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One Atomic Beam as a Detector of Classical Harmonic Vibrations with Micro Amplitudes and Low Frequencies

Yong-Yi HUANG*

MOE Key Laboratory for Nonequilibrum Synthesis and Modulation of Condensed Matter and Department of Optic Information Science and Technology, Xi'an Jiaotong University, Xi'an 710049, China

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We propose a simplest detector of harmonic vibrations with micro amplitudes and low frequencies, *i.e.* a detector consisting of one atomic beam. Here, the atomic beam is induced by a plane harmonic wave and has classical collective harmonic vibrations, whose vibrant directions are perpendicular to the wave vectors of the atomic beam. Compared with a detector consisting of an atomic Mach-Zehnder interferometer, the new detector has two advantages: (1) it is suitable for the detection of harmonic vibrations induced either by a longitudinal plane harmonic wave or by using a transverse plane harmonic wave: (2) the quantum noise fluctuation of the atomic beam is exactly zero. We present the principle for detecting classical harmonic vibrations with micro amplitudes and low frequencies by using the new detector. The frequency of a classical harmonic vibration with a micro amplitude can be evaluated by measuring the variations in the mean numbers of atoms arriving at the atomic detector.

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I. INTRODUCTION

Gravitational wave is an extremely weak wave, and it is the only unconfirmed prediction in general relativity. A plane gravitational wave with two polarization states travelling in the z direction will deform the particles around a circle in the xy plane [1], so a gravitational wave can be confirmed by measuring the oscillatory motions of particles hit by a plane gravitational wave [2], Unfortunately, this idea has not been accepted. The observations of period variations of PSR1913+16 by Hulse and Taylor in 1974 [3] indirectly confirm the existence of gravitational waves. An atom interferometer has also been designed for the detection of a gravitational wave [4, 5]. Classical harmonic vibrations are simpler, but more fundamental, than the oscillatory motions induced by a plane gravitational wave with two polarization states. The effects of an atomic beam collective classical transverse harmonic vibrations with micro amplitudes and low frequencies on the mean numbers of atoms arriving at the detectors in an atomic Mach-Zehnder interferometer have been investigated [6], and collective classical harmonic vibrations can be detected by measuring the variations of the mean numbers of atoms arriving at the detectors. However, two problems have not yet been fully solved in the detector consisting of an atomic Mach-Zehnder interferometer: (1) it does not exactly detect the classical harmonic vibrations induced by a transverse plane harmonic wave; (2) there are always quantum noise fluctuations of atomic branches. In this paper, we propose a very simple detector consisting of one atomic beam to detect classical harmonic vibrations with micro amplitudes and low frequencies. Although the detector is simple, it is suitable for detecting the harmonic vibrations induced either by a longitudinal plane harmonic wave or by a transverse plane harmonic wave, and the quantum noise fluctuation of the atomic beam is exactly zero. The paper is organized as follows. In Section II, we propose the vibrant factor F for the new detector consisting of one atomic beam. In Section III, we present the principle for detecting classical harmonic vibrations with micro amplitudes and low frequencies by using the new detector consisting of one atomic beam. In Section IV, we discuss the quantum noise fluctuation in the new detector. In Section V, we give a brief summary.

II. VIBRANT FACTOR F FOR THE NEW DETECTOR CONSISTING OF ONE ATOMIC BEAM

Our thought experiment is shown in Fig. 1 with an atomic oven, a collimator, an atomic beam and an atomic detector. The collimated atomic beam is induced by a

