



An application of Mean Escape Time and metapopulation on forestry catastrophe insurance[☆]

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HIGHLIGHTS

- We investigate a forestry catastrophe insurance model via metapopulation and Escape Time.
- Parameters are estimated with real data set of China.
- Probability of loss and its payment time are respectively investigated.
- An optimal payment time of insurance can be found.

ARTICLE INFO

Article history:

Received 23 August 2017

Received in revised form 20 November 2017

Available online 23 December 2017

Keywords:

Metapopulation

Forestry catastrophe insurance

Mean Escape Time

Forestry pest infestations and disease epidemics

ABSTRACT

A forestry catastrophe insurance model due to forestry pest infestations and disease epidemics is developed by employing metapopulation dynamics and statistics properties of Mean Escape Time (MET). The probability of outbreak of forestry catastrophe loss and the catastrophe loss payment time with MET are respectively investigated. Forestry loss data in China is used for model simulation. Experimental results are concluded as: (1) The model with analytical results is shown to be a better fit; (2) Within the condition of big area of patches and structure of patches, high system factor, low extinction rate, high multiplicative noises, and additive noises with a high cross-correlated strength range, an outbreak of forestry catastrophe loss or catastrophe loss payment due to forestry pest infestations and disease epidemics could occur; (3) An optimal catastrophe loss payment time MET due to forestry pest infestations and disease epidemics can be identified by taking proper value of multiplicative noises and limits the additive noises on a low range of value, and cross-correlated strength at a high range of value.

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1. Introduction

Insurance is designed to secure the relatively infrequent loss events in many spheres of life. Compared to other insurance products, forestry catastrophe insurance is among the most difficult to develop. Miranda [1] built a crop yield insurance model based on Johnson's [2] hedging model and applied it to Kentucky soybean farms. She assumed that the crop insurance coverage levels was the only selective choice for farmers. And farmer's utility could be maximized by minimizing his revenue

[☆] This work was supported by the National Natural Science Foundation of China (Grant No.: 11165016, 71263056, 11647078), the Science Foundation of Yunnan University of Finance and Economics (Grant No.: YC2016D07, YC2015D09) and Yunnan Province Education Department Scientific Research Fund Project (No.: 2016ZZX149).

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variance. After that, Smith, Chouinard and Baquet [3] improved Miranda’s insurance model and implemented it to wheat farmers in Montana and France. Skees, et al. [4] used an empirical model based on the coefficient of variation of a portfolio of several crops, and the results showed that nearly 29% of the aggregated regional revenue risk could be reduced. The aggregated data was used by Skees, Hazell and Miranda [5] in insurance discussion. Raushan and Gunnar combined the mean–variance(MV) approach with Second Degree Stochastic Dominance(SSD) [6] criterion for an expected utility consistent empirical procedure. Areas that are affected by forestry infestations and disease epidemics are typically huge. This raises the issue of systemic risk, or, put differently, spatial correlation of yield losses. Roberts [7] puts forth the painted apple moth *Teia* (an insect) would pose a threat to New Zealand’s forest industry and forest reserves, so the need for forestry infestations and disease epidemics mechanisms has increased dramatically. Vado Sequeira, Ligia [8] believed that forestry products are vulnerable to random changes in weather, pest infestations and disease epidemics. The interaction between various kinds of agricultural technologies, extreme weather risks and agricultural insurance were explored. However, the crop yield losses are generally difficult to determine, Multi-peril Crop Insurance (MPCI) is used to protect against different causes of yield losses [9]. MPCI calculates insured yield as a percentage of the historical average yield for insured plot. When realized yield is under the insured yield, the difference between realized yield and insured yield is paid as an indemnity [10]. A stochastic model with endogenous and exogenous periodicities is also to model the crop yield losses due to pests and diseases [11].

As a significant sector of Chinese economy, forestry industry and forestry reserves are seriously affected by forestry pest infestations and disease epidemics. Pest infestations and disease epidemics were recognized as two of the most important threats to exotic plantation forestry [12]. Su et al. [13] showed that the annual losses approximated to 88 billion Yuan, in which the direct economic losses were about 14.5 billion Yuan and the ecological losses were about 73.5 billion Yuan, resulting from some main forest pests infestations and disease epidemics from 1996 to 2001. Yan and Cai [14] analyzed Chinese forestry pest infestations incidence data, and concluded that over 14.67 billion dollars losses were caused by forestry pest infestations and disease epidemics. JIN M-T [15] believed that forest insurance is an effective method for Chinese forestry industry development and forestry reserves. Under the consideration of China’s economic and ecological benefits, to apply forestry catastrophe insurance in impacts of forestry pest infestations and disease epidemics should be addressed. By using cluster analysis method Zhao and Wang [14] quantified the risk of forestry pest infestations with an index. And an insurance product for forest pest infestations was proposed on the ground of the index. Carlson [16] suggested that the increasing availability of crop damage insurance reduced the use of pesticide in crop.

