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Controlling straight line motion up to quantum levels by quantum measurements

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ABSTRACT

A classical charged particle moves along a straight line and interacts with a silver atom in its ground state fixed at the origin of coordinates through Biot-Savart coupling. If the silver atom initially lies in a spin superposition state of valence electron, in the co-moving reference frame the projected measurements of the spin of silver's valence electron along the z direction determine the moving direction of the charged particle. The projected measurements of that spin along the x or the y direction yield straight line motion versions of Schrödinger's cat state for the charged particle. When the spin coherence state of the silver atom suffers from its environment decoherence, as long as the silver atom's spin is entangled with the trajectories of the moving charged particle, the measurements on the silver atom's spin also yield the partial superpositions of different trajectories of the moving charged particle. It is possible to control mechanical motion up to quantum level by the means of quantum measurements.

Introduction

In 1935 Einstein, Podolsky and Rosen proposed a paradox (EPR paradox) to account for the incompleteness of quantum-mechanical description of physical reality [1]. Bohr replied to EPR based on the Copenhagen interpretation of quantum mechanics [2]. Enlightened by EPR paradox, Schrödinger proposed a cat state of quantum mechanical thought experiment [3]. Schrödinger's cat thought experiment remains a defining touchstone for modern interpretations of quantum mechanics, such as the Copenhagen interpretation [4], the many-world interpretation [5], the consistent histories [6] and quantum Bayesianism [7] etc. Physicists often use the way by which each interpretation deals with Schrödinger's cat as a way of illustrating and comparing the particular features, strengths, and weaknesses of each interpretation.

The upper limits on "cat states" are obtained through searching the superpostions of relatively large objects by the standards of quantum physics. So Schrödinger's cat states attract many attentions even from its being proposed. Successful experiments have been performed and various kinds of Schrödinger's cat states have been proposed. A cat state has been achieved with photons [8], a beryllium ion has been trapped in a superposed state [9]. All the superconducting electrons in the superconducting quantum interference device flowing both ways around the loop at once yield Schrödinger's cat state [10], a piezoelectric "tuning fork" has been constructed, which can be placed into a superposition of vibrating and non-vibrating states [11]. An experiment involving a flu virus has been proposed [12], an experiment involving a bacterium and an electromechanical oscillator has also been proposed [13]. Schrödinger's cat behavior is used to create X-ray movies of atomic motion with much more detail than ever before [14].

Will Schrödinger's cat state occur on the mechanical motions of a particle? If quantum measurements on the microscopic object yield Schrödinger's cat state of a macroscopic object, our quantum measurements maybe affect the evolutions of the universe. We have proposed a planet versions of Schrödinger's cat state when we studied the interaction between a classical charged particle travelling along a circular orbit and a silver atom in its ground state at the center of the circular orbit [15]. In this paper we study a straight line motion of a charged particle, which interacts with a silver atom in its ground state fixed at the origin of coordinates through Biot-Savart coupling. The projected measurements of the spin of silver's valence electron will control the straight line motion of the charged particle. This work is novelty, maybe important. The paper is organized as follows: In Section "The system and its state evolved with time" we present the detailed calculations for the system, the system's state evolved with time is worked out in the interaction picture. In Section "Controlling the straight line motion of the particle by using quantum measurement of the spin of a silver atom", we discuss how a quantum measurement of

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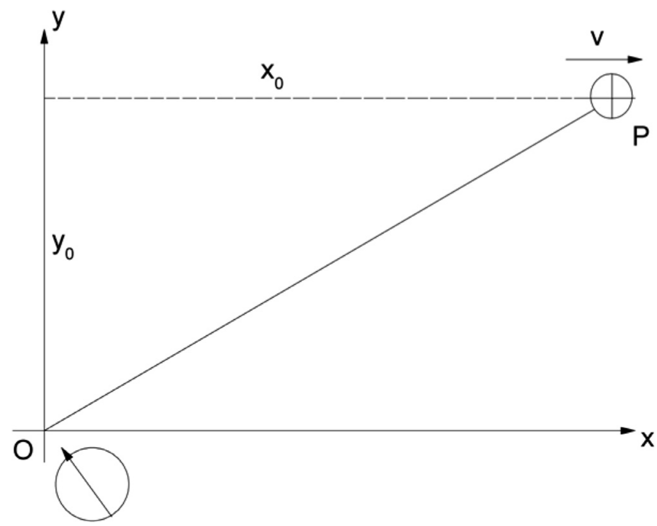

