Journal of the Korean Physical Society, Vol. 68, No. 8, April 2016, pp. 944∼948

Planet Versions of the State of Schrödinger's Cat Prepared by Using Quantum Measurements

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(Received 11 February 2016, in final form 24 March 2016)

A classical charged particle travels along a circular orbit and interacts with a silver atom in its ground state fixed at the center of the circle through Biot-Savart coupling. We find that if the silver atom initially lies in a spin superposition state of valence electron, in the co-rotating frame of reference the projected measurements of the spin of silver's valence electron along the z direction determine the rotational direction, and the projected measurements of that spin along the x or the y direction yield planet versions of the state of Schrödinger's cat.

PACS numbers: 03.65.Ta, 37.90.+j
Keywords: Quantum measurement, Biot-Savart coupling, Planet versions of state of Schrödinger's cat
DOI: 10.3938/jkps.68.944

I. INTRODUCTION

Quantum mechanics is a fundamental theory in modern physics. It is often the only tool available that reveals the individual behaviors of subatomic particles; it is also important for understanding superfluidity, superconductivity and the covalently bonding of atoms to form molecules. Many modern technological inventions are operated at a scale where quantum effects are significant. Examples include lasers, transistors (and thus microchips), electron microscopes and magnetic resonance imaging. Now efforts are being made to develop quantum cryptography, quantum computers and quantum teleportation [1].

Quantum mechanics has several counter intuitive results, such as the Heisenberg uncertainty principle [2], the entanglement of quantum states [3], Schrödinger's cat state [4], and the nonlocality of microscopic particles [5]. These fundamental results go against people's intuitions in classical physics. The most subtle and the most difficult thing in quantum mechanics is the measurement problem. In the Copenhagen interpretation of quantum mechanics, during a measurement, the system should interact with a laboratory device, so the wave function of the system immediately and randomly collapses into an eigenstate of an observable [6]. The quantum decoherence model presents an explanation for wavefunction collapse [7]. Wheeler and Zurek conclude that the heart of the Copenhagen interpretation, 'No elementary quantum phenomenon is a phenomenon until it is a registered (observed) phenomenon' was correct [8]. With their words we can understand several typical thought experiments, such as the electron interference through double slits, measurement of the silver atom's magnetic moment by using the Stern-Gerlach apparatus and Wheeler's delayed-choice experiment.

In this paper, we propose a new quantum system, in which the quantum measurement plays a crucial role. The quantum system includes a classical charged particle travelling along a circular orbit and a silver atom in its ground state at the center of the circular orbit. The charged particle interacts with the silver atom through Biot-Savart coupling. The paper is organized as follows: In Section II we present detailed calculations for our new quantum system, and the evolution of the system's state with time is worked out in the interaction picture. In Section III, we discuss how a quantum measurement of the spin of a silver atom's valence electron influences the rotations of the charged particle in a co-rotating frame of reference, and the planet version of the state of Schrödinger's cat is presented. In Section IV, we give a summary of this paper.

II. EVOLUTION OF THE NEW SYSTEM'S STATE WITH TIME

The new quantum system we study is shown in Fig. 1. A classical particle P with electric charge $+q$ travels along a circular orbit because particle P is attracted by the center of the circular orbit through a Coulomb force, a Lorentz force or a gravitational force. Here, we are not

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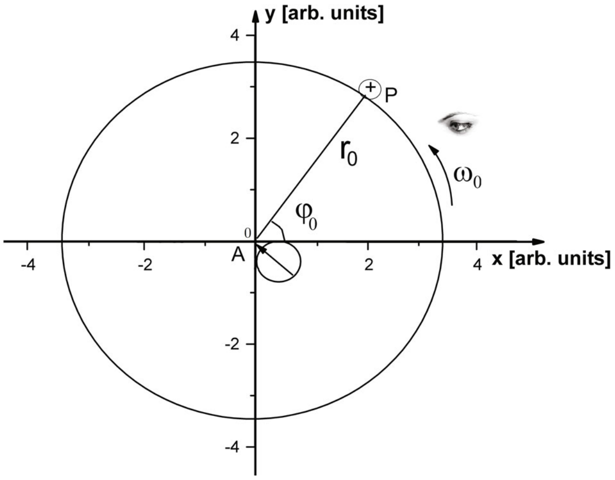

