BWFB for both low-pass filters on analysis and synthesis stages are as follows, respectively

$$\begin{cases} H_{0e}(z) = h_0 + h_2(z + z^{-1}) \\ H_{0o}(z) = h_1(z + 1) + h_3(z^2 + z^{-1}), \end{cases} \quad (4.2)$$

$$\begin{cases} G_{0e}(z) = g_0 + g_2(z + z^{-1}) \\ G_{0o}(z) = g_1(z + 1). \end{cases} \quad (4.3)$$

And according to Eq. (2.4), the polyphase matrix of the 7/5 BWFB is given by

$$\begin{aligned} \tilde{P}(z) &= \begin{bmatrix} H_{0e}(z) & G_{0e}(z) \\ H_{0o}(z) & G_{0o}(z) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ \alpha_2(1+z) & 1 \end{bmatrix} \begin{bmatrix} 1 & \alpha_3(1+z^{-1}) \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_4(1+z) & 1 \end{bmatrix} \begin{bmatrix} K & 0 \\ 0 & \frac{1}{K} \end{bmatrix}. \end{aligned} \quad (4.4)$$

The functional relations between coefficients of the 7/5 BWFB^{19–22} and their lifting parameters of lifting scheme are as follows:

$$\begin{cases} h_0 = (1 + 2\alpha_3\alpha_4)K \\ h_1 = [(1 + 2\alpha_2\alpha_3)\alpha_4 + (1 + \alpha_3\alpha_4)\alpha_2]K \\ h_2 = \alpha_3\alpha_4K \\ h_3 = \alpha_2\alpha_3\alpha_4K \\ g_0 = (1 + 2\alpha_2\alpha_3)/(2K) \\ g_1 = -\alpha_3/(2K) \\ g_2 = \alpha_2\alpha_3/(2K). \end{cases} \quad (4.5)$$

Using Eq. (4.5) and a normalization condition of wavelet filter coefficients, we can obtain that the first lifting parameter α_1 is a constant and equals zero, and then we derive all the filter coefficients for free lifting parameter α_2 as follows:

$$\begin{cases} h_0 = (2\alpha_2 + 3)/[4(2\alpha_2 + 1)] \\ h_1 = -(2\alpha_2^2 - 5\alpha_2 - 2)/[8(2\alpha_2 + 1)] \\ h_2 = (2\alpha_2 - 1)/[8(2\alpha_2 + 1)] \\ h_3 = [\alpha_2(2\alpha_2 - 1)]/[8(2\alpha_2 + 1)] \\ g_0 = (\alpha_2 + 1)/2 \\ g_1 = 1/4 \\ g_2 = -\alpha_2/4. \end{cases} \quad (4.6)$$

We combine Eqs. (2.2), (3.1), (3.5), (4.1) and (4.6) to calculate the G_{opt} with respect to free lifting parameter α_2 . $G_{\text{opt}}(\alpha_2)$ is plotted in Fig. 2, and it is known that when α_2 equals -0.3142 , $G_{\text{opt}}(\alpha_2)$ reaches the maximum value of magnitude.

4.2. Optimization of 7/5 BWFB

In order to optimize the 7/5 BWFB designed in Sec. 4.1, we first choose the interval of free lifting parameter α_2 , in which each α_2 can only define a 7/5 BWFB with step 0.01 for all the samples of this interval. We then take these 7/5 BWFBs to realize image compression automatically and very continuously. Then, the optimal 7/5 BWFB, when being applied to practical image compressions, can be found based on PSNR criterion. Where the vanishing moments N_1 and N_2 of the 7/5 BWFB are all 2, and if $k_1 = k_2 = 20$, Eq. (2.10) holds for $\alpha_2 \in [-0.11, 0.50]$. The interval of free lifting parameter α_2 can be selected by the above computational result and Eq. (2.10), so the interval $[-0.50, 0.50]$ is selected. In order to illustrate conveniently, the results in parts of the interval selected are plotted in Fig. 3.