Fig. 1. A classical charged particle P with the mass m_q and the electric charge + q travelling along a straight line interacts with a silver atom in its ground state fixed at the origin of coordinates via Biot-Savart coupling.

the spin of a silver atom's valence electron control the straight line motion of the charged particle up to quantum level, the straight line motion versions of Schrödinger's cat state are presented. In Section "The evolutions of the superpositions in the presence of spin imperfections", we discuss the evolutions of the system consisting of the silver atom's electron and the moving charged particle when the electron spin coherence suffers from the environment decoherence. In Section "Summary", we give a summary of this paper.

The system and its state evolved with time

The system we study is shown in Fig. 1. A classical charged particle with the mass m_p and the electric charge + q travels along the straight line with the velocity $\vec{v}$ in the x direction. The particle P possesses its kinetic energy. Its Hamiltonian is given by

$$\hat{H}_0 = \frac{\hat{p}_x^2}{2m_p}. \tag{1}$$

In Eq. (1), $\hat{p}_x$ is the momentum operator along the x direction, m_P is the mass of the particle. A silver atom in its ground state is fixed at the origin of coordinates; the reason why we choose a silver atom in its ground state is that the silver atom in its ground state is very simple, and its electron configuration is $[Kr]4d^{10}5s$. The magnetic moment of the silver atom is just that of the $5s$ valence electron's spin. What's more, the spin of the valence electron of a silver atom can be conveniently measured by using the Stern-Gerlach apparatus. As shown in Fig. 1, the direction of the angular momentum of the particle P with respect to the origin of coordinates is along the -z axis. According to Biot-Savart law, the magnetic field at the origin of coordinates produced by the charged particle P is given by

$$\vec{B} = \frac{q}{4\pi\varepsilon_0 m_P c^2}\frac{\vec{L}}{r_0^3}. \tag{2}$$

In Eq. (2), ε_0 is vacuum's dielectric constant, c is the light speed in vacuum, $\vec{L} = -L_z\vec{k}$ is the angular momentum of the particle P with respect to the origin of coordinates along the -z axis and $r_0 = \sqrt{x_0^2 + y_0^2}$ is the distance between the particle P and the origin of coordinates [16]. The interaction energy between the spin of the silver atom and the magnetic field produced by the particle P reads

$$\hat{H}_I = -\vec{\mu}_s\cdot\vec{B} = \frac{qq_e\hbar}{8\pi\varepsilon_0 m_P m_e c^2 r_0^3}\vec{L}\cdot\vec{\sigma} = \frac{-qq_e\hbar}{8\pi\varepsilon_0 m_P m_e c^2 r_0^3}\hat{L}_z\hat{\sigma}_z. \tag{3}$$

Actually, the spin of the silver atom is just that of the 5 s valence electron of the silver atom in its ground state, and the magnetic moment of the silver atom is $\vec{\mu}_s = -\frac{q_e\hbar}{2m_e}\vec{\sigma}$. Here, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli matrix, m_e is the mass of the electron, q_e is electric charge of an electron, $\hbar$ is the reduced Planck constant. In Eq. (3) we have $\hat{L}_z = x\hat{p}_y - y\hat{p}_x = -y_0\hat{p}_x$ because the particle's momentum p_y in the y direction is zero. Then Eq. (3) become

$$\hat{H}_I = \frac{-qq_e\hbar}{8\pi\varepsilon_0 m_P m_e c^2 r_0^3}\hat{L}_z\hat{\sigma}_z = \frac{qq_e y_0\hbar}{8\pi\varepsilon_0 m_P m_e c^2 r_0^3}\hat{p}_x\hat{\sigma}_z. \tag{4}$$

