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Optics Communications

Optics Communications 281 (2008) 2913-2917

www.elsevier.com/locate/optcom

Temporal behavior of dark spatial solitons in closed-circuit photovoltaic media

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Received 29 October 2007; received in revised form 14 December 2007; accepted 8 January 2008

Abstract

We show that the time-dependent nonlinear wave equation in closed-circuit photovoltaic media can exhibit quasi-steady-state and steady-state spatial solitons. We demonstrate that the formation time of open-circuit quasi-steady-state and open-circuit steady-state dark solitons decreases with an increase in the intensity ratio of the soliton, which is the ratio between the soliton peak intensity and the dark irradiance. We find that for the time-dependent nonlinear wave equation that exhibits only an open-circuit steady-state dark soliton, changing the electric current density J_0 does not generate quasi-steady-state dark solitons and affects the formation time of steady-state dark solitons, changing J_0 gives rise to three different time evolution regimes of the full width half maximum of the soliton's intensity. The first regime shows that the formation time of steady-state dark solitons is independent of J_0 . The second regime shows that the formation time of steady-state dark solitons decreases with an increases in J_0 and the formation time of quasi-steady-state dark solitons increases with J_0 . The third regime shows that changing J_0 are specified and solitons in the time-dependent nonlinear wave equation, of which the formation time increases with J_0 . Constant the formation time of quasi-steady-state dark solitons increases with J_0 . The third regime shows that changing J_0 and the formation time of quasi-steady-state dark solitons increases with J_0 . The third regime shows that changing J_0 enables only steady-state dark solitons in the time-dependent nonlinear wave equation, of which the formation time increases with J_0 .

PACS: 42.65.Tg; 42.65.Sf; 42.65.Hw; 42.65.Jx

Keywords: Soliton; Transient state; Photorefractive effect; Nonlinear optics

Photorefractive (PR) spatial solitons have attracted much attention because of their possible applications for optical switching and routing. At present, three types of PR bright and dark solitons are known: quasi-steady-state solitons [1–3], of which the first theoretical model was based on the PR coupling of the various spatial Fourier components of a traveling beam, and screening solitons [4–7] and photovoltaic (PV) solitons [8–10], which occur in steady-state. In particular, screening solitons require the application of an external bias field, whereas PV soli-

* Corresponding author. E-mail address: keqinglu@opt.ac.cn (K. Lu). tons can exist in PR materials with appreciable PV coefficients. Recently, steady-state screening-photovoltaic solitons have been predicted [11–13] and observed [14] in biased photorefractive–photovoltaic materials, which change into screening solitons when the bulk PV effect is neglected and PV solitons when the external field is absent. However, the major drawback of the PR effect is its low response time when compared to that of the Kerr effect. Therefore the formation time of PR solitons is longer than that of Kerr solitons. Of course, at this point, it would be of interest to know the temporal behavior of PR solitons. A non-stationary bidimensional model leading to numerical simulations has been proposed [15]. Unfortunately it does not

^{0030-4018/\$ -} see front matter \odot 2008 Elsevier B.V. All rights reserved. doi:10.1016/j.optcom.2008.01.007

provide an explicit wave propagation equation. Subsequently, a time-dependent theory for bright solitons has been developed and confirmed by experiments in which bright solitons were studied in $Bi_{12}TiO_{20}$ and $Sr_{0.61}Ba_{0.39}$ -Nb₂O₆ crystals with external applied field [16,17]. Very recently, some preliminary experiments with the temporal behavior of spatial solitons in biased photorefractive–photovoltaic materials have been demonstrated [18]. Moreover, the temporal behavior of dark solitons in open-circuit photovoltaic media has been predicted theoretically and has also been observed experimentally [19]. However, the temporal behavior of dark solitons in closed-circuit photovoltaic media has not been fully investigated yet. On the other hand, can one control the formation time of such dark solitons?