*E-mail: yyhuang@mail.xjtu.edu.cn

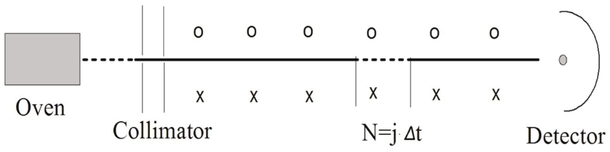

Fig. 1. Schematic of the harmonic vibration detector consisting of one atomic beam, where an atomic beam with certain velocity is produced from an oven with a stable temperature. The number of atoms arriving at the detector is measured by using a detector after the atomic beam has passed through the collimator. The crosses x (perpendicular to the paper's plane to the inside) and empty circles o (perpendicular to the paper's plane out) of the atomic beam denote the collective classical harmonic vibrations induced either by a longitudinal plane harmonic wave or by a transverse plane harmonic wave. The atomic beam is on the same wave surface and has the same phase. The collective vibrant directions are perpendicular to the wave vectors of the beam.

plane harmonic wave and is in the same wave front. Just like a detector consisting of a Mach-Zehnder interferometer [6], our new detector consisting of one atomic beam can also be used to detect the classical harmonic vibrations induced by a longitudinal plane harmonic wave. As shown in Fig. 1, our experimental setup is in the paper's plane, a longitudinal plane harmonic wave perpendicular to the paper's plane hits the atomic beam, the collective harmonic vibrations direction should be perpendicular to the paper's plane and perpendicular to the wave vector of the atomic beam. If the harmonic vibrations are induced by a transverse plane harmonic wave, the detector consisting of an atomic Mach-Zehnder interferometer can approximately be used to measure the harmonic vibrant frequencies; however, our new detector consisting of one atomic beam can exactly measure the harmonic vibrant frequencies. We let a transverse plane harmonic wave in the paper's plane be perpendicular to the atomic beam and travel from top to bottom, and vice versa, so that the collective harmonic vibrations direction is perpendicular to the wave vectors of the atomic beam. In the following, we will discuss the measurement process. One condition should be satisfied; *i.e.*, the collective harmonic vibrant directions are perpendicular to the atomic beam vectors so that the transverse collective vibrations do not couple with translational motions.

The atomic beam during the measurement time Δt is induced by a plane harmonic wave, which makes the atomic beam collectively vibrate perpendicular to the atomic wave vectors. The collective vibrations of the beam are fully decoupled from the translational motions of the beam. Given that the flux of the atomic beam is j (the number of atoms per second passing through a plane), the number of atoms in the beam during a measurement time Δt is $N = j \cdot \Delta t$. The series of N atoms are our research objects. We assume that the measurement time Δt is much smaller than the harmonic vibration period and that the efficiency of the atomic detector is unity. Let $|1>_i$ denote the ith atom within the series of N atoms incident on the detector. Following Ref. 7 the state vector of the atomic beam with atom number N is constructed by using a direct product of the individual atomic states, *i.e.*,

$$|\Phi>_N \equiv \prod_{i=1}^{N} |1>_i . \tag{1}$$

We now know that the atomic beam has a transverse collective harmonic vibration with the same phase as that induced by some plane harmonic wave and that the vibrant directions are perpendicular to the wave vectors of the atomic beam, so the collective vibrations do not couple with translational motions. The Hamiltonian of an atomic beam with the number of atoms N is given by

$$H = \hbar\Omega(A^\dagger A + 1/2) + \varepsilon_k \sum_{i=1}^{N} c_i^\dagger c_i. \tag{2}$$

In Eq. (2), $A^\dagger$ and A are, respectively, the creation and the annihilation operators of the transverse harmonic vibration, Ω is the angle frequency of the transverse harmonic vibration and is also the frequency of the plane harmonic wave, $c_i^\dagger$ and c_i are the creation and annihilation operators for the ith atom state $|1>_i$, respectively, ε_k is the kinetic energy of each atom within the series of N atoms. The time-dependent state vector of the atomic beam is written as

$$\begin{aligned} |\Psi(t)>_N &= |n(t)> \otimes e^{-iN\varepsilon_k t/\hbar} |\Phi>_N \\ &= |n(t)> \otimes e^{-iN\varepsilon_k t/\hbar} \prod_{i=1}^{N} |1>_i . \end{aligned} \tag{3}$$

In Eq. (3), $|n(t)>= e^{-iE_n t/\hbar}|n>$ is the eigenstate vector of the atomic beam's harmonic vibration, and the corresponding eigen energy is $E_n = (n + 1/2)\hbar\Omega$ with the large quantum number n. The frequency Ω of the transverse harmonic vibration is so low that the behaviors of the quantum harmonic vibration can be regarded as those of a classical harmonic oscillator.