The total energy of the system is $\hat{H} = \hat{H}_0 + \hat{H}_I$, $[\hat{H}_I, \hat{H}_0] = 0$ is obvious. Solving the Schrödinger equation in the interaction picture, where the interaction Hamiltonian is given by $\hat{H}_I(t) = e^{i\hat{H}_0 t/\hbar}\hat{H}_I e^{-i\hat{H}_0 t/\hbar} = \hat{H}_I$, is convenient and the solution of time-dependent Schrödinger equation is given by

$$|\psi\rangle = e^{-i\hat{H}_I t/\hbar}\,|\psi_0\rangle. \tag{5}$$

Given $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$, $\hat{\sigma}_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$ and $\xi \equiv \frac{qq_e y_0\hbar}{8\pi\varepsilon_0 m_P m_e c^2 r_0^3}$, we obtain $e^{-i\hat{H}_I t/\hbar} = e^{\frac{-i\xi t}{\hbar}\hat{p}_x\hat{\sigma}_z} = e^{\frac{-i\xi t}{\hbar}\hat{p}_x}|\uparrow\rangle\langle\uparrow| + e^{\frac{i\xi t}{\hbar}\hat{p}_x}|\downarrow\rangle\langle\downarrow|$, where $|\uparrow\rangle$, $|\downarrow\rangle$ are the spin states of $5s$ valence electron. The initial state of the system is assumed to be $|\psi_0\rangle = |x_0\rangle \otimes (\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$ with $|\alpha|^2 + |\beta|^2 = 1$ where $|x_0\rangle$ being the classical charged particle's position wavefunction. The superposition state $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ is the state of the silver atom. The solution of the Schrödinger equation, Eq. (5), reads

$$\begin{aligned}|\psi_t\rangle &= e^{-i\hat{H}_I t/\hbar}\,|\psi_0\rangle = \left(e^{\frac{-i\xi t}{\hbar}\hat{p}_x}|\uparrow\rangle\langle\uparrow|\right.\\ &\quad\left. + e^{\frac{i\xi t}{\hbar}\hat{p}_x}|\downarrow\rangle\langle\downarrow|\right)[|x_0\rangle(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)]\\ &= \alpha\,|x_0 + \xi t\rangle|\uparrow\rangle + \beta\,|x_0 - \xi t\rangle|\downarrow\rangle.\end{aligned} \tag{6}$$

In deriving Eq. (6), we use relationships $e^{\frac{-i\xi t}{\hbar}\hat{p}_x}|x_0\rangle = |x_0 + \xi t\rangle$ and $e^{\frac{i\xi t}{\hbar}\hat{p}_x}|x_0\rangle = |x_0 - \xi t\rangle$. $|x_0 + \xi t\rangle$ and $|x_0 - \xi t\rangle$ in Eq. (6) are the position wavefunctions of a classical charged particle as a function of time t. The operator $e^{-i\lambda\hat{p}/\hbar}$ is investigated in detail in Ref. [17], where λ is a real number and $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ is the momentum operator.

Controlling the straight line motion of the particle by using quantum measurement of the spin of a silver atom

The state of the system, Eq. (6), is the maximum entanglement state between the position wavefunction of the particle P and the spin of the silver atom. Surprisingly and interestingly the measurement of the silver atom's spin determines the particle's position in the co-moving frame of reference. For instance, after the projected measurement $\sigma_{z\uparrow}$ of the silver atom's spin along the z direction, the measured state of the system reads

$$\begin{aligned}|\psi_{z\uparrow}\rangle &\equiv |\uparrow\rangle\langle\uparrow||\psi_t\rangle = |\uparrow\rangle\langle\uparrow|[\alpha\,|x_0 + \xi t\rangle|\uparrow\rangle + \beta\,|x_0 - \xi t\rangle|\downarrow\rangle]\\ &= \alpha\,|x_0 + \xi t\rangle|\uparrow\rangle.\end{aligned} \tag{7}$$