Some research convinced evident that standard finance models are for short of covering the complicities of empirical research in the area of forests insurance, therefore new ideas and models from Physics are called for [17]. As an interdisciplinary field, Econophysics [18] applies statistical physics theories, methods and models to analyze economic and financial problems. Various econophysicists have introduced models for price fluctuations in financial markets or proposed original points of view on established models. What is more, several scaling laws was found in various economic data. Exerting the strength of quantitative analysis, Econophysics is also applied in solving insurance problems, in which uncertainty or stochastic processes and nonlinear dynamics are used. In areas of metastable systems [19,20], bistable system [21,22], Malthus–Verhulst stochastic model [23,24], randomly switching piece-wise metastable linear potential [25], the growth of tumor influenced by external fluctuations and periodic treatment [26], a self-propelled Janus particle [27], a ecological system [28–32], an energy depot model [33] and a synthetic gene circuit [34], the effects of noise on the stability of the system with the escape time and stochastic resonance were vastly used. MET is a terminology used in Physics to describe the interval of a particle in certain region, portrays the statistics properties of transit issues in nonlinearly system. The statistics properties of MET were studied in market system with stochastic volatility, especially in analysis the stability of stock price, representing the time of the stock price staying in a price range [35–37].

We employed the statistics properties of MET in catastrophe risk due to forestry pest infestations and disease epidemics. This study analyzes the appraisal of loss and method for catastrophe risk characterization due to forestry pest infestations and disease epidemics. Section 2 defines the loss model due to forestry pest infestations and disease epidemics and estimates the corresponding parameters according to an empirical study on pest incidence data of 15 cities in China. The expected value of loss payment is analyzed based on the proposed loss model in Section 3. Meanwhile, the correlation between expected value of loss payment and deductible is estimated, in addition, the probability density function (PDF) obtained from the sample data and the PDF based on proposed model are compared, and the full deductible and full indemnity are quantified respectively. In Section 4, the MET of catastrophe loss payment occurring is gained lay by the mean escape time defined as above. Conclusions and future works are presented in Section 5.

2. The loss model due to forestry pest infestations and disease epidemics

The loss caused by forestry pest infestations and disease epidemics is given based the metapopulation model proposed by Hanski, Wahlberg and Ovaskainen [38,39]. Then the simplified model can be obtained after assuming theoretically that: (i) the spatial structures of patches satisfy the coupled map lattice, and (ii) these patches are possessed of identical structural characteristics and qualities, their probabilities occupied at any time are equal to each other [38]. The loss can be defined as:

$$\frac{dl}{dt} = \frac{l^2(1-l)}{l^2 + y^2/A^2} - \frac{e}{A^b}l - l\xi(t) + \eta(t), \quad l \in [0, 1], \quad (1)$$

where $l(t)$ is the rate that patch is occupied by forestry pest infestations and disease epidemics at time t ; $y = 1/(fc^{1/2})$, and f is the structure factor of patches, $f = \sum_{j \neq i} e^{-d_{ij}}$, $1/a$ gives the average migration distance and d_{ij} is the distance between

patches i and j ; A is the area of patch and b is a parameter of the system; c and e represent the colonization and extinction rate parameters, $\xi(t)$ and $\eta(t)$ are the Gaussian white noise respectively as following properties:

$$\begin{aligned} \langle \eta(t) \rangle &= \langle \xi(t) \rangle = 0, \\ \langle \eta(t)\eta(t') \rangle &= 2D\delta(t - t'), \\ \langle \xi(t)\xi(t') \rangle &= 2\alpha\delta(t - t'), \\ \langle \eta(t)\xi(t') \rangle &= 2\lambda\sqrt{D\alpha}\delta(t - t'), \end{aligned} \tag{2}$$

D and α denoting noise intensities and λ being a cross-correlated strength.

The potential

$$V(l) = \frac{1}{2}\left(1 + \frac{e}{A^b}\right)l^2 - l + \frac{y}{A} \arctan\left(\frac{A}{y}l\right) - \frac{y^2}{2A^2} \ln\left(l^2 + \frac{y^2}{A^2}\right) \tag{3}$$

corresponding to Eq. (2) has an unstable state at $l_u = 0$ and a non-trivial equilibrium state at l_s of the system in $l \in [0, 1]$. l_s denotes the steady-state value of $\langle l(t) \rangle$ and be calculated from the equation $f(l_s) = 0$ with $l_s \in (0, 1]$. The non-trivial equilibrium state is given by Eq. (1) with the parameters in Ref. [38]:

$$l_s = \frac{1 + \sqrt{1 - 4\left(1 + \frac{e}{A^b}\right)\frac{ey^2}{A^{b+2}}}}{2\left(1 + \frac{e}{A^b}\right)}. \tag{4}$$

By the Novikov theorem and the Fox approach, the approximate Fokker–Planck equation (AFPE) [40–42] can be obtained that:

$$\frac{\partial P(l, t)}{\partial t} = -\frac{\partial}{\partial l}A(l)P(l, t)dl + \frac{\partial^2}{\partial l^2}B(l)P(l, t)dl, \quad l \in [0, 1], \tag{5}$$

where $P(l, t)$ is the probability distribution function, $A(l)$ and $B(l)$ respectively read:

$$A(l) = f(l) + Dl - \lambda\sqrt{D\alpha}, \tag{6}$$

$$B(l) = Dl^2 - 2\lambda\sqrt{D\alpha}l + \alpha, \tag{7}$$

where

$$f(l) = \frac{l^2(1-l)}{l^2 + y^2/A^2} - \frac{e}{A^b}l. \tag{8}$$

The stationary PDF of Eq. (5) is expressed as:

$$P_{st}(l) = \frac{N}{B(l)} \exp(-U(l)/D), \tag{9}$$

where N is a normalization constant, $U(l)$ is the modified potential which given by :

$$U(l) = -\int_0^l \frac{\frac{z^2(1-z)}{z^2 + y^2/A^2} - \frac{e}{A^b}z}{z^2 - 2\lambda\sqrt{\alpha/D}z + \alpha/D} dz. \tag{10}$$

