Precise spectrum reconstruction of the Fourier transforms imaging spectrometer based on polarization beam splitters

Wenyi Ren,[1,2,3] Chunmin Zhang,[1,2,*] Chenling Jia,[1,2] Tingkui Mu,[1,2] Qiwei Li,[1,2] and Lin Zhang[1,2]

[1]School of Science, Xi'an Jiaotong University, Xi'an 710049, China

[2]MOE Key Laboratory for Non-Equilibrium Synthesis and Modulation of Condensed Matter, Xi'an Jiaotong University, Xi'an 710049, China

[3]Xi'an Institute of Optics and Precision Mechanics, Chinese Academy of Science (CAS), Xi'an 710119, China

*Corresponding author: zcm@mail.xjtu.edu.cn

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A method was proposed to precisely reconstruct the spectrum from the interferogram taken by the Fourier transform imaging spectrometer (FTIS) based on the polarization beam splitters. Taken the FTISs based on the Savart polariscope and Wollaston prism as examples, the distorted spectrums were corrected via the proposed method effectively. The feasibility of the method was verified via simulation. The distorted spectrum, recovered from the interferogram taken by the polarization imaging spectrometer developed by us, was corrected.

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Fourier transform imaging spectroscopy, based on the two beam interference phenomenon, is a powerful spectroscopic technique in the ultraviolet, visible, and infrared regions [1,2]. There are several approaches for the Fourier transform imaging spectrometer (FTIS), such as modified Michelson interferometer, modified Sagnac interferometer, liquid crystal technology, and the schemes based on the polarization beam splitters, such as the Wollaston prism (WP), Savart polariscope (SP), etc. [3–10]. The spectrum theoretically can be reconstructed from the interferogram via inverse Fourier transform (IFT) [11]. During the data processing, it is necessary to correct the errors introduced by the dispersion of the beam splitters [12–14]. A reference light should be pre-detected in the correction methods proposed in [15–17]. A method to the moving-optical-wedge Fourier transform spectrometer (FTS), which does not need reference light, is proposed in [18]. In this Letter, a generalized method is developed for the precise spectrum reconstruction of the FTIS based on the polarization beam splitters.

According to the Fourier transform spectroscopy [11], the recorded interferogram $I_R(\Delta)$ for an ideal FTS is

$$I_R(\Delta) = \frac{1}{2}\int_0^{+\infty} B(\sigma)[1 + \cos(2\pi\sigma\Delta)]\mathrm{d}\sigma, \tag{1}$$

where Δ is the optical path difference (OPD), σ is the wavenumber, $B(\sigma)$ is the spectrum. If we subtract the constant term in Eq. (1) and extend $B(\sigma)$ to include negative frequencies by defining $B(-\sigma) = B(\sigma)$, we get

$$I(\Delta) = \frac{1}{4}\int_{-\infty}^{+\infty} B(\sigma)\exp(-j2\pi\sigma\Delta)\mathrm{d}\sigma. \tag{2}$$

The OPD is a function of the spatial coordinate x and wavenumber σ [12–14]. Generally, the sampling interval of the OPD is $\delta\Delta = 1/(\sigma_{\max} - \sigma_{\min})$ [11]. $\delta\Delta$ also can be given by the channel number N as $\Delta_{\max}(\sigma_{\max})/N$. Due to the dispersion effect of birefringent material, the sampling intervals vary with the wavenumber. The traditional Fourier transform is only suitable for the uniform sampled signal. Thereby, a distorted spectrum will be obtained via IFT.

If we denote $\Delta(\sigma_{\max})$ as L, $\Delta(\sigma)$ can be given as

$$\Delta(\sigma) = \kappa(\sigma)L, \tag{3}$$

where $\kappa(\sigma)$ is the ratio of the OPD at σ to $\sigma_{\max}$. Inspired by the method proposed in [14], Eq. (2) can be rewritten as

$$I(L) = \frac{1}{4}\int_{-\infty}^{+\infty} B(\sigma)\exp(-j2\pi\sigma\kappa(\sigma)L)\mathrm{d}\sigma. \tag{4}$$

The distorted spectrum B_D can be gotten via the IFT of $I(L)$ as

$$\begin{aligned} B_D(u) &= 4\int_{-\infty}^{+\infty} I(L)\exp(j2\pi uL)\mathrm{d}L \\ &= 4\int_{-\infty}^{+\infty}\frac{1}{4}\int_{-\infty}^{+\infty} B(\sigma)\exp(-j2\pi\sigma\kappa(\sigma)L)\mathrm{d}\sigma \\ &\quad\times\exp(j2\pi uL)\mathrm{d}L \\ &= \int_{-\infty}^{+\infty} B(\sigma)\mathrm{d}\sigma\delta[\sigma\kappa(\sigma) - u]. \end{aligned} \tag{5}$$