In our thought experiment, we can measure the valence electron's spin state of a silver atom by using a mini Stern-Gerlach apparatus at the origin of coordinates. Eq. (7) implies that after the projected measurement $\sigma_{z\uparrow}$ of the silver atom's spin, the charged particle P travels along the x direction, which is our intuitive expectation. If we perform the projected measurement$\sigma_{z\downarrow}$ of the silver atom's spin, the measured state of the system is written as

$$\begin{aligned}|\psi_{z\downarrow}\rangle &\equiv |\downarrow\rangle\langle\downarrow||\psi_t\rangle = |\downarrow\rangle\langle\downarrow|[\alpha\,|x_0 + \xi t\rangle|\uparrow\rangle + \beta\,|x_0 - \xi t\rangle|\downarrow\rangle]\\ &= \beta\,|x_0 - \xi t\rangle|\downarrow\rangle.\end{aligned} \tag{8}$$

The charged particle P travels along the –x direction after the projected $\sigma_{z\downarrow}$ measurement of the silver atom's spin.

It is worthy of noting that the Eqs. (7) and (8) are in the co-moving frame of reference with the velocity $\vec{v}$ the same as that of the charged particle P. In the laboratory frame of reference the Eqs. (7) and (8) read, respectively,

$$|\psi_{z\uparrow}\rangle_{lab} = \alpha\ |x_0 + (v + \xi)t\rangle|\uparrow\rangle. \tag{9}$$

$$|\psi_{z\downarrow}\rangle_{lab} = \beta\ |x_0 + (v-\xi)t\rangle|\downarrow\rangle. \tag{10}$$

Eqs. (9) and (10) show that the charged particle P always travels along the x direction, no matter whether the projected measurements $\sigma_{z\uparrow}$ or $\sigma_{z\downarrow}$ of the silver atom's spin are performed. From the Eqs. (9) and (10) we can control mechanical motion, for instance the straight line motion, up to quantum level by using quantum measurements. Actually given the parameters the velocity$v = 100$ m/s, $x_0 = y_0 = 1m$, $r_0 = \sqrt{2}m$ for a flying hydrogen ion we have $\xi = \frac{qq_e y_0 \hbar}{8\pi\varepsilon_0 m_P m_e c^2 r_0^3} = 6.0 \times 10^{-23} m/s$ up to quantum level, much smaller than the velocity v.

In what follows we discuss the problems still in the co-moving frame of reference with the velocity $\vec{v}$ and our starting point is Eq. (6). What happens if the measurements $\sigma_{x\uparrow}$ or $\sigma_{y\uparrow}$ of the silver atom's spin along the x or the y direction are performed? The eigenstates of σ_x and σ_y are, respectively, written as

$$|\varphi_{x\uparrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle), |\varphi_{x\downarrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle),$$
$$|\varphi_{y\uparrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle), |\varphi_{y\downarrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - i|\downarrow\rangle).$$

From the above formula, we obtain

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle + |\varphi_{x\downarrow}\rangle), |\downarrow\rangle = \frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle - |\varphi_{x\downarrow}\rangle)$$
$$|\uparrow\rangle = \frac{1}{\sqrt{2}}(|\varphi_{y\uparrow}\rangle + |\varphi_{y\downarrow}\rangle), |\downarrow\rangle = \frac{1}{i\sqrt{2}}(|\varphi_{y\uparrow}\rangle - |\varphi_{y\downarrow}\rangle). \tag{11}$$

Substituting Eq. (11) into Eq. (6), the state of the system, Eq. (6), can be rewritten as

$$\begin{aligned} |\psi_t\rangle &= \alpha\ |x_0 + \xi t\rangle|\uparrow\rangle + \beta\ |x_0 - \xi t\rangle|\downarrow\rangle \\ &= \alpha\ |x_0 + \xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle + |\varphi_{x\downarrow}\rangle) + \beta\ |x_0 - \xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle - |\varphi_{x\downarrow}\rangle), \\ &= \alpha\ |x_0 + \xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{y\uparrow}\rangle + |\varphi_{y\downarrow}\rangle) + \beta\ |x_0 - \xi t\rangle\frac{1}{i\sqrt{2}}(|\varphi_{y\uparrow}\rangle - |\varphi_{y\downarrow}\rangle). \end{aligned} \tag{12}$$

