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REGULAR PAPER

Classical harmonic vibrations with micro amplitudes and low frequencies monitored by quantum entanglement

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Abstract We study the entanglement dynamics of the two two-level atoms coupled with a single-mode polarized cavity field after incorporating the decoupled atomic centers of mass classical harmonic vibrations with micro amplitudes and low frequencies. We discover a new quantum mechanical measurement effect for the entanglement dynamics. We propose a quantitative vibrant factor to modify the concurrence of the two atomic states. When the vibrant frequencies are very low, we obtain that: (1) the factor depends on the relative vibrant displacements and the initial phases rather than the absolute amplitudes, and reduces the concurrence to three orders of magnitude; (2) the concurrence increases with the increase of the initial phases; (3) the frequency of the harmonic vibration can be obtained by measuring the maximal value of the concurrence during a small measurement time. These results indicate that the extremely weak classical harmonic vibrations can be monitored by the entanglement of quantum states. The effect reported in the paper always works well as long as the internal degrees of freedom of the system (regardless of unitary evolution or non-unitary evolution with time) are decoupled with the external classical harmonic vibrations of atomic centers of mass.

Keywords The vibrant factor · Micro amplitudes · Low frequencies · Entanglement concurrence

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1 Introduction

A gravitational wave is the only direct unconfirmed prediction in general relativity. Because of its good coherence and strong penetrability, the detection of a gravitational wave is very important to the modern astronomy. From the analysis of the gravitational waves we can obtain the inner core variations of the supernova from in a supernova explosion, confirm the existence of a black hole and even research the early universe in the Big Bang [1]. A plane gravitational wave causes a time-dependent strain in space, with an oscillating quadrupolar strain pattern that is transverse to the wave's propagation direction, expanding space in one direction while contracting it along the orthogonal direction [2]. Although a gravitational wave is an extremely weak wave, people attempt to detect the gravitational waves with different methods. The gravitational wave can be confirmed by Weber in 1969 through measuring the oscillatory motions of Weber bars [3] (unfortunately, the results are not accepted). The observations of the period decreasing of PSR1913 + 16 by Hulse and Taylor in 1974 [4] indirectly confirm the existence of a gravitational wave. In order to directly detect a gravitational wave, people are launching huge detection projects for gravitational waves, such as LIGO [5], Virgo [6], GEO600 [7], KAGRA [8], AIGO [9], eLISA [10, 11], etc. An atomic interferometer is even designed for the detection of gravitational waves [12–15].

The entanglement of quantum states discovered by Einstein–Podolsky–Rosen (EPR) [16] and Schrödinger [17] is one of the strangest phenomena in quantum mechanics. Entanglement as a new resource can not only be applied to information field [18], such as quantum state teleportation, quantum cryptography, quantum dense coding, quantum computing, etc., but also provide new angles of view, such

as the emergence of classicality [19], disordered systems [20], superconductivity [21] and superradiance [22], etc. To meet the experimental needs, several quantitative descriptions of entanglement have been proposed, for instance Peres–Horodecki theorem [23, 24], Duan criteria [25], Simon criteria [26], Wootters concurrence [27], negativity [28, 29] and entanglement witness [30, 31], etc.

Entanglement is versatile; can it be used to detect gravitational waves? A good question. A plane gravitational wave causes a time-dependent strain in space, with an oscillating quadrupolar strain pattern that is transverse to the wave's propagation direction, expanding space in one direction while contracting it along the orthogonal direction. So we can extract some information of a harmonic vibration from the entanglement of a system to detect gravitational waves. In this paper we study the influence of the classical harmonic vibrations of atomic centers of mass on the entanglement concurrence between the two two-level atoms in a single-mode polarized cavity field, i.e., Jaynes–Cummings model. In Sect. 2 Wootters concurrence of the atomic states are calculated in a toy thought system. In Sect. 3 we pay close attention to the vibrant factor for the classical harmonic vibrations of atomic centers of mass, which gives a modification of the concurrence of the two atomic states. We also discuss how classical harmonic vibrations are monitored by entanglement concurrence. In Sect. 4 a summary is presented.