Parameters in Eq. (1) or (9) are estimated by minimizing the mean-square errors: $\sum_i (P^{Data}(l) - P^{Model}(l))^2$, where $P^{Data}(l)$ is the probability densities derived from the real loss data by using kernel density estimation, and $P_t^{Model}(l)$ is the probability densities calculated from Eq. (9) or simulated from Eq. (1). Incidence data of forest pests of 15 cities in China from 2005 to 2014 are used. The data comes from China Knowledge Resource Integrated Database. The loss ratios are obtained by dividing the yearly incidence of forest pests by the historical average yields. The data set consists of 150 samples. The mean μ of loss is 0.09789, the standard deviation σ is 0.08325, the minimum is 0.00335, the Median is 0.06814 and the maximum is 0.44182. The following estimates are obtained: $\hat{A} = 2.30836$, $\hat{b} = 0.744386$, $\hat{y} = 0.192071$, $\hat{e} = 8.48628$, $\hat{D} = 8.02888$, $\hat{\alpha} = 0.0127618$ and $\hat{\lambda} = 0.944168$. Based on the above parameters estimated using the sample loss data, we show the comparison of the probability density function P_{appr} vs. loss between real data and theoretical result in Fig. 1. The P_{appr} of real data is calculated by the kernel density estimation with the bandwidth 0.01. The analytical result is computed from the Eqs. (9) and (10). The simulated result is obtained from Eqs. (1) and (2). Relatively agreement between real data and theoretical results can be observed in Fig. 1.

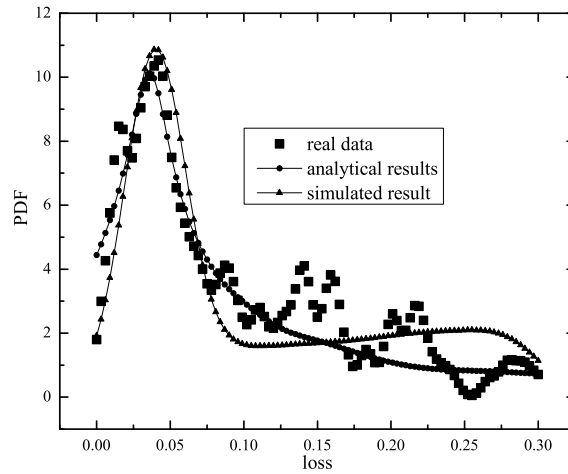


Fig. 1. A comparison of P_{appr} between real data and theoretical result.

3. The indemnity of forestry catastrophe insurance

The insurance company pays the indemnity x whenever forest losses l due to pests exceed the deductible q . For simplicity, the indemnity of forestry catastrophe insurance is defined as

$$x = \begin{cases} P_H & \text{if } l \geq l_H \\ P_H \frac{l - q}{l_H - q} & \text{if } q < l < l_H \\ 0 & \text{if } l \leq q \end{cases} , \tag{11}$$

l_H is the upper limit loss for the indemnity, P_H is the upper limit of indemnity (maximum loss payment). The loss l of x can be written as

$$l = \frac{l_H - q}{P_H}x + q, \quad x \in (0, P_H). \tag{12}$$

Finally, from Eqs. (9) and (12), we obtain the following PDF of x :

$$\begin{aligned} P_x(x) &= P_{st}(l(x))|h'(x)| \\ &= \frac{(l_H - q)N}{P_H B(\frac{l_H - q}{P_H}x + q)} \exp(-U(\frac{l_H - q}{P_H}x + q)/D), \quad x \in (0, P_H). \end{aligned} \tag{13}$$

The probability of not paid is

$$P_x(x = 0) = \int_0^q P_{st}(l)dl. \tag{14}$$

After a huge disasters occur, the probability of full compensation is

$$P_x(x = 1) = \int_{l_H}^1 P_{st}(l)dl. \tag{15}$$

Consider $\mu \pm \sigma$ range as the range of insurance indemnity, $q = \mu - \sigma$ and $l_H = \mu + \sigma$, i.e., $q = 0.01464$ and $l_H = 0.18114$ based on the data of above section. Fix $P_H = 1$ for 100%.

Fig. 2 describes different PDF of the indemnity x under various deductible q (see Eqs. (1) and (11)). We increase the deductible q from 0.005 to 0.045, a minimum peak value of expected loss payment is gained. It indicates that a threshold q is existed where has the worst loss payment stability. 0.025 is the thresholded value of q in Fig. 2. Varying deductible q , and the peak of PDF of the indemnity x gradually shifts towards the left side, which means the corresponding expected value of loss payment is reduced gradually. On one hand, the thresholded q implies the existence of a catastrophe risk due to forestry pest infestations and disease epidemics, which may trigger an extreme loss payment. We have to admit that insurance companies are facing high loss payment risk, which is determined by the unpredictability of pest forestry pest infestations and disease epidemics and the uncontrollability of forestry economy. On the other hand, the higher deductible set by insurance companies, the less loss payment is required. Insurance companies might have the tendency to increase deductible in order to reduce loss payment. Overall, Fig. 2 suggests that insurance companies should concern the catastrophe