It is assumed that $\tau = \sigma\kappa(\sigma)$, $\sigma = g(\tau)$ and $d\sigma = g'(\tau)d\tau$, Eq. (5) can be rewritten as

$$B_D(u) = \int_{-\infty}^{+\infty} B[g(\tau)]g'(\tau)\delta(\tau - u)\mathrm{d}\tau = B[g(u)]g'(u), \tag{6}$$

$$g'(u) = \frac{d\sigma}{du} = \frac{d\sigma}{\kappa(\sigma)d\sigma + \sigma\kappa'(\sigma)d\sigma} = \frac{1}{\kappa(\sigma) + \sigma\kappa'(\sigma)}. \tag{7}$$

Based on Eqs. (4)–(7), it can be obtained that

$$B(\sigma) = B_D(u)/g'(u) = [\kappa(\sigma) + \sigma\kappa'(\sigma)]B_D(u),$$
$$\sigma = u/\kappa(\sigma). \tag{8}$$

According to Eq. (8), the amplitude and wavenumber of the spectrum are rescaled by the factors $\kappa(\sigma) + \sigma\kappa'(\sigma)$ and $\kappa(\sigma)$. The distorted spectrum can be corrected based on Eq. (8). $\kappa(\sigma)$ can be obtained via Eq. (3) provided that the OPD of the FTIS is known. If there is no dispersion, $\kappa(\sigma)$ is 1 constantly. The spectrum is without distortion.

For instance, the OPDs of FTISs based on SP and WP are [19]

$$\Delta_{\text{Savart}}(x, \sigma) = \sqrt{2}\frac{n_o^2(\sigma) - n_e^2(\sigma)}{n_o^2(\sigma) + n_e^2(\sigma)}\frac{x}{f}t, \tag{9}$$

$$\Delta_{\text{Wollaston}}(x, \sigma) = 2x[n_o(\sigma) - n_e(\sigma)]\tan\theta, \tag{10}$$

where $n_o(\sigma)$ and $n_e(\sigma)$ are the refractive indices of the birefringent crystal, t is the thickness of a single Savart plate, x is the spatial dimension, f is the focal length of imaging lens, θ is the apex angle of the prism. Then, it can be obtained that

$$\kappa_{\text{Savart}}(\sigma) = \frac{n_o^2(\sigma_{\max}) + n_e^2(\sigma_{\max})}{n_o^2(\sigma_{\max}) - n_e^2(\sigma_{\max})}\frac{n_o^2(\sigma) - n_e^2(\sigma)}{n_o^2(\sigma) + n_e^2(\sigma)}, \tag{11}$$

$$\kappa_{\text{Wollaston}}(\sigma) = \frac{n_o(\sigma) - n_e(\sigma)}{n_o(\sigma_{\max}) - n_e(\sigma_{\max})}. \tag{12}$$

In this Letter, this method shown in Eq. (8) is utilized to reconstruct the spectrum precisely from the interferogram taken by the polarization interference imaging spectrometer (PIIS) [20,21] as shown in Fig. 1. Lenses L_1 and L_2 compose a pretelescope. P_1 and P_2 are the polarizer and analyzer. An incident beam is split into two beams via SP. The two beams interference and image on CCD, which is located on the back focal plane of the imaging lens L_3. The OPD between the two beams is given by Eq. (9).

As shown in Fig. 2, we have simulated an interferogram based on the PIIS with the specification as: the wavelength range is from 480 to 960 nm, the channel number N is 256, the spectral resolution $\delta\sigma$ is 40.7 cm^{-1}, the thickness of a single Savart plate is 12 mm.

As proposed in [22], OPD linearly increases with the incident angle while the incident angle is less than 5°. The maximal OPD, $\Delta_{\max}(\sigma)$, is obtained at the maximal incident angle. The maximal OPD of the system is obtained at $\sigma_{\max}$.

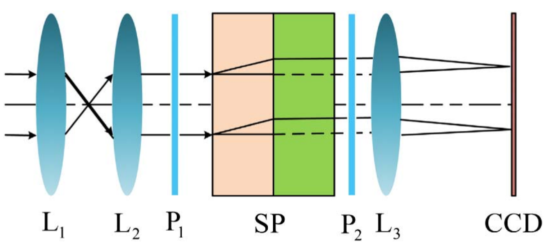

Fig. 1. (Color online) Schematic of the FTIS based on SP.

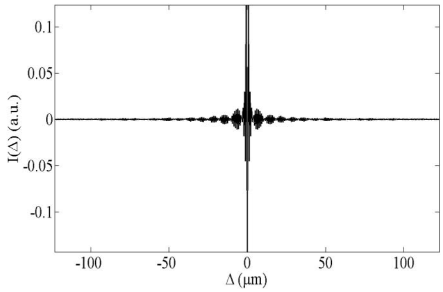

Fig. 2. Simulated interferogram.