In this paper, we present that the time-dependent nonlinear wave equation in closed-circuit photovoltaic media can exhibit quasi-steady-state and steady-state spatial solitons. We show that the formation time of open-circuit quasi-steady-state and open-circuit steady-state dark solitons decreases with an increase in the intensity ratio of the soliton, which is the ratio between the soliton peak intensity and the dark irradiance. We find that for the time-dependent nonlinear wave equation that exhibits only an open-circuit steady-state dark soliton, changing the electric current density J_0 does not generate quasi-steady-state dark solitons and affects the formation time of steady-state dark solitons. When the intensity ratio is much smaller than unity, the time-dependent nonlinear wave equation exhibits only steady-state dark solitons. We also find that for the time-dependent nonlinear wave equation that exhibits an open-circuit quasi-steady-state dark soliton, changing J_0 gives rise to three different time evolution regimes of the full width half maximum (FWHM) of the soliton's intensity. The first regime shows that the formation time of steady-state dark solitons increases with J_0 whereas the formation time of quasi-steady-state dark solitons is independent of J_0 . The second regime shows that the formation time of steady-state dark solitons decreases with an increases in J_0 and the formation time of quasisteady-state dark solitons increases with J_0 . The third regime shows that changing J_0 enables only the steady-state dark soliton in the time-dependent nonlinear wave equation, of which the formation time increases with J_0 .

To start, let us consider an optical beam that propagates in a photovoltaic material along the z axis and is allowed to diffract only along the x direction. In this case, the photorefractive space-charge field formed by this inhomogeneous beam can be obtained from the time-dependent bandtransport model. In a photovoltaic medium, the model is represented by the following set of equations [10,16,19]: ΔM^+

$$\frac{\partial N_{\rm D}}{\partial t} = (sI + \beta_{\rm T}) \left(N_{\rm D} - N_{\rm D}^+ \right) - \gamma n N_{\rm D}^+,\tag{1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial J}{\partial x} = 0, \tag{2}$$

$$\rho = e \left(N_{\rm D}^+ - N_{\rm A} - n \right), \tag{3}$$

$$\frac{\partial}{\partial x}(\varepsilon_0\varepsilon_r E_s) = \rho,\tag{4}$$

$$\widehat{J} = e\mu nE_s + \mu k_{\rm B}T \frac{\partial n}{\partial x} + \kappa s \left(N_{\rm D} - N_{\rm D}^+\right)I,\tag{5}$$

where $N_{\rm D}$ is the total donor number density, $N_{\rm D}^+$ is the number density of ionized donors, N_A is the number density of negatively charged acceptors that compensate for the ionized donors, n is the electron number density, $\beta_{\rm T}$ is the thermal excitation rate of the electrons, s is the photo excitation cross section, ρ is the charge density, \hat{J} is the electric current density, I = I(x,z) is the power density profile of the optical beam, which is defined by $I = |A(x,z)|^2$, A(x,z) is the slowly varying amplitude of the optical field defined by $E(x,z) = A(x,z) \exp[i(kx - \varpi t)] + c.c$ $(k = 2\pi n_e/s)$ λ_0 is the propagation constant, λ_0 is the common free-space wavelength, $n_{\rm e}$ is the unperturbed extraordinary index of refraction, and ϖ is the optical frequency). The other parameters are the photovoltaic constant κ , the electric charge e, the electron mobility μ , the recombination coefficient γ , the absolute temperature T, Boltzmann's constant $k_{\rm B}$, the dielectric constant ε_r of the material, and the permittivity of the vacuum ε_0 .