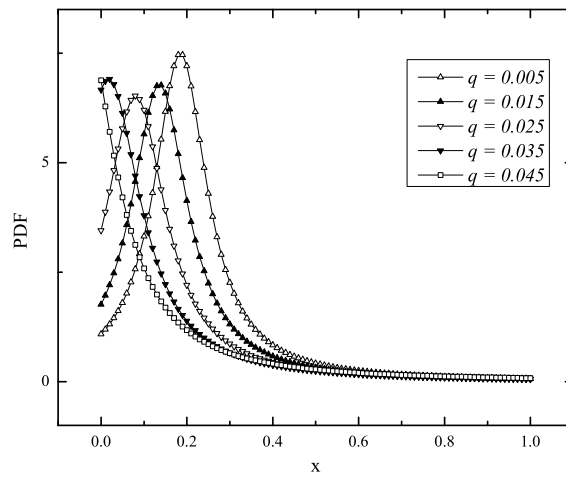


Fig. 2. PDF of the indemnity x of the forestry catastrophe insurance with varying the deductible q .

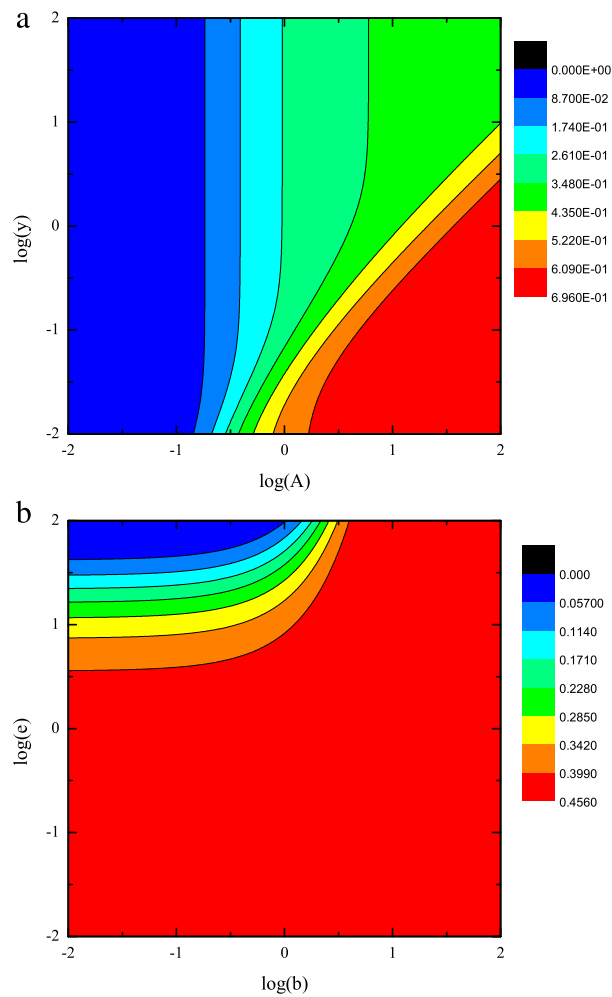


Fig. 3. The contour of the probability of full indemnity (Eq. (15)) vs. $\log(y)$ and $\log(A)$ in (a) and vs. $\log(b)$ and $\log(e)$ in (a).

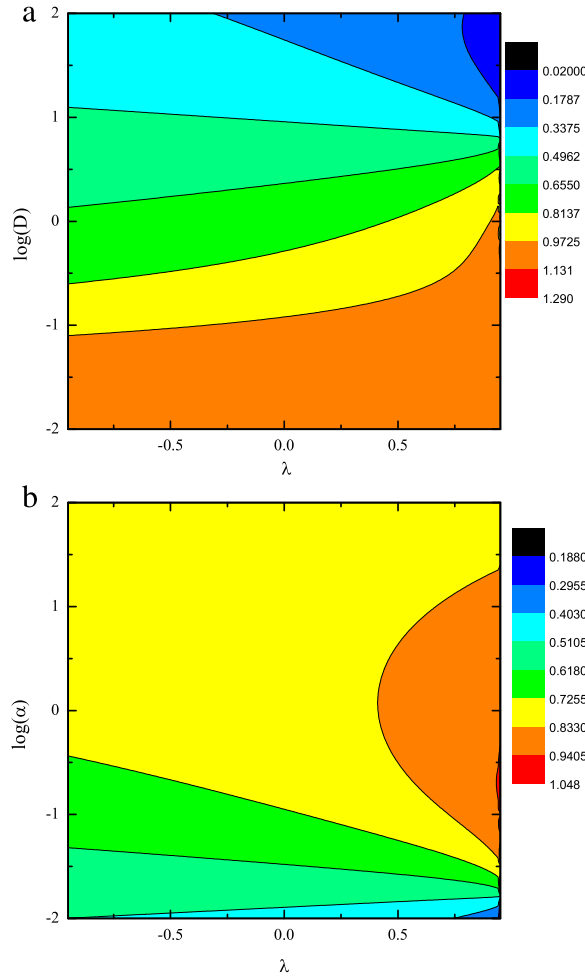


Fig. 4. The contour of the probability of full indemnity (Eq. (15)) vs. λ and $\log(D)$ in (a) and vs. λ and $\log(\alpha)$ in (a).

risk when implementing this forestry catastrophe insurance product, and then optimize the deductible q as a balance between maximum insurance revenue and minimum loss payment risk.