We have the number operators $n_i = c_i^\dagger c_i$, where the eigenvalue n is 1. The operator c obeys the commutation relationships $c_i c_j^\dagger \pm c_j^\dagger c_i = \delta_{ij}$, where the minus or plus sign indicates Bose or Fermi statistics. However, the statistics are neglected in this article because we assume that the density of the atomic beam is not large, and that there is one atom at a time within a single coherence length. Actually, for an atomic beam, we roughly define the de Broglie wavelength as the single coherence length of an atom. The typical atom density range for high-temperature metal atomic beams is from 10^8 up to 10^{13} atoms/cm^3 , and the upper limit is determined by the capability of the source [8]. For instance, for a silver beam with 10^{13} atoms/cm^3 from a 1000 K oven, we evaluate the distance between two silver atoms to be about 0.5 μm, which is much larger than the de Broglie wavelength of silver (about 0.01nm). Thus we may neglect Bose or Fermi statistics of the atoms in our atomic beam.

The number operator N of the atomic beam is given by

$$N = \sum_{i=1}^{N} n_i. \tag{4}$$

The expectation value $< N >_N$ of this number operator is written as

$$\begin{aligned} < N >_N &= _N< \Psi(t)|N|\Psi(t) >_N \\ &= _N< \Phi|\sum_{i=1}^{N} n_i|\Phi >_N \cdot \int_{\Delta t} < n(t)|x >< x|n(t) > dx \\ &= \sum_{i=1}^{N} {}_i < 1|n_i|1 >_i \cdot \int_{\Delta t} < n|x >< x|n > dx \\ &= N \int_{\Delta t} < n|x >< x|n > dx \end{aligned} \tag{5}$$

where $N = j \cdot \Delta t$ has been declared above. Because the measurement time Δt is much smaller than the vibration period $T = 2\pi/\Omega$, the mean number of atoms in the detector has a correction $F \equiv \int_{\Delta t} < n|x >< x|n > dx$. Here, we define the corrected coefficient F of the mean number of atoms arriving at the detector as the vibrant factor F because it comes from the collective vibrations induced by the plane harmonic wave. The vibrant factor F quantitatively describes the effects of the collective harmonic vibrations on the mean number of atoms in the detector and includes the frequency of the collective harmonic vibration. From the vibrant factor F, it becomes possible to confirm the existence of harmonic vibrations, especially vibrations with micro amplitudes, by measuring variations in the mean number of atoms arriving at the detector.

Under the condition of low frequency, the probability density of a quantum oscillator with a large quantum number quickly oscillates around the probability density of a classical oscillator. When we need to obtain the probability of the classical oscillator, the probability density of a quantum oscillator $< n|x >< x|n >$ with a large quantum number n may be equivalent to the probability density of a classical harmonic oscillator. Given $\alpha = \sqrt{M\Omega/\hbar}$ and $\xi = \alpha x$, we obtain the classical vibrant equation of the atomic beam $\xi = \sqrt{2n+1}\sin(\Omega t + \delta)$, where n is a large quantum number, δ is the initial phase of the atomic beam, and $\sqrt{2n+1}$ denotes the absolute amplitude. The classical harmonic oscillator's probability density is $w(\xi) = \frac{1}{\pi\sqrt{(2n+1)-\xi^2}}$, and $w(\xi)$ increases with increasing displacement ξ in $\xi \in [0, \sqrt{2n+1}]$. We do not consider the $\xi \in [-\sqrt{2n+1}, 0]$ region because the probability density is an even function of the reduced position ξ in $[-\sqrt{2n+1}, \sqrt{2n+1}]$ and is symmetric in both $[-\sqrt{2n+1}, 0]$ and $[0, \sqrt{2n+1}]$, as seen from the probability density $w(\xi) = \frac{1}{\pi\sqrt{(2n+1)-\xi^2}}$. During a short measurement time Δt, we conveniently use the probability density expression of the classical harmonic oscillator rather than the probability density of a quantum harmonic oscillator and obtain