Fig. 1. A classical charged particle P travelling along a circular orbit interacts with a silver atom A in its ground state fixed at the center of the circle through Biot-Savart coupling.

concerned with what attracts the charged particle P; we simply assume that the particle P travels along a circular orbit with a radius r_0 at angular velocity of ω_0. The energy of the particle P traveling along the circular orbit includes kinetic energy and equivalent potential energy, and the equivalent potential energy should be spherically symmetric. The Hamiltonian is given by

$$\hat{H}_0 = \frac{\hat{p}^2}{2m_P} + V(r). \tag{1}$$

In Eq. (1), $\hat{p}$ is the momentum operator, m_P is the mass of the particle P and $V(r)$ is the equivalent potential energy.

A silver atom 'A' in ground state is fixed at the center of the circular orbit; the reason we choose a silver atom in its ground state is that the silver atom in its ground state is very simple, and its electron configuration is $[Kr]4d^{10}5s$. The magnetic moment of the silver atom is just that of the $5s$ valence electron's spin. What's more, the spin of the valence electron of a silver atom can be conveniently measured by using the Stern-Gerlach apparatus. As shown in Fig. 1, the direction of the angular momentum of the particle P is along the z axis. The magnetic field at the center of the circle produced by the particle P is given by

$$\overrightarrow{B} = \frac{q}{4\pi\varepsilon_0 m_P c^2}\frac{\overrightarrow{L}}{r_0^3}. \tag{2}$$

In Eq. (2), ε_0 is vacuum's dielectric constant, c is the light speed in vacuum and $\overrightarrow{L} = L_z\overrightarrow{k}$ is the angular momentum of the particle P along the z axis. The interaction energy between the spin of the silver atom and the magnetic field produced by the particle P is given by [9]

$$\begin{aligned}\hat{H}_I = -\overrightarrow{\mu}_s \cdot \overrightarrow{B} &= \frac{qq_e\hbar}{8\pi\varepsilon_0 m_P m_e c^2 r_0^3}\hat{\overrightarrow{L}}\cdot\hat{\overrightarrow{\sigma}} \\ &= \frac{qq_e\hbar}{8\pi\varepsilon_0 m_P m_e c^2 r_0^3}\hat{L}_z\cdot\hat{\sigma}_z. \end{aligned}\tag{3}$$

Actually, the spin of the silver atom is just the spin of the 5s valence electron of the silver atom in its ground state, and the magnetic moment of the silver atom is $\overrightarrow{\mu}_s = -\frac{q_e\hbar}{2m_e}\overrightarrow{\sigma}$. Here, $\overrightarrow{\sigma}$ is the Pauli matrix, m_e is the mass of the electron, q_e is electric charge of a electron, $\hbar$ is the reduced Planck constant. According to the Biot-Savart law, a charged particle travelling along a circular orbit will produce a magnetic field (Eq. (2)) at the center of the circle. The magnetic moment of a silver atom in its ground state, *i.e.*, the spin magnetic moment of the 5s valence electron of silver atom, at the center of the circle has an additional energy (Eq. (3)) in the magnetic field produced by the charged particle. Thus we call Eq. (3) the Biot-Savart coupling. The total energy of the system is $\hat{H} = \hat{H}_0 + \hat{H}_I$. Because of $[\hat{L}_z,\hat{H}_0] = [\hat{\sigma}_z,\hat{H}_0] = 0$, we obtain $[\hat{H}_I,\hat{H}_0] = 0$. Resolving the Schrödinger equation in the interaction picture, where the interaction Hamiltonian is given by $\hat{H}_I(t) = e^{i\hat{H}_0 t/\hbar}\hat{H}_I e^{-i\hat{H}_0 t/\hbar} = H_I$, is convenient and the solution of time-dependent Schrödinger equation is given by