Fig. 3 graphs the contour of the probability of full indemnity (as given in Eq. (15)) based on the proposed insurance model. It also shows the range of parameters, within which the catastrophe risk or catastrophe loss payment occurs. The increase of $\log(A)$ and the decrease of $\log(y)$, and also the increase of $\log(b)$ and the decrease of $\log(e)$ lead to the rising of the probability of full indemnity, as portrayed in Figs. 3(a) and 3(b), respectively. Based on the definitions of each parameter, experimental results indicate that higher probability of full indemnity are determined by greater insured area of patch A , bigger the structure factor of patches f , lower the extinction rate e , greater the system factor b . The increase of patches area and structure factor add the systematic risk due to forestry pest infestations and disease epidemics, which demands higher probability of full indemnity (see Fig. 3(a)). From the point of view of forestry catastrophe insurance, this implies the existence of a catastrophic risk caused by forestry pest infestations and disease epidemics. It can also be observed from Fig. 3(b) that with the increase of system factor b , and the decrease of extinction rate e , the systematic risk due to forestry pest infestations and disease epidemic is raised dramatically, which requires an accelerated growth of the probability of full indemnity. This suggests that the systematic risk is the key to reduce the catastrophe risk due to pest infestations and disease epidemics, and then control the probability of full indemnity. In order to lower the cost of loss, insurance companies should properly regulate the area and structure factor of insured patches, extinction rate of forestry pest, moreover, lower the system factor.

Figs. 4(a) and 4(b) show the probability of full indemnity Eq. (15) with noise intensity D and α respectively. And both figures take cross-correlated strength λ into consideration. Fig. 4(a) graphs that with the increase of λ and decrease of D , the probability of full indemnity is monotonically increasing, but does not reach extreme high probability of full indemnity. Fig. 4(b) presents that when λ is small, the probability of full indemnity is low, however, when λ is big, the probability of full indemnity is increasing, and a threshold value of λ exists. From that threshold value on, the probability of full indemnity

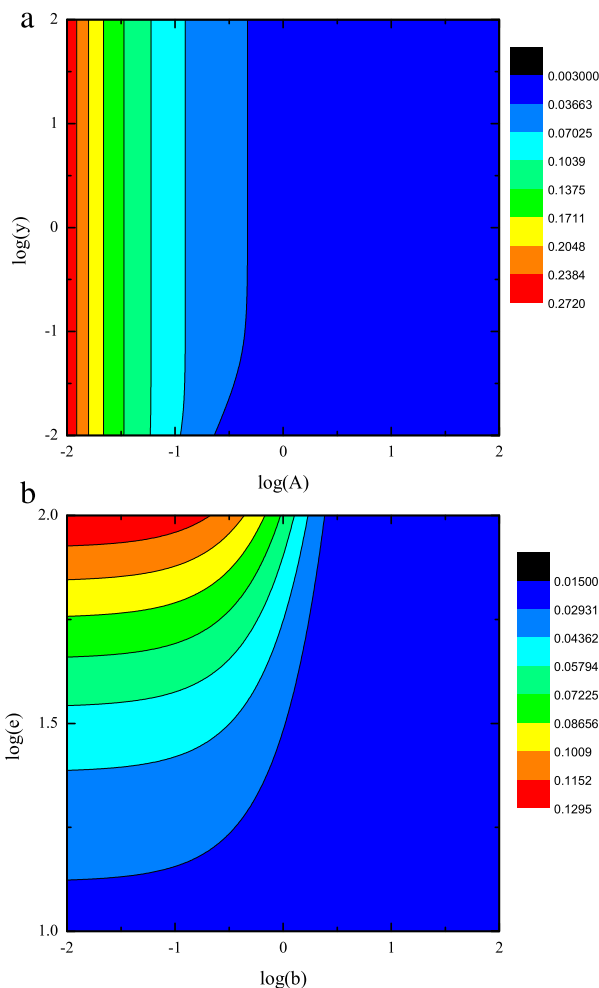


Fig. 5. The contour of the probability of zero indemnity (Eq. (14)) vs. $\log(A)$ and $\log(y)$ in (a) and vs. $\log(b)$ and $\log(e)$ in (b).

keeps growing with the increase of λ , while the increase or decrease of a cannot affect the growth trend of the probability of full indemnity. The experimental results mean the noise intensity D and the cross-correlated strength λ affect the forestry loss payment positively, but would not trigger the catastrophe risk. As for the noise intensity a and the cross-correlated strength λ , only when λ reaches a certain threshold value, λ causes the forestry loss payment risk to increase, however a has no effect on forest loss payment risk, only when a is small, and λ achieves the extreme value around 1, the catastrophe risk would happen. It proves that it is proper to consider the noise intensity D , a , and cross-correlated λ in the proposed model. By examining the noise intensity and cross-correlated strength, a threshold value is able to recognize, which could help insurance companies to control loss payment risk under certain realm. It indicates multiplicative noises have higher probability than additive noises in causing catastrophe risk or catastrophe loss payment. Only under strong correlation of multiplicative and additive noises, additive noises can trigger the happen of catastrophe risk or catastrophe loss payment.