2 The calculations of Wootters concurrence

The simple thought system we study is shown in Fig. 1. The two equal two-level atoms, A atom at $z_A = -z_0$ and B atom at $z_B = z_0$, are coupled with a single-mode cavity field polarized along y direction, which runs along z direction. The two atoms are controlled to harmonically vibrate along x direction by some drive, for instance the two atoms are induced by a gravitational wave and will harmonically vibrate in the plane perpendicular to the gravitational wave vector. Under the rotating-wave approximation the system Hamiltonian is written as

$$H = H_0 + H',$$

$$H_0 = \frac{1}{2}\hbar\omega_A\sigma_A^z + \frac{1}{2}\hbar\omega_B\sigma_B^z + \hbar\omega a^\dagger a + \hbar\Omega_A(C_A^\dagger C_A + 1/2) + \hbar\Omega_B(C_B^\dagger C_B + 1/2),$$

$$H' = \hbar g \sum_{i=A,B} [a\sigma_i^+ \exp(ikz_i) + a^\dagger\sigma_i^- \exp(-ikz_i)].$$

H_0 describes the energy levels of the two atoms and the cavity and the vibrations of the atomic centers of mass, H' describes the atom–field interactions. In the total Hamiltonians of H_0 and H', $a^\dagger, a$ are the bosonic operators of the single-mode field with the frequency ω; $\sigma^z = |e><e| - |g><g|$, $\sigma^+ = |e><g|$ and $\sigma^- = |g><e|$ are respectively Pauli operator, raising and lowering operators of the two-level atoms where $|e>$ is the exciting state and $|g>$ is the ground state; The energy level difference of atom A (atom B) is $\hbar\omega_A$ ($\hbar\omega_B$); $C_A^\dagger, C_A$ and $C_B^\dagger, C_B$ are the bosonic operators for the harmonic vibrations of the centers of mass of atoms A and atoms B with the frequencies Ω_A, Ω_B; g is the coupling coefficient and k is the wavenumber of the cavity field. In our thought experiment we assume that the conditions $\omega_A = \omega_B = \omega$, $\Omega_A = \Omega_B \equiv \Omega$ are satisfied. The vibrations of the two atoms are assumed to have very small amplitudes, the effect of the vibrations of the atomic centers of mass on the atom energy levels can be neglected. The recoil motions of A and B atoms along y direction are ignored when a photon is absorbed or emitted by A (or B) atoms, without loss of generality we do not take the coupling between the vibrations of the atomic centers of mass and the single-mode polarized cavity field into account.

Several authors have studied the question that the two two-level atoms are coupled with a single-mode cavity field [32–34]. Now we study the same problem after incorporating the classical harmonic vibrations of atomic centers of mass with micro amplitudes and low frequencies. In the conditions $\omega_A = \omega_B = \omega$, we have $[H_0, H'] = 0$. The Hamiltonian in the interaction picture is given by $H_I(t) = e^{iH_0t/\hbar}H'e^{-iH_0t/\hbar}$. After considering the condition $[H_0, H'] = 0$, we get the Hamiltonian in the interaction picture $H_I(t) = H'$ independent of the time t. In order to emphasize the influence of classical vibrations on the entanglement concurrence of the two atoms in a cavity, we only study the unitary evolution of the system, which is governed by von Neumann equation $\frac{d\rho_I}{dt} = -\frac{i}{\hbar}[H', \rho_I]$. Actually the effect discussed in the following always works well as long as the internal degrees of freedom of the system (regardless of unitary evolution or non-unitary

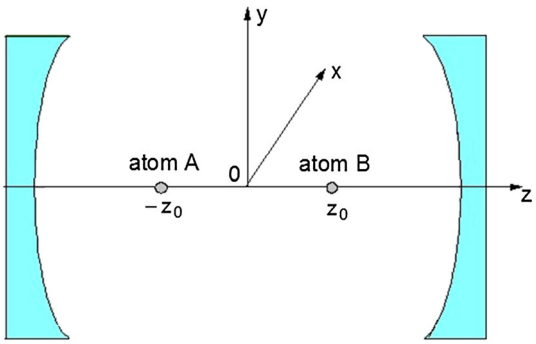

Fig. 1 Schematic of a setup illustrates how the classical harmonic vibrations of atomic centers of mass with micro amplitudes and low frequencies affect the concurrence between two two-level atoms coupled with a single-mode cavity field

evolution with time) are decoupled with the external classical harmonic vibrations of atomic centers of mass. The solution of von Neumann equation is given by

$$\rho_{\mathrm{I}}(t) = U(t)\rho_{\mathrm{I}}(0)U^{\dagger}(t), \tag{1}$$

where the time evolution operator is $U(t) = \exp[-iH't/\hbar]$ and $\rho_{\mathrm{I}}(0)$ is the density operator of the initial state in the interaction picture. The reduced density operator for the two atoms is given by

$$\rho_{\mathrm{I}}(t)_{\mathrm{atoms}} = Tr_{\mathrm{cavity+vibrations}}[U(t)\rho_{\mathrm{I}}(0)U^{\dagger}(t)]. \tag{2}$$