From Eq. (12), the projected measurements $\sigma_{x\uparrow}$ and $\sigma_{y\uparrow}$of the silver atom's spin yield

$$\begin{aligned} &|\varphi_{x\uparrow}\rangle\langle\varphi_{x\uparrow}|\left[\alpha\ |x_0 + \xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle + |\varphi_{x\downarrow}\rangle) + \beta\ |x_0 - \xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{x\uparrow}\rangle - |\varphi_{x\downarrow}\rangle)\right] \\ &\qquad = \frac{1}{\sqrt{2}}(\alpha\ |x_0 + \xi t\rangle + \beta\ |x_0 - \xi t\rangle)|\varphi_{x\uparrow}\rangle, \\ &|\varphi_{y\uparrow}\rangle\langle\varphi_{y\uparrow}|\left[\alpha\ |x_0 + \xi t\rangle\frac{1}{\sqrt{2}}(|\varphi_{y\uparrow}\rangle + |\varphi_{y\downarrow}\rangle) + \beta\ |x_0 - \xi t\rangle\frac{1}{i\sqrt{2}}(|\varphi_{y\uparrow}\rangle - |\varphi_{y\downarrow}\rangle)\right] \\ &\qquad = \frac{1}{\sqrt{2}}(\alpha\ |x_0 + \xi t\rangle - i\beta\ |x_0 - \xi t\rangle)|\varphi_{y\uparrow}\rangle. \end{aligned} \tag{13}$$

We observe that after the projected measurements $\sigma_{x\uparrow}$ and $\sigma_{y\uparrow}$ of the silver atom's spin, the spins are expected to be $|\varphi_{x\uparrow}\rangle$ and$|\varphi_{y\uparrow}\rangle$, respectively; however, the charged particle P is in macroscopic position superposition states $\frac{1}{\sqrt{2}}(\alpha\ |x_0 + \xi t\rangle + \beta\ |x_0 - \xi t\rangle)$ and $\frac{1}{\sqrt{2}}(\alpha\ |x_0 + \xi t\rangle - i\beta\ |x_0 - \xi t\rangle)$, respectively. The macroscopic superposition states are the typical Schrödinger-cat states; here we name them as the *straight line motion versions of Schrödinger-cat state*. Is it possible? The classical charged particle P travels not only along the x direction but also along the –x direction at the same time, which is very surprising in our classical physics intuition. In quantum mechanics, the macroscopic straight line motion versions of Schrödinger's cat state indeed exist, they are prepared by quantum measurements.

The evolutions of the superposions in the presence of spin imperfections

From the previous discussions we know that if the silver atom is in the pure superposition state $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$, the system initial state is $|\psi_0\rangle = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |x_0\rangle$, then the state evolved with time is given by$|\psi_t\rangle = \alpha\ |x_0 + \xi t\rangle|\uparrow\rangle + \beta\ |x_0 - \xi t\rangle|\downarrow\rangle$, i.e. Eq. (6), in the interaction picture. Eq. (6) is the maximal entangled state of the silver atom's spin and the trajectories of the moving charged particle, the subsequent measurements on the silver atom's spin yield the superpositions of different trajectories of the moving charged particle. The entanglement between the silver atom's spin and the trajectories of the moving charged particle is necessary for the preparations of the superpositions of the trajectories. The silver atom's spin as a single qubit always suffers from the decoherence of its environment, its pure superposition state will become an imperfect state described by a density matrix. In the interaction picture the initial state of the system is $\rho_I(0)$, the state evolved with time obeys

$$\rho_I(t) = U(t)\rho_I(0)U^\dagger(t) \tag{14}$$

where the evolution operator is $U(t) = e^{-i\hat{H}_I t/\hbar}$. As discussed in the previous sections, the useful expressions for the evolution operator are listed: $e^{-i\hat{H}_I t/\hbar} = e^{\frac{-i\xi t}{\hbar}\hat{p}_x}|\uparrow\rangle\langle\uparrow| + e^{\frac{i\xi t}{\hbar}\hat{p}_x}|\downarrow\rangle\langle\downarrow|$, $e^{\frac{-i\xi t}{\hbar}\hat{p}_x}|x_0\rangle = |x_0 + \xi t\rangle$ and $e^{\frac{i\xi t}{\hbar}\hat{p}_x}|x_0\rangle = |x_0 - \xi t\rangle$.