For the completeness of this experiment and comparison, Figs. 5 and 6 addressed the case of zero indemnity, which is different from Figs. 3 and 4 with respect to full indemnity. The contour of the probability of zero indemnity (as given in Eq. (14)) affected by $\log(y)$ and $\log(A)$, $\log(b)$ and $\log(e)$ are graphed in Figs. 5(a) and 5(b), respectively. Fig. 5(a) shows the lower the area of patches A , the higher the probability of zero indemnity, meanwhile, the probability of zero indemnity is very sensitive towards smaller area range of patches. However, the change of the structure of patches y does not affect the probability of zero indemnity. It proves that the loss cost can be achieved by set the area of patches in a small area range. In comparison, with the increase of system factor b , and the increase of extinction rate e , the probability of zero indemnity decreases monotonically (as given in Fig. 5(b)). It indicates the significance of system factor and the extinction rate in reducing the loss cost of insurance companies. This result is similar to with the result showed in Fig. 3(b). The difference is the structure of patches do have influence on the probability of full indemnity (see Fig. 3(a)), but not for the probability of zero indemnity (see Fig. 5(b)).

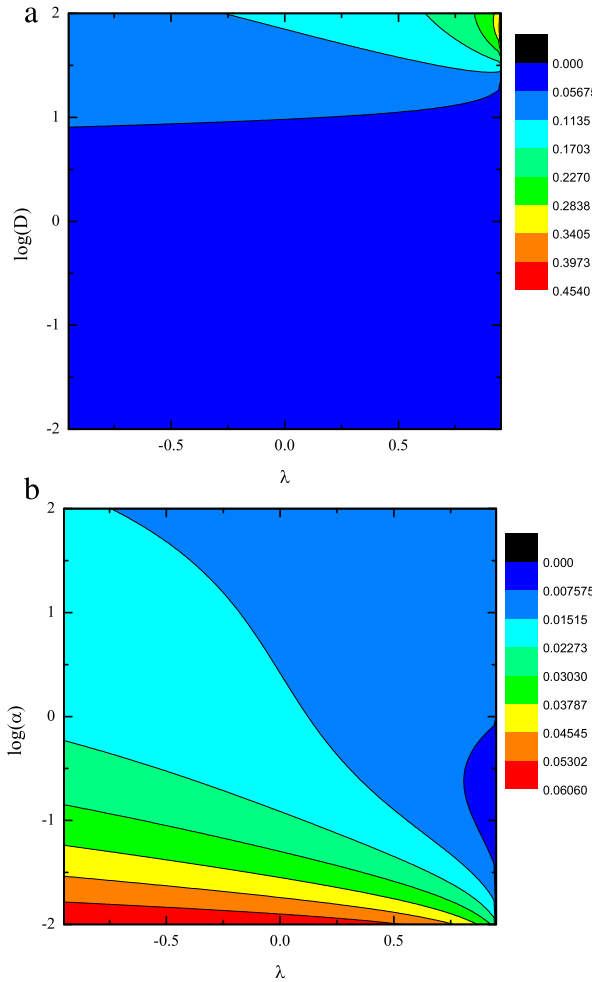


Fig. 6. The contour of the probability of zero indemnity (Eq. (14)) vs. λ and $\log(D)$ in (a) and vs. λ and $\log(\alpha)$ in (a).

Figs. 6(a) and 6(b) show how the probability of zero indemnity Eq. (14) is affected by noise intensity D and a , respectively. And cross-correlated strength λ are considered in both figures. Fig. 6(a) graphs that with the increase of both λ and D , the probability of zero indemnity increases slightly, but under a low realm. Fig. 6(b) presents that the probability of zero indemnity decreases gradually long with the increase of λ and reduce of a . What is more, when a is small, the probability of zero indemnity is much higher. It indicates for insurance companies to reduce loss cost, the additive noises and cross-correlated strength should be controlled under small numeral ranges.

4. The catastrophe loss payment time MET

From Eqs. (5) to (10), the catastrophe loss payment time MET is gained by the mean escape time [38,43]:

$$\begin{aligned}
 \text{MET}(P_0 \rightarrow P_H) &= \int_{P_0}^{P_H} \frac{dy}{B(\frac{l_H - q}{P_H}y + q)P_x(y)} \int_{P_0}^y P_x(z)dz \\
 &= \int_{P_0}^{l_H} \frac{dy}{B(l)P_{st}(y)} \int_{P_0}^y P_{st}(z)dz,
 \end{aligned}
 \tag{16}$$

here P_0 is the initial payment. Considering the zero payment with no catastrophe loss, we usually set $P_0 = 0$ as following discussion.

As explained in Section 1, mean escape time is employed in our forestry catastrophe insurance. Catastrophe loss payment time MET Eq. (16) is defined as the average time period of catastrophe loss payment occurring with respect to the variety from zero indemnity to full indemnity. Under the presumption of invariable time period of insurance payment affected