The trace in Eq. (2) includes the traces over both the cavity field and the vibrations of the atomic centers of mass. Taken $U(t) = \exp[-iH't/\hbar]$ into account, $\rho_{\mathrm{I}}(t)_{\mathrm{atoms}}$ is written as

$$\rho_{\mathrm{I}}(t)_{\mathrm{atoms}} = Tr_{\mathrm{cavity+vibrations}}[e^{-iH't/\hbar}\rho_{\mathrm{I}}(0)e^{iH't/\hbar}]. \tag{3}$$

$e^{-iH't/\hbar}$ is exactly worked out in the atomic basis $\{|ee>, |eg>, |ge>, |gg>\}$ similarly in ref [33], where $|e>$ is excited state and $|g>$ is ground state, i.e.,

In deriving Eq. (6) we have traced over the cavity field and inserted the completeness $\int |\xi_1><\xi_1| \otimes |\xi_2><\xi_2| \mathrm{d}\xi_1 \mathrm{d}\xi_2 = I$ into Eq. (3) to trace over the vibrations, where ξ_1, ξ_2 denotes the positions of the two atomic centers of mass in x direction. The sense of the time interval Δt subscript in Eq. (6) will be presented in what follows. Quantum number n, m will be very large under the low frequencies condition of $\Omega << g, \Omega << \omega$, $F \equiv \int_{\Delta t} <\psi_n|\xi_1><\xi_1|\psi_n> \mathrm{d}\xi_1 \times \int_{\Delta t} <\psi_m|\xi_2><\xi_2|\psi_m> \mathrm{d}\xi_2$ can be regarded as classical harmonic oscillators probabilities [35], The integrand in the integral vibrant factor is just right the probability density of each classical harmonic oscillator $w(\xi_{1,2}) = \psi^*_{1,2}\psi_{1,2} = <\psi|\xi_{1,2}><\xi_{1,2}|\psi>$. The defined factor F is called by the vibrant factor in this paper, because the vibrant factor comes from the trace over classical harmonic vibrations of the atomic centers of mass in deriving the concurrence. During a short measurement time interval Δt less than the period T, the vibrant factor

$$\mathrm{e}^{-iH't/\hbar} = \begin{pmatrix} 2g^2a(C-\Theta)a^+ + 1 & -igaSe^{ikz_0} & -igaSe^{-ikz_0} & 2g^2a(C-\Theta)a \\ -igSa^+e^{-ikz_0} & (\cos\Gamma t + 1)/2 & (\cos\Gamma t - 1)e^{-2ikz_0}/2 & -igSae^{-ikz_0} \\ -igSa^+e^{ikz_0} & (\cos\Gamma t - 1)e^{2ikz_0}/2 & (\cos\Gamma t + 1)/2 & -igSae^{ikz_0} \\ 2g^2a^+(C-\Theta)a^+ & -iga^+Se^{ikz_0} & -iga^+Se^{-ikz_0} & 2g^2a^+(C-\Theta)a + 1 \end{pmatrix} \tag{4}$$

Here $\Gamma^2 = \Theta^{-1} = 2g^2(2a^+a + 1)$ and C, S are defined by $C = \Theta\cos\Gamma t$ and $S = \Gamma^{-1}\sin\Gamma t$.

Without loss of generality, we study a typical initial state and give the modification of the vibrations of atomic centers of mass on the entanglement concurrence between the two atomic states. The initial state is

$$\rho_I(0) = |gg><gg| \otimes |1><1| \otimes |\psi_n\psi_m><\psi_n\psi_m|, \tag{5}$$

where $|g>$ is the ground state of an atom, $|1>$ and $|\psi_n\psi_m>$ denote the one photon state and the eigenstates of the harmonic oscillators of the two atomic centers of mass. Considering Eq. (4) and substituting $\rho_I(0)$ from Eq. (5) into Eq. (3), we obtain

$$\begin{aligned} \rho_I(t)_{atoms}|_{\Delta t} = & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\sin^2\sqrt{2}gt}{2} & \frac{\sin^2\sqrt{2}gt}{2} & 0 \\ 0 & \frac{\sin^2\sqrt{2}gt}{2} & \frac{\sin^2\sqrt{2}gt}{2} & 0 \\ 0 & 0 & 0 & \cos^2\sqrt{2}gt \end{pmatrix}_{\Delta t} \\ & \times \int_{\Delta t} <\psi_n|\xi_1><\xi_1|\psi_n> \mathrm{d}\xi_1 \\ & \times \int_{\Delta t} <\psi_m|\xi_2><\xi_2|\psi_m> \mathrm{d}\xi_2 \end{aligned} \tag{6}$$

traced over classical vibrations is less than unity, which is a remarkable result. The vibrant factor less than unity brings a quantitative modification on the entanglement concurrence.