Without loss of generality, we study the evolution of the initial state,

$$\rho_I(0) = (p\ |\Omega\rangle\langle\Omega| + (1-p)I/2) \otimes |x_0\rangle\langle x_0| \tag{15}$$

In Eq. (15) $|\Omega\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$ is the silver atom's spin coherence state, $I = |\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|$ is the unit matrix and the real parameter $p \in [0, 1]$ is the percentage of the spin's coherence state. The silver atom's spin state is the 'isotropic state' of a single qubit, the spin and the trajectory of the charged particle are initially separated. According to Eq. (14), we obtain the state of the system evolved with time in the basis $\{|\uparrow,x_0 + \xi t\rangle, |\uparrow,x_0 - \xi t\rangle, |\downarrow,x_0 + \xi t\rangle, |\downarrow,x_0 - \xi t\rangle\}$

$$\rho_I(t) = \begin{pmatrix} 1/2 & 0 & 0 & p/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ p/2 & 0 & 0 & 1/2 \end{pmatrix}. \tag{16}$$

In order to prepare the superpositions of the trajectories of the charged particle, we need to know whether there exists the entanglement of Eq. (16) between the silver atom's spin and the trajectories of the moving charged particle or not. The entanglement is described by the concurrence (one of entanglement of formations) $C(\rho_I(t)) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})$, where the quantities λ_i are the eigenvalues of the matrix $\rho_I(t)(\sigma_A^y \otimes \sigma_B^y)\rho_I^*(t)(\sigma_A^y \otimes \sigma_B^y)$ arranged in decreasing order. $\rho_I^*(t)$ is the elementwise complex conjugation of $\rho_I(t)$, and $\sigma_A^y \otimes \sigma_B^y$ is the direct product of Pauli matrix expressed [18]. The eigenvalues of the matrix $\rho_I(t)(\sigma_A^y \otimes \sigma_B^y)\rho_I^*(t)(\sigma_A^y \otimes \sigma_B^y)$ are, respectively, $\lambda_1 = (1 + p)^2/4$, $\lambda_2 = (1-p)^2/4$, $\lambda_{3,4} = 0$, so the concurrence $C(\rho_I(t))$ is p.

As long as the real parameter $p \neq 0$, the entanglement between the silver atom's spin and the trajectories of the moving charged particle always exists, then the subsequent measurements on the silver atom's spin should prepare the superpositions of different trajectories of the moving charged particle. For instance we perform a projection measurement $P_{x\uparrow} = |\varphi_{x\uparrow}\rangle\langle\varphi_{x\uparrow}|$ on the silver atom's spin with $|\varphi_{x\uparrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$, the state of the system, i.e. Eq. (16), after the measurement will become $\widetilde{\rho}_{x\uparrow} = \frac{P_{x\uparrow}\rho_I(t)P_{x\uparrow}}{Tr[P_{x\uparrow}\rho_I(t)P_{x\uparrow}]}$. Omitting the non-essential coefficient $Tr[P_{x\uparrow}\rho_I(t)P_{x\uparrow}]$, the state of the system is given by

$$\widetilde{\rho}_{x\uparrow} = |\varphi_{x\uparrow}\rangle\langle\varphi_{x\uparrow}| \otimes \left(p\ |\Xi_x\rangle\langle\Xi_x| + \frac{1-p}{2}I\right) \tag{17}$$