To establish a time-dependent relation between the space-charge field and the optical intensity, let us recall that in typical photorefractive materials $N_{\rm D}$ or $N_{\rm A} \gg n$ and $N_{\rm D}^+ \gg n$. Thus Eq. (3) yields $\rho \approx e(N_{\rm D}^+ - N_{\rm A})$. Substituting the expression we have just found for ρ into Eq. (4) yields $\varepsilon_0 \varepsilon_r \partial E / \partial x = e(N_D^+ - N_A)$. When $x \to \pm \infty$, $E(x \to \pm \infty, z) =$ E_0 (constant) [5,10], and thus $\partial E/\partial x = 0$. Form $\varepsilon_0 \varepsilon_r \partial E / \partial x = e(N_D^+ - N_A)$ we obtain that $N_D^+ = N_A$. Moreover, Eq. (1) describes the process of charge generation and recombination and gives the characteristic carrier's recombination time $1/\gamma n$ characterizing the response time for free-electron buildup. Eqs. (2) and (5) express the current distribution continuity and give the dielectric response time $\varepsilon_0 \varepsilon_r / e \mu n$, which is associated with the dynamics of the ionic charge buildup. In typical photorefractive materials, $\tau = (1/\gamma n)/(\varepsilon_0 \varepsilon_r / e \mu n) = e \mu / \varepsilon_0 \varepsilon_r \gamma \ll 1$. For example, its maximum value is 2×10^{-4} in LiNbO₃ and 1×10^{-2} in BaTiO₃ [16]. Therefore, Eq. (1) can be considered as a steady-state. Under these conditions and from Eq. (1) the electron number density can be obtained and it is given by

$$n = \frac{s(I+I_d)(N_{\rm D}-N_{\rm A})}{\gamma N_{\rm A}},\tag{6}$$

where $I_d = \beta_T / s$. Substituting $N_D^+ = N_A$ and Eq. (6) into Eq. (5) leads to

$$\widehat{J} = \frac{e\mu s(N_{\rm D} - N_{\rm A})}{\gamma N_{\rm A}} \left[(I + I_d) E_s + \frac{k_{\rm B} T}{e} \frac{\partial I}{\partial x} + E_{\rm p} I \right],\tag{7}$$

where $E_{\rm p} = \kappa \gamma N_{\rm A}/e\mu$ is the photovoltaic field constant. Substitution of Eqs. (4) and (7) into Eq. (2) yields

$$T_d I_d \frac{\partial^2 E_s}{\partial x \partial t} + \frac{\partial [(I+I_d)E_s]}{\partial x} + \frac{k_{\rm B}T}{e} \frac{\partial^2 I}{\partial x^2} + E_{\rm p} \frac{\partial I}{\partial x} = 0, \tag{8}$$

where $T_d = (\epsilon_0 \epsilon_r / e\mu) [\gamma N_A / \beta_T (N_D - N_A)]$. The integral of Eq. (8) leads to

$$T_{d}I_{d}\frac{\partial E_{s}}{\partial t} + (I+I_{d})E_{s} + \frac{k_{\rm B}T}{e}\frac{\partial I}{\partial x} + E_{\rm p}I = C_{1}, \qquad (9)$$

where C_1 is an integration constant. In steady stare and at $x \to \pm \infty$, $I(x \to \pm \infty) = I_{\infty}$ and $E_s(x \to \pm \infty) = E_0$. From Eq. (9) the integration constant C_1 in these regions $(x \to \pm \infty)$ can be determined and is given by $C_1 = (I_{\infty} + I_d)E_0 + E_pI_{\infty}$. By assuming that the optical intensity *I* varies slowly with time, the integral of Eq. (9) leads to

$$E_{s} = \left(E_{0}\frac{I_{\infty} + I_{d}}{I + I_{d}} - E_{p}\frac{I - I_{\infty}}{I + I_{d}} - \frac{k_{B}T}{e}\frac{1}{I + I_{d}}\frac{\partial I}{\partial x}\right) + C_{2}\exp\left(-\frac{I + I_{d}}{T_{d}I_{d}}t\right),$$
(10)

where C_2 is an integration constant. Substituting the initial condition that $E_s = 0$ into Eq. (10) leads to $C_2 = -E_0$ $(I_{\infty} + I_d)/(I + I_d) + E_p(I - I_{\infty})/(I + I_d) + [(k_B T/e)/(I + I_d)]$ $(\partial I/\partial x)$. Substituting the expression we have just found for C_2 into Eq. (10) yields the following relation:

$$E_{s} = \left(E_{0}\frac{I_{\infty} + I_{d}}{I + I_{d}} - E_{p}\frac{I - I_{\infty}}{I + I_{d}} - \frac{k_{B}T}{e}\frac{1}{I + I_{d}}\frac{\partial I}{\partial x}\right) \times \left[1 - \exp\left(-\frac{I + I_{d}}{T_{d}I_{d}}t\right)\right].$$
(11)