$$|\psi\rangle = e^{-i\hat{H}_I t/\hbar}|\psi_0\rangle. \tag{4}$$

Given $\hat{L}_z = -i\hbar\frac{\partial}{\partial\varphi}$, $\hat{\sigma}_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$ and $\xi \equiv \frac{qq_e\hbar}{8\pi\varepsilon_0 m_P m_e c^2 r_0^3}$, we obtain $e^{-i\hat{H}_I t/\hbar} = e^{\frac{-i}{\hbar}\xi t\hat{L}_z\hat{\sigma}_z} = \left[1 + \left(e^{\frac{-i}{\hbar}\xi t\hat{L}_z} - 1\right)|\uparrow\rangle\langle\uparrow|\right]\left[1 + \left(e^{\frac{i}{\hbar}\xi t\hat{L}_z} - 1\right)|\downarrow\rangle\langle\downarrow|\right]$. The initial state of the system is given by $|\psi_0\rangle = \varphi_0\rangle(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$, with $|\alpha|^2 + |\beta|^2 = 1$:*i.e.* $|\varphi_0\rangle$ being the classical charged particle's position wavefunction in the co-rotating frame of reference with the angular velocity ω_0. The superposition state $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ is the state of the silver atom. The solution of the Schrödinger equation, Eq. (4), reads

$$\begin{aligned}|\psi\rangle &= e^{-i\hat{H}_I t/\hbar}|\psi_0\rangle \\ &= \left[1 + \left(e^{\frac{-i}{\hbar}\xi t\hat{L}_z} - 1\right)|\uparrow\rangle\langle\uparrow|\right]\left[1 + \left(e^{\frac{i}{\hbar}\xi t\hat{L}_z} - 1\right)|\downarrow\rangle\langle\downarrow|\right][\varphi_0\rangle(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)] \\ &= \alpha|\varphi_0 + \xi t\rangle|\uparrow\rangle + \beta|\varphi_0 - \xi t\rangle|\downarrow\rangle. \end{aligned}\tag{5}$$

In deriving Eq. (5), we use $e^{\frac{-i}{\hbar}\xi t\hat{L}_z}|\varphi_0\rangle = |\varphi_0+\xi t\rangle$ and $e^{\frac{i}{\hbar}\xi t\hat{L}_z}|\varphi_0\rangle = |\varphi_0-\xi t\rangle$ when the commutation relations, $[\varphi,\hat{L}_z] = i\hbar$ and $[\varphi, e^{\mp\frac{i}{\hbar}\xi t\hat{L}_z}] = \pm t e^{\mp\frac{i}{\hbar}\xi t\hat{L}_z}$, are respectively employed. $|\varphi_0+\xi t\rangle$ and $|\varphi_0-\xi t\rangle$ in Eq. (5) are the position wavefunctions of a classical charged particle as a function of time t. The operator $e^{-i\lambda\hat{p}/\hbar}$ is investigated in detail in Ref. [10], where λ is a real number and $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ is the momentum operator. As we know, the angular quantities in a circular orbit are essentially equivalent to the translational quantities along a straight line, so our calculations of Eqs. (3)−(5) are valid.

III. DETERMINATION OF THE ROTATION OF A CHARGED PARTICLE BY USING QUANTUM MEASUREMENTS OF THE SPIN OF A SILVER ATOM

The state of the system, (Eq. (5)), is the largest entanglement state between the circular orbit of the particle P and the spin of the silver atom. Surprisingly and interestingly the spin measurement of the silver atom determines the direction of the particle's rotation in the co-rotating frame of reference. For instance, after the projected measurement $\sigma_{z\uparrow}$ of the silver atom's spin projected along the z direction, the measured state of the system reads

$$\begin{aligned}|\uparrow\rangle\langle\uparrow||\psi\rangle &= |\uparrow\rangle\langle\uparrow|[\alpha|\varphi_0+\xi t\rangle|\uparrow\rangle + \beta|\varphi_0-\xi t\rangle|\downarrow\rangle] \\ &= \alpha|\varphi_0+\xi t\rangle|\uparrow\rangle. \end{aligned} \tag{6}$$