The results show that when the free lifting parameter α_2 equals 0.10, the maximum PSNR value of the reconstructed image are obtained in terms of statistical ideas, and this α_2 uniquely defines a 7/5 BWFB by Eq. (4.6). In addition, one

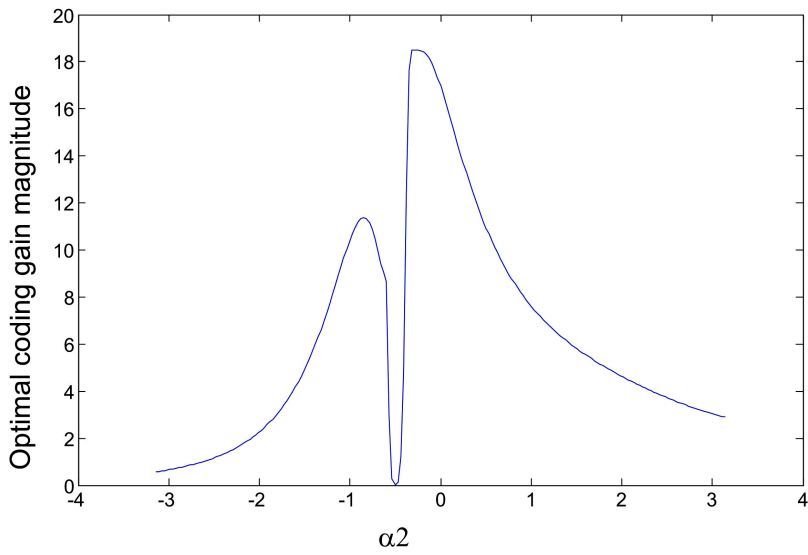


Fig. 2. Free parameter α_2 for the 7/5 BWFB.

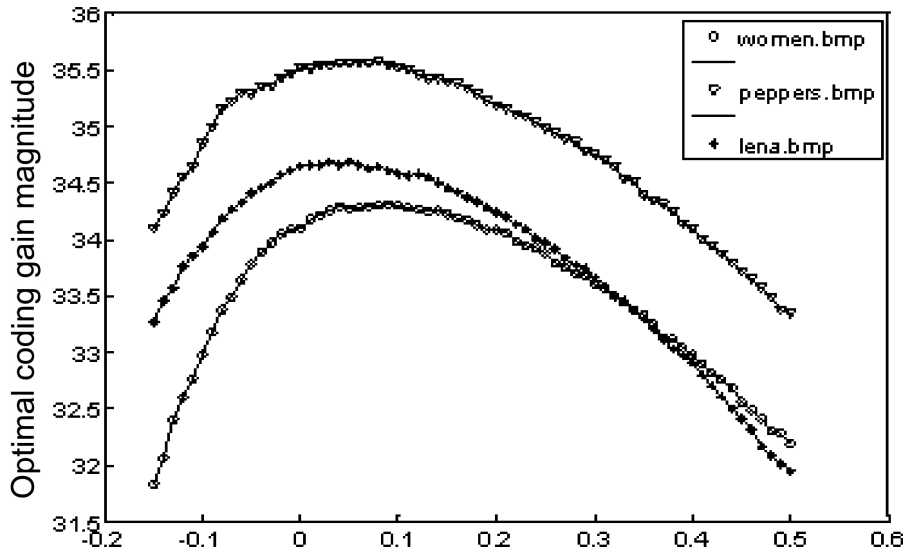


Fig. 3. Free parameter α_2 for 7/5 BWFB (C.R. = 16 : 1).

result of the objective comparison results for image compression performance using different test images and different filters are shown in Table 1.

The coefficients of 7/5 BWFB and its lifting parameters are shown in Tables 2 and 3.

The coefficients of the 7/5 filter in JPEG2000 part II and its lifting parameters are shown in Tables 4 and 5.

The objective comparison results for image compression performances using the 7/5 BWFB are shown in Table 6.

The subjective comparison results are excellent between 7/5 BWFB and 7/5 filter in the JPEG2000 standard as shown in Fig. 4.