In deriving Eq. (6) with substituting from Eqs. (4) and (5) into Eq. (3), we are sure that the vibrant factor always occurs as long as the internal degrees of freedom of the system (regardless of unitary evolution or non-unitary evolution with time) are decoupled with the external classical harmonic vibrations of atomic centers of mass. Because of the decoupling between the external vibrations and the internal degree of freedom of the system, the whole system state should be the direct product between the state of the internal degree of freedom and the state of the external vibrations. The states of the internal degree of freedom and the external vibrations independently, simultaneously evolve with time. The time evolution of the external vibrations yields the vibrant factor. So the vibrant factor is used to characterize the external independent harmonic vibrations of the two atoms, it can not disappear by renormalization. Please note that the vibrant factor F is less than unity during a short measurement time Δt, the reason will be presented as follows. The condition $\Omega << g, \Omega << \omega$ can be satisfied in laboratory. In fact the typical frequency Ω

of mechanical vibration due to a gravitational wave is about 10^3 Hz [36], a strong coupling coefficient g can arrive at 10^6 Hz [37], and the resonant ω is higher than the coupling coefficient g.

In order to calculate the classical harmonic oscillators' probabilities, we write the Hamiltonian of a classical oscillator $H = p^2/2M + M\Omega^2 x^2/2$. Given $\alpha = \sqrt{M\Omega/\hbar}$ and $\xi = \alpha x$ we obtain the classical motion equations of the two atomic centers of mass: $\xi_1 = \sqrt{2n_1+1}\sin(\Omega t + \delta_1)$ and $\xi_2 = \sqrt{2n_2+1}\sin(\Omega t + \delta_2)$, where $\sqrt{2n_1+1}$ and $\sqrt{2n_2+1}$ are the classical amplitudes with their large quantum numbers n, m, δ_1, δ_2 are the initial phases of the two atomic centers of mass. The classical harmonic oscillator probability density is $w(\xi) = <\psi|\xi><\xi|\psi> = \frac{1}{\pi\sqrt{(2n+1)-\xi^2}}$, and $w(\xi)$ increases with the increase of the displacement ξ in $\xi \in [0, \sqrt{2n+1}]$. We do not consider $\xi \in [-\sqrt{2n+1}, 0]$ because of the classical harmonic oscillator probability density's symmetry between $[0, \sqrt{2n+1}]$ and $[-\sqrt{2n+1}, 0]$.

We work out $\int_{\Delta t} <\psi_n|\xi_1><\xi_1|\psi_n> \mathrm{d}\xi_1 \times \int_{\Delta t} <\psi_m|\xi_2><\xi_2|\psi_m> \mathrm{d}\xi_2$ during a short measurement time Δt, i.e.,

$$
\begin{aligned}
F &\equiv \int_{\xi_{10}}^{\xi_{10}+\zeta_1} w(\xi_1)\mathrm{d}\xi_1 \int_{\xi_{20}}^{\xi_{20}+\zeta_2} w(\xi_2)\mathrm{d}\xi_2 \\
&= \frac{1}{\pi^2}\left[\arcsin\left(\frac{\zeta_1}{\sqrt{2n_1+1}} + \sin\delta_1\right) - \delta_1\right] \\
&\quad \times \left[\arcsin\left(\frac{\zeta_2}{\sqrt{2n_2+1}} + \sin\delta_2\right) - \delta_2\right] \qquad (7)
\end{aligned}
$$

where ζ_1, ζ_2 denote the absolute displacements of the two atomic centers of mass, and δ_1, δ_2 are the initial phases, that is, $\xi_{10} = \sqrt{2n_1+1}\sin\delta_1, \xi_{20} = \sqrt{2n_2+1}\sin\delta_2$ with ξ_{10}, ξ_{20} denoting the initial displacements. Δt should meet the condition $1/g << \Delta t << 1/\Omega$. The condition guarantees that the integral upper limit and lower limit in Eq. (7) are $[\xi_{10}, \xi_{10}+\zeta_1], [\xi_{20}, \xi_{20}+\zeta_2]$ rather than $(-\infty, +\infty)_1$, $(-\infty, +\infty)_2$ and that the vibrant factor F is always less than unity. During a short measurement time Δt the relative displacements are very small and satisfy the relationships $0 < \zeta_1/\sqrt{2n_1+1} << 1$, $0 < \zeta_2/\sqrt{2n_2+1} << 1$.