In our thought experiment, we can measure the valence electron's spin state of a silver atom by using a mini Stern-Gerlach apparatus at the center of the circle, which can be realized provided that the radius of the circular orbit is large enough. Equation (6) implies that after the projected $\sigma_{z\uparrow}$ projective measurement of the silver atom's spin, the charged particle P still travels counterclockwise along the circular orbit, which is our intuitive expectation. However, what happens if we perform the projected $\sigma_{z\downarrow}$ measurement of the silver atom's spin? Now the measured state of the system is given by

$$\begin{aligned}|\downarrow\rangle\langle\downarrow||\psi\rangle &= |\downarrow\rangle\langle\downarrow|[\alpha|\varphi_0+\xi t\rangle|\uparrow\rangle + \beta|\varphi_0-\xi t\rangle|\downarrow\rangle] \\ &= \beta|\varphi_0-\xi t\rangle|\downarrow\rangle. \end{aligned} \tag{7}$$

The charged particle P initially travels counterclockwise along the circular orbit after the projected $\sigma_{z\downarrow}$ measurement of the silver atom's spin, and surprisingly, the charged particle P travels clockwise along circular orbit!

This counter-intuition phenomenon can be easily understood in the Copenhagen interpretation of quantum mechanics. The system should interact with a laboratory device, so people's measurement changes the system's state. Because the charged particle is coupled with the silver atom via the Biot-Savart interaction, the measurement of the silver atom's spin should influence the rotation of the charged particle; specifically, the rotational directions are changed.

However, the measurement of $\sigma_{x\uparrow}$ or $\sigma_{y\uparrow}$ of the silver atom's spin projected along the x or the y direction results in a dilemma for the Copenhagen interpretation of quantum mechanics, just as the Schrödinger's cat state does. Actually, the eigenstates of σ_x and σ_y are, respectively, written as

$$\begin{aligned}|\varphi_{x\uparrow}\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle),\ |\varphi_{x\downarrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle), \\ |\varphi_{y\uparrow}\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle),\ |\varphi_{y\downarrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - i|\downarrow\rangle).\end{aligned}$$

From the above formula, we obtain

$$\begin{aligned}|\uparrow\rangle &= \frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle + |\varphi_{x\downarrow}\rangle),\ |\downarrow\rangle = \frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle - |\varphi_{x\downarrow}\rangle), \\ |\uparrow\rangle &= \frac{1}{\sqrt{2}}(|\varphi_{y\uparrow}\rangle + |\varphi_{y\downarrow}\rangle),\ |\downarrow\rangle = \frac{1}{i\sqrt{2}}(|\varphi_{y\uparrow}\rangle - |\varphi_{y\downarrow}\rangle).\end{aligned} \tag{8}$$

From Eq. (8), the state of the system, Eq. (5), can be rewritten as

$$\begin{aligned}|\psi\rangle &= \alpha|\varphi_0+\xi t\rangle|\uparrow\rangle + \beta|\varphi_0-\xi t\rangle|\downarrow\rangle \\ &= \alpha|\varphi_0+\xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle + |\varphi_{x\downarrow}\rangle) + \beta|\varphi_0-\xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle - |\varphi_{x\downarrow}\rangle) \\ &= \alpha|\varphi_0+\xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{y\uparrow}\rangle + |\varphi_{y\downarrow}\rangle) + \beta|\varphi_0-\xi t\rangle\frac{1}{i\sqrt{2}}(|\varphi_{y\uparrow}\rangle - |\varphi_{y\downarrow}\rangle).\end{aligned} \tag{9}$$

From Eq. (9), the projected measurements $\sigma_{x\uparrow}$ and $\sigma_{y\uparrow}$ of the silver atom's spin yield

$$\begin{aligned}&|\varphi_{x\uparrow}\rangle\langle\varphi_{x\uparrow}|\left[\alpha|\varphi_0+\xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle + |\varphi_{x\downarrow}\rangle) + \beta|\varphi_0-\xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle - |\varphi_{x\downarrow}\rangle)\right] \\ &= \frac{1}{\sqrt{2}}(\alpha|\varphi_0+\xi t\rangle + \beta|\varphi_0-\xi t\rangle)|\varphi_{x\uparrow}\rangle,\end{aligned} \tag{10}$$

$$|\varphi_{y\uparrow}\rangle\langle\varphi_{y\uparrow}|\left[\alpha|\varphi_0+\xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{y\uparrow}\rangle+|\varphi_{y\downarrow}\rangle)+\beta|\varphi_0-\xi t\rangle\frac{1}{i\sqrt{2}}(|\varphi_{y\uparrow}\rangle-|\varphi_{y\downarrow}\rangle)\right]$$
$$=\frac{1}{\sqrt{2}}(\alpha|\varphi_0+\xi t\rangle-i\beta|\varphi_0-\xi t\rangle)|\varphi_{y\uparrow}\rangle. \quad (11)$$