As shown in Fig. 3(a), $\kappa(\sigma)$ is obtained based on Eq. (3). Herein, the OPD at σ_i is given by

$$\Delta(\sigma_i) = \kappa(\sigma_i)L, \quad i = 1, \cdots, N, \tag{13}$$

where σ_i and $\delta\sigma$, respectively, are given by $\sigma_i = \sigma_{\min} + \delta\sigma(i-1)$ and $(\sigma_{\max} - \sigma_{\min})/N$. Based on Eq. (8), the wavenumber of the distorted spectrum is

$$u_i = \kappa(\sigma_i)\sigma_i. \tag{14}$$

The original and distorted wavenumbers are plotted as the curves shown in Fig. 3(b). Obviously, u is smaller than σ when σ is smaller than $\sigma_{\max}$. The wavenumber range is extended toward the small wavenumber.

As shown in Fig. 4, the distorted spectrum was obtained via IFT. The location has a shift toward the small wavenumber (the long wavelength). It is consistent with

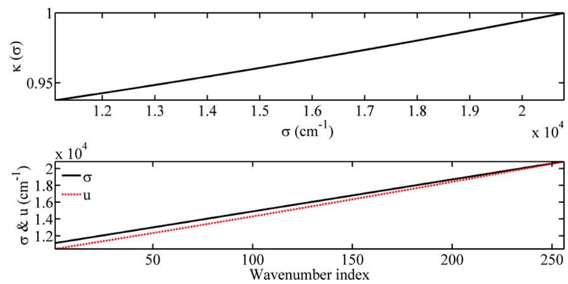

Fig. 3. (Color online) (a) $\kappa(\sigma)$ and (b) the original (the black solid curve) and distorted (the red dashed curve) wavenumbers.

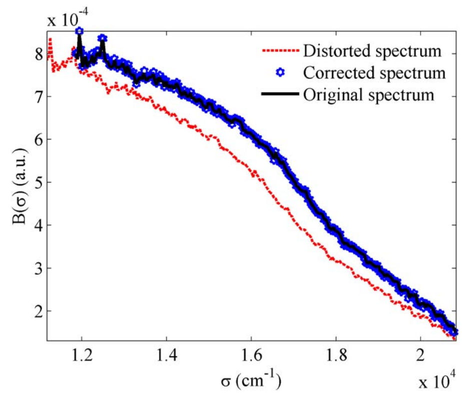

Fig. 4. (Color online) Corrected result of the FTIS based on SP. The black solid, red and blue curves, respectively, are the original, distorted, and corrected spectrums.

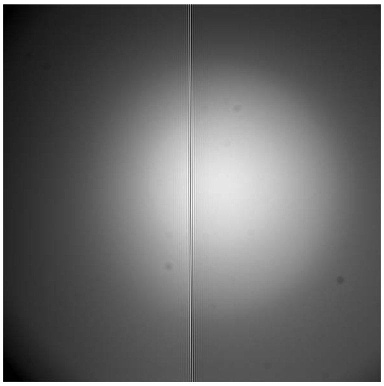

Fig. 5. Interferogram taken by the PIIS.

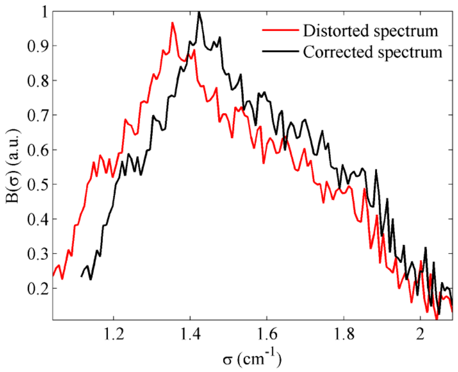

Fig. 6. (Color online) Corrected result of PIIS.

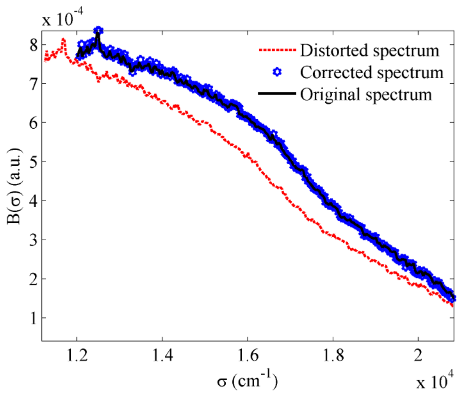

Fig. 7. (Color online) Corrected result of the FTIS based on WP.

the shift of the distortion wavenumber as shown in Fig. 3(b). The amplitude of the reconstructed spectrum is also changed and lower than the original spectrum. According to Eq. (8), the distorted spectrum was corrected. The corrected spectrum is agreement with the original spectrum very well.

As shown in Fig. 5, a polychromatic interferogram was taken by the PIIS based on SP developed by our group [13,20,21]. Via IFT, a distorted spectrum was obtained as shown in Fig. 6. A corrected spectrum was obtained by the proposed method.

The feasibility of the method for the FTIS based on WP was also verified via simulation. The corrected result is shown in Fig. 7. The spectrum was corrected effectively.

In conclusion, we have proposed a precise spectrum reconstruction method for the FTIS based on the polarization beam splitters. Taken the FTIS based on SP and WP as examples, the distorted spectrums were corrected effectively. The feasibility of the method was verified through simulation. A spectrum reconstructed from the experimental interferogram was corrected.

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