$$\begin{aligned} F(\delta) &= \int_{\xi_0}^{\xi_0+\zeta} w(\xi) d\xi \\ &= \frac{1}{\pi}[\arcsin(\frac{\zeta}{\sqrt{2n+1}} + \sin\delta) - \delta], \end{aligned} \tag{6}$$

where ζ denotes the absolute displacement of the classical vibration during the measurement time Δt, and δ is the initial phase, *i.e.*, $\xi_0 = \sqrt{2n+1}\sin\delta$, with ξ_0 being the initial displacement at the initial time $t = 0$. Equation (6) is the fundamental formula in this article and was firstly derived in Ref. 9. It is the condition that the measurement time $\Delta t \ll T$ makes the integral's upper limit and lower limit in Eq. (6) be $[\xi_0, \xi_0 + \zeta]$ rather than $(-\infty, +\infty)$ and the relative displacement be small, *i.e.*, $0 < \zeta/\sqrt{2n+1} \ll 1$.

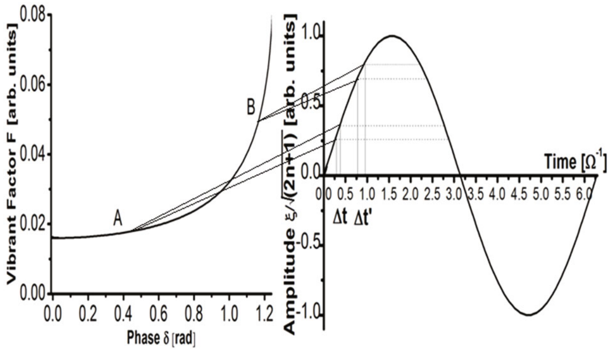

Fig. 2. Vibrant factor F versus the initial phase δ with the relative displacement $\zeta/\sqrt{2n+1} = 0.05$ (the left panel). The right panel corresponding to the vibrant factor F is the time-varying harmonic vibrant displacement with the same frequency Ω, reproduced from Ref. 6. A dot in the left panel corresponds to a measurement time Δt with an initial phase δ, and it takes different measurement times Δt at different phases δ to keep the relative displacement $\zeta/\sqrt{2n+1}$ constant. For instance, dot A and dot B in the left panel have different initial phases and correspond to different displacements in the right panel. If the relative displacement $\zeta/\sqrt{2n+1}$ is kept constant (the measurement time in dot A is less than the time in dot B, *i.e.*, $\Delta t < \Delta t'$), and the vibrant factor F(A) is less than F(B) because of the probability in dot A being less than the probability in dot B as seen from Eq. (6).

The factor F versus the initial phase δ is shown in Fig. 2 for the relative displacement $\zeta/\sqrt{2n+1} = 0.05$. Here, $\zeta/\sqrt{2n+1} = 0.05$ is not a particular choice, but is just an example; the maximum of the relative displacement is unity. Actually, we study the vibrant factor F during one period T and require the measurement time Δt be less than one period T. We see, from Fig. 2 and Eq. (6), (1) the vibrant factor F depends on the relative vibrant displacement $\zeta/\sqrt{2n+1}$ and the initial phase δ rather than the absolute vibrant amplitude $\sqrt{2n+1}$, and (2) the vibrant factor F increases with increasing initial phase δ. The reason is that different initial phases δ correspond to different displacements ξ, and the probability density $w(\xi)$ increases with increasing displacement ξ.