Because of the essential equivalence between the angular quantities in a circular orbit and the translational quantities along a straight line, we obtain the orthogonality of the position wavefunction, *i.e.*, $\langle\varphi|\varphi'\rangle=\delta(\varphi-\varphi')$, from the orthogonality of the translational position wavefunction, *i.e.*, $\langle x|x'\rangle=\delta(x-x')$. When $t>0$, we have the orthogonality $\langle\varphi_0+\xi t|\varphi_0-\xi t\rangle=0$. We observe that after the projected measurements $\sigma_{x\uparrow}$ and $\sigma_{y\uparrow}$ of the silver atom's spin, as expected, the spins are, respectively, in $|\varphi_{x\uparrow}\rangle$ and $|\varphi_{y\uparrow}\rangle$; however, the charged particle P is, respectively, in macroscopic superposition states between the counterclockwise circular orbit $\frac{1}{\sqrt{2}}(\alpha|\varphi_0+\xi t\rangle+\beta|\varphi_0-\xi t\rangle)$ and $\frac{1}{\sqrt{2}}(\alpha|\varphi_0+\xi t\rangle-i\beta|\varphi_0-\xi t\rangle)$ the clockwise circular orbit and The macroscopic superposition states between the counterclockwise circular orbit and the clockwise circular orbit are typical Schrödinger-cat states; here, we name them as the *planet versions of Schrödinger-cat state*. Is it possible? The classical charged particle P travels not only along a counterclockwise circular orbit but also along a clockwise circular orbit, which is very strange in our classical physics intuition. In quantum mechanics, the macroscopic planet versions of Schrödinger's cat state are, indeed, prepared by a quantum measurement!

IV. SUMMARY

We have studied a classical charged particle travelling along a circular orbit and interacting with a silver atom in its ground state fixed at the center of the circle through Biot-Savart coupling. We calculate in detail the evolution of the new state of the quantum system with time. We observe that if the silver atom initially lies in a superposition state of the valence electron spin, the projected measurements $\sigma_{z\uparrow}$ or $\sigma_{z\downarrow}$ of the silver atom spin along the z direction determine the rotational directions of the charged particle P. The projected measurements $\sigma_{x\uparrow}$ or $\sigma_{y\uparrow}$ of the silver atom's spin along the x or the y directions can yield planet versions of the Schrödinger's cat state of the charged particle P. Please note that the projected measurements do not change the classical orbit of the charged particle P. Actually, the calculations are performed in an interaction picture, so the observer lies in the frame of reference co-rotating with the angular velocity ω_0. In the co-rotating frame of reference, the observer 'sees' that projected measurements determine the directions of the charged particle or yield the macroscopic superposition states between the counterclockwise orbit and the clockwise orbit of Schrödinger's cat state.

To our knowledge, we, for the first time, report that the rotations of a classical charged particle flying along circular orbit can be influenced by using quantum measurement on the spin of another particle, *i.e.*, a silver atom. Needless to say, measurements are very important in quantum mechanics, and do change the system's state. An elementary quantum phenomenon is a registered (observed) phenomenon. Compared to other states of Schrödinger's cat, such as photons [11], beryllium ions [12], the electrons in a superconducting quantum interference device [13], a flu virus [14], a bacterium [15], a piezoelectric tuning fork [16], a superconductor [17], and a nanoscale magnet [18], the planet versions of the state of Schrödinger's cat also look very strange and are impossible in the classical world, however, these states can be prepared through quantum measurements, so they, indeed, exist in the quantum world!

ACKNOWLEDGMENTS

This work is supported by the State Key Program of National Natural Science Foundation of China (Grant No. 41530422); the National Natural Science Foundation of China (Grant No. 61540018; 61275184, 61405153)the National Major Project (Grant No. 32-Y30B08-9001-13/15); the National High Technology Research and Development Program of China 863 (Grant No. 2012AA121101) and the Fundamental Research Funds for the Central Universities.

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and the origin of coordinate is still at the center of the circle with a silver atom A.
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