From Eqs. (6) and (7), we can obtain the concurrence $C(\rho_I) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})$, where the quantities λ_i are the eigenvalues of the matrix $\rho_I(\sigma_A^y \otimes \sigma_B^y)\rho_I^*(\sigma_A^y \otimes \sigma_B^y)$ arranged in decreasing order. ρ_I^* is the elementwise complex conjugation of ρ_I in the atomic basis $\{|ee>, |eg>, |ge>, |gg>\}$, and $\sigma_A^y \otimes \sigma_B^y$ is the direct product of Pauli matrix expressed in the same basis [38]. The concurrence is calculated as

$$
\begin{aligned}
C(\rho_I)_{\Delta t} = \frac{1}{\pi^2}&\left[\arcsin\left(\frac{\zeta_1}{\sqrt{2n_1+1}} + \sin\delta_1\right) - \delta_1\right] \\
&\times \left[\arcsin\left(\frac{\zeta_2}{\sqrt{2n_2+1}} + \sin\delta_2\right) - \delta_2\right] \times \sin^2\sqrt{2}gt|_{\Delta t} \qquad (8)
\end{aligned}
$$

The classical vibrations of the two atomic centers of mass are decoupled with the intrinsic motions of the two atomic states in a cavity field, why will the vibrations of the centers of mass modulate the concurrence of the two atomic states? Physically the present situation resembles the amplitude modulation in radio. The concorrence between the two atomic states varies periodically with time, it resembles carrier wave with high frequency; the centers of mass of the two atoms will simultaneously and independently vibrate with time as well, they resemble signal wave with low frequency. Within a period of the vibrations of the centers of mass of the two atoms we measure the entanglement concurrence, we find that the vibrations of the centers of mass of the two atoms bring a modulated vibrant factor, it resemble the modulated amplitude, please see Eq. (8). In a word, the reason why the vibrations of centers of mass modulate the concurrence between the two atomic states is that the two motions (the vibrations of centers of mass and the intrinsic motion of the two atomic states coupled with the cavity field) are decoupled and simultaneously evolve with time, the whole state is the direct product between the state of the internal degree of freedom and the state of the external vibrations. We should not be too much surprised of the fact that the measurement of concurrence has a true effect on entanglement concurrence when the two atoms have classical harmonic vibrations. The measurement of concurrence happens during a short time interval Δt is less than the period of the external vibrations of the centers of mass of the two atoms. The trace over the vibrations is less than unity and the so-called vibrant factor appears, the vibrant factor brings a quantitative modification on the measured concurrence. The vibrant factor is just one more constraint to the entanglement concurrence when the measurement of concurrence is performed.

3 The vibrant factor and classical harmonic vibrations monitored by entanglement concurrence

Wootters concurrences Eq. (8) have a vibrant factor for the classical harmonic vibrations of the atomic centers of mass

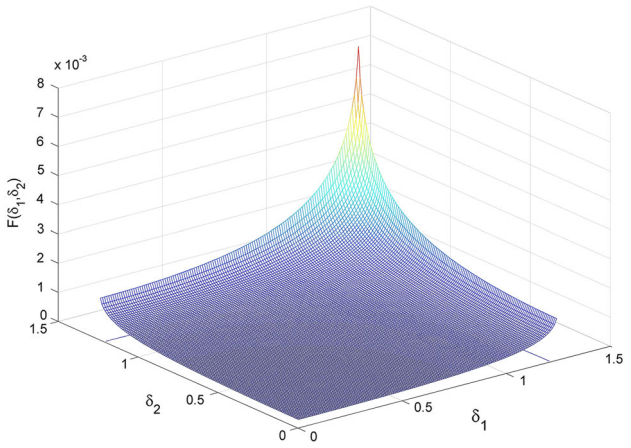

Fig. 2 The vibrant factor F is versus the initial phases δ_1, δ_2, with $\zeta_1/\sqrt{2n_1+1} = \zeta_2/\sqrt{2n_2+1} = 0.05$

$$F(\delta_1,\delta_2) = \frac{1}{\pi^2}\left[\arcsin\left(\frac{\zeta_1}{\sqrt{2n_1+1}} + \sin\delta_1\right) - \delta_1\right] \times \left[\arcsin\left(\frac{\zeta_2}{\sqrt{2n_2+1}} + \sin\delta_2\right) - \delta_2\right] \quad (9)$$