where $|\Xi_x\rangle = \frac{1}{\sqrt{2}}(|x_0 + \xi t\rangle + |x_0 - \xi t\rangle)$ is the coherence superposition state of the trajectory of the moving charged particle and $I = |x_0 + \xi t\rangle\langle x_0 + \xi t| + |x_0 - \xi t\rangle\langle x_0 - \xi t|$ is the unit matrix of the moving charged particle. The state of the moving charged particle after the projection measurement $P_{x\uparrow} = |\varphi_{x\uparrow}\rangle\langle\varphi_{x\uparrow}|$ on the silver atom's spin will become an imperfect state, however, including the coherence superposition of the trajectories. If the projection measurement on the silver atom's spin is $P_{y\uparrow} = |\varphi_{y\uparrow}\rangle\langle\varphi_{y\uparrow}|$ with $|\varphi_{y\uparrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle)$, omitting the non-essential coefficient $Tr[P_{y\uparrow}\rho_I(t)P_{y\uparrow}]$, the state of the system, i.e.

Eq. (16), after the measurement is given by

$$\widetilde{\rho}_{y\uparrow} = |\varphi_{y\uparrow}\rangle\langle\varphi_{y\uparrow}| \otimes \left(p\ |\Xi_y\rangle\langle\Xi_y| + \frac{1-p}{2}I\right) \tag{18}$$

where $|\Xi_y\rangle = \frac{1}{\sqrt{2}}(|x_0+\xi t\rangle - i\,|x_0-\xi t\rangle)$ is the coherence superposition state of the trajectory of the moving charged particle and $I = |x_0+\xi t\rangle\langle x_0+\xi t| + |x_0-\xi t\rangle\langle x_0-\xi t|$ is the unit matrix of the moving charged particle. The state of the moving charged particle after the projection measurement $P_{y\uparrow} = |\varphi_{y\uparrow}\rangle\langle\varphi_{y\uparrow}|$ on the silver atom's spin will also become an imperfect state, however, including the coherence superposition of the trajectories. We do not consider the influence of the particle's environment on its motions, as the environment only changes the particle's velocity, however, does not change the structures of the system's quantum states which are only determined by the interaction between the silver atom and the charged particle.

Summary

We have studied a classical charged particle travelling along a straight line and interacting with a silver atom in its ground state fixed at the origin of coordinates through Biot-Savart coupling. We calculate in detail the evolution of the system's state with time. If the silver atom initially lies in a superposition state of the valence electron spin, the projected measurements $\sigma_{z\uparrow}$ or $\sigma_{z\downarrow}$ of the silver atom spin along the z direction control the straight line motion velocity of the charged particle P up to quantum level. The projected measurements $\sigma_{x\uparrow}$ or $\sigma_{y\uparrow}$ of the silver atom's spin along the x or the y directions yield the straight line motion versions of the Schrödinger's cat state of the charged particle P. Please note that the projected measurements do not change the direction of the flying charged particle P. Actually, the calculations are performed in an interaction picture, so the observer lies in the frame of reference co-moving with the velocity v. In the co-moving frame of reference, the observer 'sees' that projected measurements control the flying directions of the charged particle or yield the macroscopic superposition states of Schrödinger's cat state. When the spin coherence state of the silver atom $\alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$ suffers from its environment decoherence, the coherence superposition state will become an imperfect state described by a density matrix. We obtain the system's state evolved with time and find that as long as the percentage of the spin coherence state is not zero, the entanglement between the silver atom's spin and the trajectories of the moving charged particle always exists, then the measurements on the silver atom's spin yield the superpositions of different trajectories of the moving charged particle. The straight line motion versions of the Schrödinger's cat state also look very strange and are impossible in the classical world, however, these states can be prepared through quantum measurements, so they indeed exist in the quantum world.

Needless to say, measurements are very important in quantum mechanics, and do change the system's states. Wheeler summarized the heart of the Copenhagen interpretation, 'No elementary quantum phenomenon is a phenomenon until it is a registered (observed) phenomenon' [19]. This paper and our previous work [15] make it possible that we control mechanical motion up to quantum level by the means of quantum measurements.

Acknowledgments

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