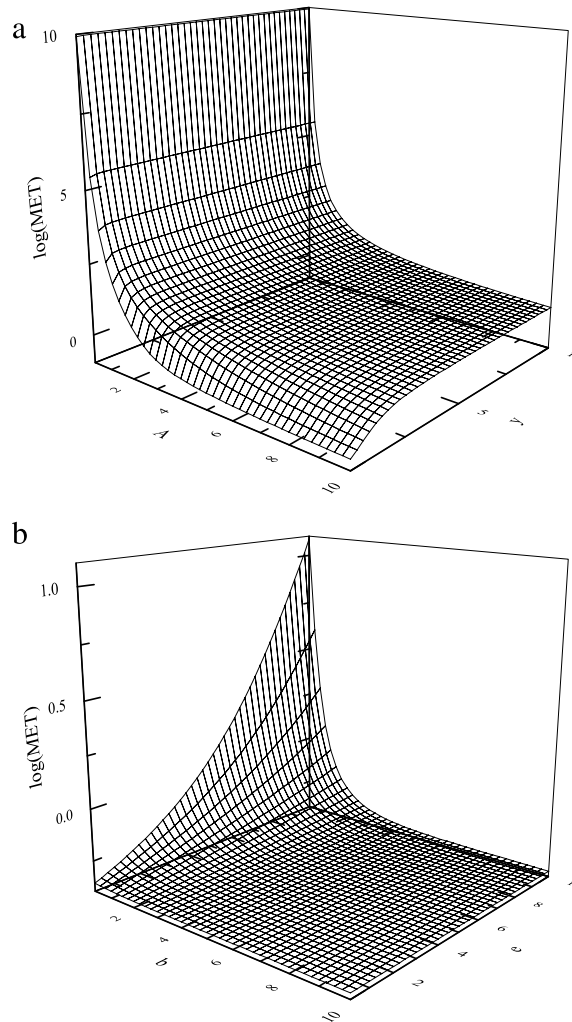


Fig. 7. The catastrophe loss payment time MET (Eq. (16)) vs. $\log(A)$ and $\log(y)$ in (a) and vs. $\log(b)$ and $\log(e)$ in (a).

by insurance companies side. In another word, this article excludes the time differences of loss payment due to insurance companies claims and quality and efficiency of service. So the catastrophe loss payment time MET is also an approximate indicator of the occurrences of forestry catastrophe risk due to forestry pest infestations and disease epidemic. How A and y , b and e affect the catastrophe loss payment time MET is graphed in Figs. 7(a) and 7(b), respectively. Experiments are conducted and Fig. 7(a) shows the influence of y on catastrophe loss payment time MET is limited, but positive. A has a negative influence on catastrophe loss payment time MET, and the influence is obvious, when A is small. Fig. 7(b) graphs system factor b influences catastrophe loss payment time MET negatively, and extinction rate has a positive influence on catastrophe loss payment time MET. When b is small and e is big, both influence increase significantly. From the perspective of forestry catastrophe insurance, the area of patches should be controlled under a small range, and with the increase of structure factor of patches and the decrease of the area of patches, a long catastrophe loss payment time MET is achieved, which would give longer time for loss cost risk management, and in addition, the capital investment and capital turnover derived from premium income are benefited from the extended catastrophe loss payment time MET.

Figs. 8(a) and 8(b) graph the influence of $\log(D)$ and $\log(a)$ on catastrophe loss payment time MET, respectively. Cross-correlated strength λ is given different reading values. Fig. 8(a) describes that higher λ leads to longer catastrophe loss payment time MET and greater noises. As an optimal control model, with the increase of D , catastrophe loss payment time MET is optimized to maximum, and then decrease monotonically, and finally all of them converge to 0. Fig. 8(b) shows that when a is small, higher λ causes longer MET of catastrophe loss payment, however with the increase of a , all MET of catastrophe loss payment are monotonically decreasing, and the convergence value is 0. Based on this experimental results, it indicates an optimal catastrophe loss payment time MET can be achieved by varying multiplicative noises and Cross-correlated strength, also, limiting additive noise is helpful for maintaining a higher catastrophe loss payment time MET. The

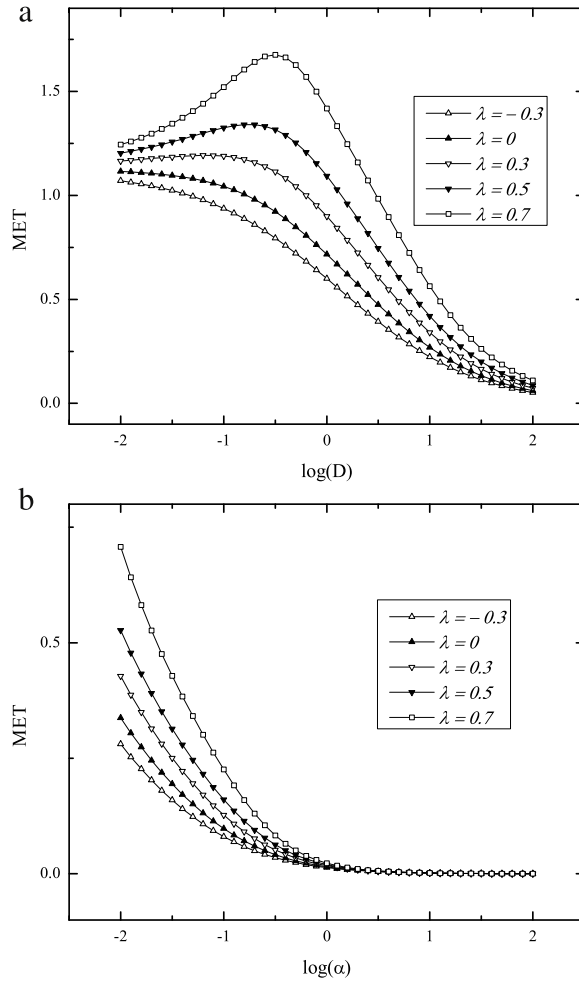


Fig. 8. The catastrophe loss payment time MET (Eq. (16)) vs. λ and $\log(D)$ in (a) and vs. λ and $\log(\alpha)$ in (a).

empirical meaning of optimal catastrophe loss payment time MET is profound. It gives insurance companies an quantified reference for premium pricing and insurance clauses designing etc., for the purpose of minimizing loss cost and maximizing premium incomes.