The factor F versus the initial phases δ_1, δ_2 is shown in Fig. 2, which is our main result. Seen from Fig. 2 and Eq. (9) we obtain three results: (1) The vibrant factor will increase with the increase of the initial phases δ_1, δ_2. The reason is that the initial phases δ_1, δ_2 correspond to the different displacements ξ_1, ξ_2, the probability density $w(\xi)$ increases with the increase of displacement ξ. (2) The vibrant factor for the vibrations of the atomic centers of mass depends on the relative vibrant displacements $\zeta_1/\sqrt{2n_1+1}, \zeta_2/\sqrt{2n_2+1}$ and the initial phases δ_1, δ_2, rather than the absolute vibrant amplitudes $\sqrt{2n_1+1}, \sqrt{2n_2+1}$.

(3) The harmonic vibrations of atomic centers of mass greatly reduce the concurrence to three orders of magnitude. Because in the condition of $\Omega << g, \Omega << \omega$ the harmonic oscillator probabilities are not normalized during a short measurement time Δt, the probabilities within the small relative displacements $\zeta_1/\sqrt{2n_1+1}, \zeta_2/\sqrt{2n_2+1}$ are of course much smaller than unity, please see Eq. (7). Actually the relative displacements $\zeta_1/\sqrt{2n_1+1}$, $\zeta_2/\sqrt{2n_2+1}$ depends on arbitrary short measurement time Δt, which satisfies the condition $1/g \ll \Delta t \ll 1/\Omega$. Given $\zeta_1/\sqrt{2n_1+1} = \zeta_2/\sqrt{2n_2+1} = 0.05$ and $g = 10^6$Hz, $\Omega = 10^3$Hz, we have $\sin\delta_1 = \sin\delta_2 = 0.95$, i.e., $\delta_1 = \delta_2 \simeq 1.25$ maximally due to the fact $\zeta/\sqrt{2n+1} + \sin\delta = 1$. Substituting $\delta_1 = \delta_2 \simeq 1.25$ into $F(\delta_1,\delta_2)$, we obtain that $F(\delta_1,\delta_2)$ maximum is about 0.01.

Now we discuss how to monitor the classical harmonic vibrations of the atomic centers of mass by measuring the entanglement concurrence between the two two-level atoms in a single mode cavity during a short measurement time. Without considering the classical harmonic vibrations of the atomic centers of mass, the concurrence versus time between the two two-level atoms in a single mode cavity is very simple, i.e., $C(\rho_I)_0 = \sin^2\sqrt{2}gt$. After incorporating the harmonic vibrations of the atomic centers of mass, the concurrence is a product between vibrant factor F and $C(\rho_I)_0$, i.e., Eq. (8). Figure 3 vividly shows this picture. The selection of a measurement time Δt is very subtle, $1/g << \Delta t << 1/\Omega$ is required. It indicates that Δt is much less than the period $2\pi/\Omega$ of the classical harmonic vibrations, however, much larger than the period $\pi/(\sqrt{2}g)$ of the $C(\rho_I)_0$. Because the maximal value of $C(\rho_I)_0$ is unity, theoretically we can obtain the vibrant factor F by measuring the maximal value of the concurrence $C(\rho_I)$ during a small measurement time Δt. In one period $2\pi/\Omega$ of the classical harmonic vibrations, once we obtain the vibrant factor F versus the initial phases δ_1, δ_2 of the two atomic centers of mass, i.e., the left panel F in Fig. 3, we confirm the existence of the classical harmonic vibrations of the two atomic centers of mass. It is very valuable that the vibrant factor F is independent of the absolute amplitudes, which implies that we can confirm the existence of the classical harmonic vibrations with extremely micro amplitudes.