During the measurement time $\Delta t \ll T$, the relative displacement $\zeta/\sqrt{2n+1}$ is much smaller than unity, and the integral's upper limit and lower limit in Eq. (6) is $[\xi_0, \xi_0+\zeta]$ rather than $(-\infty, +\infty)$, so the vibrant factor F is less than unity. What's more, the vibrant factor F reduces the mean number of atoms in the detector by two orders of magnitude. For instance, given $\frac{\zeta}{\sqrt{2n+1}}=0.05$, we have $\sin\delta = 0.95$, *i.e.*, $\delta \approx 1.25$ maximally from the fact $\frac{\zeta}{\sqrt{2n+1}} + \sin\delta = 1$. Substituting $\delta \approx 1.25$ into $F(\delta)$, we find that the $F(\delta)$ maximum is about 0.1. If $\frac{\zeta}{\sqrt{2n+1}} = 1$ is satisfied in Eq. (6), we have $\sin\delta = 0$. The maximal value of $F(\delta)$ is 0.5, which corresponds to one-half period. In a whole period, we have $F(\delta) = 1$; however, this is a trivial result because we can't extract useful information.

III. DETECTION PRINCIPLE FOR CLASSICAL HARMONIC VIBRATIONS WITH MICRO AMPLITUDES AND LOW FREQUENCIES WITH THE NEW DETECTOR CONSISTING OF ONE ATOMIC BEAM

Given $\zeta/\sqrt{2n+1} = 0.05$ in the detection process for the number of atoms, the mean number of atoms arriving at the detector is given by

$$_N < \Psi|N|\Psi >_N = N \cdot F(\delta), \tag{7}$$

where $F(\delta) = \frac{1}{\pi}[\arcsin(0.05+\sin\delta) - \delta]$ and $N = j \cdot \Delta t$. As seen from Fig. 2, the larger the initial phase δ becomes, the longer the measurement time Δt is needed to keep the same relative displacement $\zeta/\sqrt{2n+1}$ constant. Maybe, it is difficult and not practical to use this method to measure the mean number of atoms arriving at the detector during a short measurement time Δt; however, theoretically once the left curve for F versus the phase δ in Fig. 2 is obtained, the classical collective harmonic vibrations of the atomic beam can be verified. How does one obtain the vibrant factor F ? Without the collective classical harmonic vibrations, the mean number of atoms arriving at the detector is written as

$$< \Phi|\sum_{i=1}^{N} n_i|\Phi >_N = \sum_{i=1}^{N} {}_i < 1|n_i|1 >_i = N. \tag{8}$$

Obviously we can get the curve of the vibrant factor F after comparing the modified results from Eq. (7) with those from Eq. (8).

Another practical measurement of the mean number of atoms is an equal-time-interval measurement; *i.e.*, the measurement time Δt is kept constant. As shown in Fig. 2, a relative vibrant displacement $\zeta/\sqrt{2n+1}$ contains two to-and-fro processes in one-half period, so we need to divide the vibrant factor $F(\delta)$ in Eq. (6) by 2 in a practical measurement process. If we let the measurement time Δt be constant, we surprisingly obtain $\frac{F(\delta)}{2} = \frac{\Delta t}{T}$, independent of the initial phase δ. The mean number of atoms arriving at the detector is given by

$$_N < \Psi|N|\Psi >_N = j \cdot \Delta t \cdot \frac{\Delta t}{T}. \tag{9}$$

As for the multi-period measurement, we have to replace Δt by $m\Delta t$ and T by mT, where m is the number of periods. As seen from Eq. (9), the vibrant factor's corrections are invariant and independent of the number of measurement periods. Obviously we can evaluate the frequency T of the atomic beam after comparing the modified results of Eq. (9) with those of Eq. (8).