Table 1. Compression performance in the reconstructed image in PSNR/dB (C.R = 16:1)

Table.I-1					
Free Parameter	0.00	0.01	0.02	0.03	0.04
women.bmp	34.106045	34.176065	34.216773	34.255802	34.292583
peppers.bmp	35.522636	35.509923	35.549724	35.536380	35.558725
lena.bmp	34.653684	34.659786	34.662632	34.690112	34.664104

Table.I-2					
Free Parameter	0.05	0.06	0.07	0.08	0.09
women.bmp	34.277509	34.285669	34.295206	34.300209	34.308034
peppers.bmp	35.360044	35.567384	35.558003	35.570439	35.552238
lena.bmp	34.692596	34.661845	34.628390	34.643379	34.609546

Table.I-3					
Free Parameter	0.10	0.11	0.12	0.13	0.14
women.bmp	34.300399	34.281561	34.264215	34.247024	34.253355
peppers.bmp	35.522906	35.504107	35.458867	35.419809	35.430412
lena.bmp	34.581566	34.569732	34.580034	34.549179	34.500686

Table.I-4						
Free Parameter	0.15	0.16	0.17	0.18	0.19	0.20
women.bmp	34.220783	34.187325	34.174147	34.129254	34.088997	34.088363
peppers.bmp	35.394931	35.381370	35.344401	35.287850	35.232024	35.183696
lena.bmp	34.452617	34.422441	34.376072	34.335217	34.285304	34.240994

Table 2. The coefficients for the 7/5 BWFB.

k	Analysis Filters		Synthesis Filters	
	Low-Pass Filter	High-Pass Filter	Low-Pass Filter	High-Pass Filter
	h_k	\tilde{h}_k	g_k	\tilde{g}_k
0	$\frac{2}{3}$	$\frac{11}{20}$	$\frac{11}{20}$	$\frac{2}{3}$
± 1	$\frac{31}{120}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{31}{120}$
± 2	$-\frac{1}{16}$	$-\frac{1}{40}$	$-\frac{1}{40}$	$-\frac{1}{16}$
± 3	$-\frac{1}{120}$			$\frac{1}{120}$

Table 3. Lifting parameters for the 7/5 BWFB.

Parameters	Values
α_2	$\frac{1}{10}$
α_3	$-\frac{5}{12}$
α_4	$\frac{6}{25}$
K	$\frac{5}{6}$

Table 4. The coefficients for the 7/5 filter.

k	Analysis Filters		Synthesis Filters	
	Low-Pass Filter	High-Pass Filter	Low-Pass Filter	High-Pass Filter
	\hat{h}_k	\hat{h}_k	\hat{g}_k	\hat{g}_k
0	$\frac{79}{116}$	$\frac{27}{50}$	$\frac{27}{50}$	$\frac{79}{116}$
± 1	$\frac{373}{1450}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{373}{1450}$
± 2	$-\frac{21}{232}$	$-\frac{1}{50}$	$-\frac{1}{50}$	$-\frac{21}{232}$
± 3	$-\frac{21}{2900}$			$\frac{21}{2900}$

Table 5. Lifting parameters for the 7/5 filter.

Parameters	Values
α_2	$\frac{2}{25}$
α_3	$-\frac{175}{406}$
α_4	$\frac{609}{2500}$
K	$\frac{25}{29}$

Table 6. Results of comparison between 7/5 BWFB and 7/5 filter in PSNR/dB.

Compressions Ratios (C.R.)	8:1	16:1	32:1	64:1	128:1
7/5 BWFB $\alpha_2 = 0.10$	37.718118	34.581566	31.718073	28.952086	27.271790
7/5 Filter $\alpha_2 = 0.08$	37.725914	34.643379	31.691242	28.951855	27.347160

5. Conclusions

This paper presented a novel approach, which adopted both coding gain criteria and PSNR criteria to design and optimize the BWFB, respectively. Moreover, using the approach, 7/5 BWFB applied to image compression were proposed. The 7/5



(a) Original



(b) 7/5 BWFB C.R = 32:1



(c) 7/5 Filters C.R = 32:1

Fig. 4. Subjective comparison using 7/5 BWFB and 7/5 filter.

BWFBs not only have lower computational complexity but also are more suitable for VLSI hardware implementation than 7/5 filter in the JPEG2000 standard part II.