If we keep each measurement time Δt to be constant, we obtain the vibrant factor F as

$$F(\delta_1,\delta_2) = 4\frac{\Delta t^2}{T^2}, \quad (10)$$

where T is the frequency of the classical harmonic vibration. Equation (10) is even independent of the initial phases. In practice we should divide Eq. (10) by 4,

$$\frac{F(\delta_1,\delta_2)}{4} = \frac{\Delta t^2}{T^2}, \quad (11)$$

because each relative vibrant displacement $\zeta_{1,2}/\sqrt{2n_{1,2}+1}$ contains two to-and-fro processes in one-half period shown in Fig. 3. Equation (8) in practice now reads

$$C(\rho_I)_{\Delta t} = \left(\frac{\Delta t}{T}\right)^2 \times \sin^2\sqrt{2}gt|_{\Delta t}. \quad (12)$$

We can obtain the vibrant factor F by measuring the maximal value of the concurrence $C(\rho_I)$ during a small measurement time Δt, so we acquire the frequency of a classical harmonic vibration even with an amplitude tending to zero.

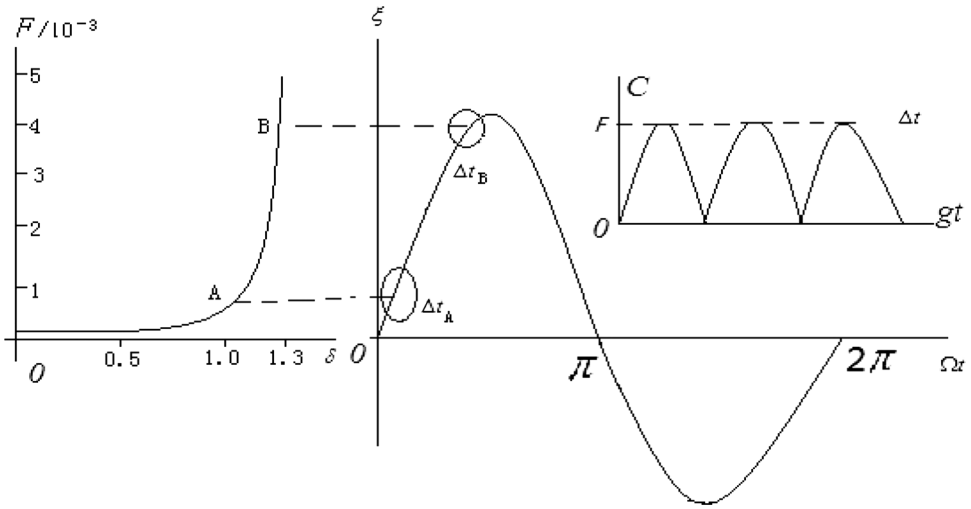

Fig. 3 It is shown the process of the classical harmonic vibrations of the atomic centers of mass monitored by the entanglement concurrence between the two two-level atoms in a single mode cavity. The vibrant factor F versus the two atomic initial phases δ_1, δ_2 lies in the *left panel*, with $\delta_1 = \delta_2$ and $\zeta_1/\sqrt{2n_1+1} = \zeta_2/\sqrt{2n_2+1} = 0.05$ without loss of generality. The harmonic amplitude with time in the *right panel* corresponds to the left vibrant factor. The inset in the *right panel* is the concurrence C versus gt after considering the harmonic vibrations of the atomic centers of mass. The maximum of the concurrence C is the vibrant factor F, rather than unity. A *dot* in the *left panel* corresponds to a measurement time Δt with an initial phase Δ, and it takes a different time Δt at a different phase Δ to keep the relative displacement $\zeta/\sqrt{2n+1}$ to be constant. The measurement time interval is chosen such that Δt is less than the period of the harmonic vibration, larger than the period of the entanglement concurrence. For instance, *dot A* and *dot B* in the *left panel* have different initial phases and correspond to different displacements in the *right panel*. If the relative displacement $\zeta/\sqrt{2n+1}$ is kept to be constant, then the measurement time in *dot A* is less than the time in *dot B*, i.e., $\Delta t_A < \Delta t_B$. The vibrant factor F(A) is less than F(B) because of the probability density in *dot A* is less than the probability density in *dot B* seen from Eq. (7)

4 Summary

In conclusion we have studied the entanglement dynamics of the two two-level atoms coupled with a single-mode polarized cavity field after incorporating the classical harmonic vibrations of the atomic centers of mass. When the vibrations of the two atomic centers of mass are classical and harmonic due to some external drive and the vibrations are decoupled with the internal degrees of freedom of the two atoms, the entanglement concurrence of the two two-level atoms coupled with a single-mode polarized cavity field will be modulated by the vibrant factor during the shorter measurement time than the period of the classical harmonic vibrations. This is actually a new quantum mechanics measurement effect.