IV. QUANTUM NOISE FLUCTUATION OF THE ATOMIC BEAM

Without considering the harmonic vibrations of the two atomic branches, when the beams intensity is so low that there is only one atom at a time within a single coherence length, the quantum noise fluctuations in the detector consisting of an atomic Mach-Zehnder interferometer are given by [7]

$$< \Delta N_{A,B} >_0 = \frac{\sqrt{N}}{2} \sin\varphi_{\alpha\beta}, \tag{10}$$

where we have $N = j \cdot \Delta t$ and $\varphi_{\alpha\beta} \equiv k(l_\alpha - l_\beta)$, with k being the atomic wave vector, and l_α and l_β being the path lengths through the α and β branches, respectively. Given $\varphi_{\alpha\beta} \neq 0$, quantum noise fluctuations always exist in the two detectors of a Mach-Zehnder interferometer.

If the quantum noise fluctuations of the beams are larger than the number of atoms arriving at the detector during a small measurement time Δt, the number of atoms arriving at the detector will not be stable, so measuring the frequency of the beams accurately would be difficult. The thought setup needs a stable beam with very small, even zero, quantum noise fluctuations, which is very difficult to realize in a detector consisting of an atomic Mach-Zehnder interferometer. We will illustrate that there is exactly zero quantum noise fluctuation in the new detector consisting of one atomic beam. Without the classical harmonic vibrations, the quantum noise fluctuation in the beam is given by

$$
\begin{aligned}
< \Delta N^2 > &=< \Phi|N^2|\Phi >_N -(< \Phi|N|\Phi >_N)^2 \\
&=< \Phi|\sum_{i=1}^{N} n_i \sum_{i=j}^{N} n_j|\Phi >_N -(< \Phi|\sum_{i=1}^{N} n_i|\Phi >)^2 \\
&= \sum_{i=1}^{N} < 1|n_i|1 >_i \sum_{j=1,j\neq i}^{N} < 1|n_j|1 >_j \\
&\quad + \sum_{i=1}^{N} < 1|n_i^2|1 >_i -N^2 \\
&= N(N-1) + N - N^2 = 0. \qquad (11)
\end{aligned}
$$

Physically there is not a quantum noise fluctuation in one atomic beam because the atoms have only one way to arrive at the atomic detector, and there are not 'which way' choices of the atoms in one atomic beam. Because of the existence of zero quantum noise fluctuation of the atomic beam, we do not consider the effects of quantum noise fluctuations on the number of atoms arriving at the detector. Thus there are not limits on the measurement time Δt. Due to the quantum noise fluctuations in the detector consisting of an atomic Mach-Zehnder interferometer, the measured number of atoms arriving at a detector during a very small time Δt may be submerged in the quantum noise fluctuations of the branches.

V. SUMMARY

In conclusion, the simplest detector for classical harmonic vibrations with micro amplitudes and low frequencies, *i.e.*, a detector consisting of one atomic beam is proposed. The new detector has two advantages: (1) it is suitable for the detection of classical harmonic vibrations induced either by a longitudinal plane harmonic wave or by a transverse plane harmonic wave; (2) the quantum noise fluctuation of the atomic beam is exactly zero. The new detector is simpler, but more practical, than the detector consisting of an atomic Mach-Zehnder interferometer. In an equal-time-interval measurement the vibrant factor F is independent both of the absolute amplitude and of the initial phase, as seen from Eq. (9), so a classical harmonic vibration is always detected by measuring the mean variation in the number of atoms arriving at the atomic detector, no matter how small the vibrant amplitude is. The present results may provide a new principle for detecting a gravitational wave.

What about a realistic detector for classical harmonic vibrations? The detection efficiency for atoms should be very high; close to unity is best. If the thought setup is further improved to detect vibrations induced by a gravitational wave, we speculate that the setup will not be very large and that it should be in vacuum and be laid in a satellite to receive the vibrations induced by a gravitational wave.

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