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References

1. M. Antonini, M. Barlaud and I. Daubechies, Image coding using wavelet transform, *IEEE Trans. Image Process.* **1**(2) (1992) 205–220.
2. JPEG2000 Image Coding System, Part 1 (Core), ISO/IEC Int'l. Standard 15444-1, ITU-T Rec. T.800, Int'l. Org. for Standardization (2003).
3. JPEG2000 Image Coding System, Part 2 (Extensions) ISO/IEC Int'l. Standard 15444-2, ITU-T Rec. T.800, Int'l. Org. for Standardization (2004).
4. W. Sweldens, The lifting scheme: A new philosophy in biorthogonal wavelet constructions, in *Wavelet Applications in Signal and Image Processing 3*, eds. A. F. Laine and M. Unser, Vol. 2569 (SPIE, 1995), pp. 68–79.
5. W. Sweldens, The lifting scheme: A custom-design construction of biorthogonal wavelets, *J. Appl. Comput. Harmon. Anal.* **3**(2) (1996) 186–200.
6. W. Sweldens, The lifting scheme: A construction of second generation wavelets, *SIAM J. Math. Anal.* **29**(2) (1997) 511–546.
7. I. Daubechies and W. Sweldens, Factoring wavelet transforms into lifting steps, *J. Fourier Anal. Appl.* **4**(3) (1998) 247–269.
8. A. Cohen, I. Daubechies and J. C. Feauveau, Biorthogonal bases of compactly supported wavelets, *J. Commun. Pure Appl. Math.* **45**(5) (1992) 485–560.
9. J. D. Villasenor, B. Belzer and J. Liao, Wavelet filter evaluation for image compression, *IEEE Trans. Image Process.* **4**(8) (1995) 1053–1060.
10. Y. Shang, L. Z. Li and B. Wah, Optimization design of biorthogonal filter banks for image compression, *Inform. Sci.* **132** (2001) 23–51.
11. D. Taubman, High performance scalable image compression with EBCOT, *IEEE Trans. Image Process.* **9**(7) (2000) 1158–1170.
12. A. H. Tewfik, D. Sinha and P. Jorgensen, On the optimal choice of a wavelet for signal representation, *IEEE Trans. Inform. Theory* **38**(2) (1992) 747–765.
13. R. R. Coifman and M. V. Wickerhauser, Entropy-based algorithms for best basis selection, *IEEE Trans. Inform. Theory* **38**(3) (1992) 713–718.
14. P. P. Vaidyanathan and A. Kirac, Results on optimal biorthogonal filter banks, *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.* **45**(8) (1998) 932–947.
15. S. Ohno and H. Sakai, Optimization of filter banks using cyclostationary spectral analysis, *IEEE Trans. Signal Process.* **44**(11) (1996) 2718–2725.
16. P. Moulin, *et al.*, Theory of rate-distortion optimal, constrained filterbanks-application to IIR and FIR biorthogonal design, *IEEE Trans. Signal Process.* **48**(4) (2000) 1120–1132.
17. J. Tuqan and P. P. Vaidyanathan, Statistically optimum pre- and postfiltering in quantization, *IEEE Trans. Circuits Syst. II, Analog Digit. Process.* **44**(1) (1997) 1015–1031.
18. I. Djokovic and P. P. Vaidyanathan, On optimal analysis/synthesis filters for coding gain maximization, *IEEE Trans. Signal Process.* **44**(5) (1996) 1276–1279.
19. G. Yang and S. Guo, A new lifting scheme for image compression, in *The International Workshop on Intelligence Computing in Pattern Analysis/Synthesis (IWICPAS2006)*, ed. X. Lan, Proceeding LNCS, Vol. 4153 (Springer-Verlag 2006), pp. 465–474.
20. G. Yang, N. Zheng, S. Guo *et al.*, Extensible JPEG2000 image compression systems, in *Proc. 2005 IEEE Int. Conf. on Industrial Technology (ICIT2005)*, Hong Kong (December 2005), pp. 1376–1380.
21. G. Yang, N. Zheng *et al.*, Research on cluster of 7/5-tap wavelet filters and their image compression performance, *J. Xi'an Jiaotong Univ.* **39**(6) (2005) 628–632.
22. G. Yang, N. Zheng *et al.*, Optimal wavelet filter design for remote sensing image compression, *J. Electronics (China)* **24**(2) (2007) 276–284.