Interestingly and surprisingly, when the external degrees of freedom of the centers of mass motions are fully decoupled with the internal degrees of freedom of the system, in some conditions the decoupled motions of atomic centers of mass indeed have the effects on the internal entanglement of atoms. The conditions are that the classical harmonic vibrations frequencies of atomic centers of mass are very low, i.e., their period is rather large, and that experimenters measure the entanglement concurrence of atomic internal freedoms during a short time interval less than one period. Of course, if experimenters measure the entanglement during a whole period, the decoupled motions of atoms do not have any effect on the entanglement concurrence of the internal freedoms. With John A. Wheeler's style [39], the new quantum mechanical measurement effect states *interaction without interaction*. 'without interaction' means that there are decoupling between centers of mass motion and the internal entanglement of atoms, 'interaction' means the entanglement concurrence between the states of the atoms depends on their respective centers of mass motions during a shorter measurement time interval than the period of the vibrations. The effect presented in the paper always works well as long as the internal degrees of freedom of the system (regardless of unitary evolution or non-unitary evolution with time) are decoupled to the external classical harmonic vibrations of atomic centers of mass. The new quantum mechanical measurement effect originates in nature from the non-unity trace of a classical harmonic oscillator during a shorter time interval than its period.

The classical harmonic vibrations of centers of mass reduce the concurrence to three orders of magnitude. The concurrence is sensitively modified by the initial phases and relative displacements rather than the absolute vibrant amplitudes under the condition $\Omega << g, \Omega << \omega$. The larger the initial phases become, the larger the concurrence becomes. Measuring phase-varying entanglement concurrence can confirm

the existence of micro vibrations. If we keep each measurement time Δt to be constant, we even obtain the frequency of the harmonic vibrations by measuring the maximal value of the concurrence during a small time Δt. Besides the entanglement concurrence of the two two-level atoms in a single-mode cavity modulated by the vibrant factor due to the measurement effect of classical vibrations, we have found another physical quantity, i.e., the mean number of atoms reaching an atomic detector, modulated by the vibrant factor due to the measurement effect with classical vibrations [40, 41]. The motion of the atoms can be used to control the entanglement concurrence, so the entanglement yields the information about the harmonic motions of the atoms. Moreover the vibrant factor F is independent of the absolute amplitudes and initial phases, the entanglement of atoms maybe provide another new way to detecting gravitational waves.

How does one directly measure the concurrence of two atomic states in a cavity? Mintert et al. proposed a method to directly measure entanglement of a pure state $|\psi\rangle$ through a single projection measurement on it twofold copy $|\psi\rangle \otimes |\psi\rangle$ [42]. If a state is available in a twofold copy, the state's concurrence is given as the expectation value of a single, suitably defined, self-adjoint operator, defined with respect to the twofold copy [43]. In a single run of an experiment, if a measurement on one of the duplicate subsystem reveals an antisymmetric or symmetric state, then the other duplicate subsystem is projected onto an antisymmetric or symmetric state. Concurrence can be measured with a single measurement on only one of the twin subsystems, as long as one deals with pure states. Moreover Mintert et al. also presented observable lower bounds of the squared concurrence for arbitrary bipartite mixed states by suitable, local parity measurements on its twofold copy [44].

Cavity QED can be employed for the measurement of concurrence of a two-atom entangled state, repeatedly produced in some experimental setup, proposed in [45, 46]. In the first step, one transfers the atomic state to a two-mode cavity field, initially in the vacuum state $|00\rangle$. This can be done by letting each of the atoms in the entangled state be resonant with one of the modes, so that if the atom enters the cavity in the upper state $|e\rangle$, it leaves one photon in the corresponding mode, exiting the cavity in the lower state $|g\rangle$,

$$(\alpha|ee\rangle + \beta|eg\rangle + \gamma|ge\rangle + \Delta|gg\rangle) \otimes |00\rangle \\ \rightarrow |gg\rangle \otimes (\alpha|11\rangle + \beta|10\rangle + \gamma|01\rangle + \Delta|00\rangle)$$

Next, one sends a second set of entangled atoms, prepared in the same state as the first one, through the cavity. The aim now is to determine the probability of finding one of the atoms, together with one of the cavity modes, in the Bell state $|\psi^-\rangle = (|g1\rangle - |e0\rangle)/\sqrt{2}$. This can be accomplished by using the technique in [47]. The result yields the concurrence of the two-atom state.

The concurrence of two photons has been experimentally directly measured by using linear optics method [48–51]. However, the experimentally measured concurrence of two atoms in a cavity has not been reported nowadays.

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