The results shown in Fig. 8(a) are related to the noise enhanced stability phenomenon. For the reader to understand the dynamics, we show the catastrophe loss payment time MET (Eq. (16)) vs. $\log(D)$ for the different initial conditions in Fig. 9. Obviously, the nonmonotonic behavior is also found in Fig. 9. The same as in Ref. [20], as $P_0 \rightarrow 0$, the peak of the nonmonotonic behavior increases, i.e., the noise enhanced stability phenomenon is enhanced.

In this section, we employ mean escape time to describe Catastrophe loss payment time and find some nonmonotonic behaviors related to the noise enhanced stability phenomenon. In the proposed model of Section 2, we only consider the Gaussian noise. However, the stochastic Langevin equation driven by the Lévy noise describes the stochastic system more well in some condition [44]. For some real data, the proposed model with the Lévy noise can give the exact results for PDF of loss and MET of Catastrophe loss payment. Readers can do further research on this in the future.

5. Conclusion

Various forest insects and diseases can cause tremendous losses to forests. The degree of damage is affected by forest insect infestations and disease epidemics in a dynamic way over time. A forest loss model caused by insect infestations and disease epidemics is developed in this paper. Metapopulation dynamics is used to model the forest catastrophe loss caused by forest insects and diseases. The stochastic metapopulation model used is similar to Hanski, Wahlberg and Ovaskainen's [38,39] model. Multiplicative and additive noises are considered. The proposed model is estimated with real data in China from 2005 to 2014. The PDF obtained from the sample loss data is compared with the PDF derived from the proposed model. The analytical results is shown to be a better fit than simulated results.

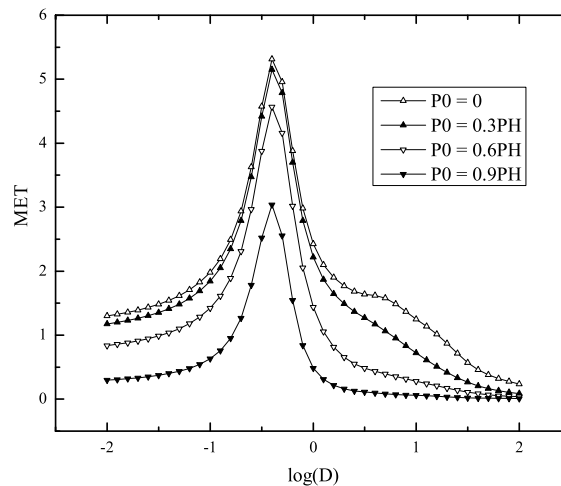


Fig. 9. The catastrophe loss payment time MET (Eq. (16)) vs. $\log(D)$ with different initial conditions P_0 . Other parameters are the same as the estimated values in Section 2.

The probability of full indemnity and the probability of zero indemnity are studied, respectively. The experimental results shows a higher probability of occurrences of catastrophe risk or catastrophe indemnity due to forestry pest infestations and disease epidemics can be triggered by the bigger area of patches and structures of patches, higher multiplicative noises and lower extinction rate. Among these ecological factors, system factor and extinction rate have greater influence on the probability of catastrophe risk or catastrophe indemnity occurring. In comparison, the same ecological factors, multiplicative and additive noises are employed in the probability of zero indemnity shows similar results. How multiplicative and additive noises affect the probability of full indemnity and the probability of zero indemnity are discussed separately in the contours. It implies that multiplicative noises correlates to the probability of occurrences of catastrophe risk or catastrophe indemnity positively with the increase of the cross-correlated strength, and the influence of multiplicative noises is greater than the additive noises. The additive noises only produce effect at a high range of cross-correlated strength value. From the perspective of forestry catastrophe insurance, outbreaks of forestry catastrophe loss or catastrophe loss payment due to forestry pest infestations and disease epidemics can be recognized from the following aspects: big area of patches and structure of patches, high system factor, low extinction rate, high multiplicative and additive noises with a high cross-correlated strength range, where insurance companies should take special notice.

Resonance of average time period of forestry catastrophe loss payment outbreak is analyzed by using the mean escape time in this study. It can be observed that as patch size becomes larger, and the structure factor of patches become smaller, the catastrophe loss payment time MET decreases monotonically. It also shows with the increase of extinction rate and decrease of system factor, the catastrophe loss payment time MET increases dramatically. Experimental results suggest that a long catastrophe loss payment time MET could be achieved by small patch size and big structure factor of patches, high extinction rate and long system factor. Moreover, study of correlations of the catastrophe loss payment time MET with multiplicative and additive noises are conducted when taking various value of λ (from negatively correlated to positively correlated). That is, with the increase of D , the catastrophe loss payment time MET grows from increase to decrease. Meanwhile a maximum catastrophe loss payment time MET is optimized with proper value of D and value of λ . The increase of value of λ does not affect the monotonic decrease trend of catastrophe loss payment time MET. The results shows it is possible to identify an optimal catastrophe loss payment time MET by taking proper value of D and limits a on a low range of value, and λ at a high range of value. The optimal value is helpful for the characterization of catastrophe risk and the appraisal of catastrophe loss due to forestry pest infestations and disease epidemics. Taking it as a meaningful reference, insurance companies could eventually realize the goal of loss cost minimizing and premium incomes maximizing.

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