In the above equation the constant field E_0 can be determined from the potential condition $\oint E_s \cdot dl = 0$. Substituting Eq. (11) into Eq. (7) and transforming the electric current density into dimensionless form yield

$$J = \left(J_0 - \frac{I}{I_d} - \frac{k_{\rm B}T}{eI_d E_{\rm p}} \frac{\partial I}{\partial x}\right) \left[1 - \exp\left(-\frac{I + I_d}{T_d I_d}t\right)\right] + \frac{k_{\rm B}T}{eI_d E_{\rm p}} \frac{\partial I}{\partial x} + \frac{I}{I_d},$$
(12)

where $J_0 = (E_0/E_p)(1 + \delta) + \delta$, $\delta = I_\infty/I_d$, and $J = \hat{J}/[e\mu s(N_D - N_A)/\gamma N_A]E_pI_d$. Under strong bias conditions, all the terms associated with the process of diffusion $(k_BT/e \text{ term})$ can be neglected in Eqs. (11) and (12) [5]. In this case Eqs. (11) and (12) yield the following results:

$$E_{s} = \left(E_{0}\frac{I_{\infty} + I_{d}}{I + I_{d}} - E_{p}\frac{I - I_{\infty}}{I + I_{d}}\right) \left[1 - \exp\left(-\frac{I + I_{d}}{T_{d}I_{d}}t\right)\right],\tag{13}$$

$$J = \left(J_0 - \frac{I}{I_d}\right) \left[1 - \exp\left(-\frac{I + I_d}{T_d I_d}t\right)\right] + \frac{I}{I_d}.$$
 (14)

Evidently, Eq. (14) shows that J_0 is the steady-state electric current density. Substituting J_0 into Eq. (13) and transforming the space-charge field into dimensionless form yield

$$\widehat{E}_s = \frac{J_0 - \frac{I}{I_d}}{\frac{I}{I_d} + 1} \left[1 - \exp\left(-\frac{I + I_d}{T_d I_d}t\right) \right],\tag{15}$$

where $\hat{E}_s = E_s/E_p$. This space-charge field gives rise to, via the electro-optic Pockels effect, the refractive index variation in the photovoltaic material. The propagation of the optical field in the photovoltaic material with a low index

modulation can be described by the scalar wave propagation equation

$$\left(\frac{\partial}{\partial z} - \frac{\mathrm{i}}{2k} \frac{\partial^2}{\partial x^2}\right) A(x, z) = \frac{\mathrm{i}k}{n_{\rm e}} \Delta n(E_s) A(x, z), \tag{16}$$

where $\Delta n(E_s) = -(1/2)n_e^3 r_{\rm eff} E_s$ is the perturbation in the refractive index and $r_{\rm eff}$ is the effective electro-optic coefficient. We look for soliton solutions of the form

$$A(x,z) = u(x) \exp(i\Gamma z) \sqrt{I_d},$$
(17)

where Γ is the soliton propagation constant and u(x) is the normalized amplitude. Moreover, for simplicity, let us adopt the following dimensionless coordinate $\xi = x/d$, where $d = (\pm 2kb)^{-1/2}$ is the characteristic length scale and $b = (k/n_e)(1/2)n_e^3 r_{\rm eff} E_p$ is the parameter that characterizes the strength and the sign of the optical nonlinearity. By employing this transformation and by substituting Eqs. (15) and (17) into Eq. (16), we find the following time-dependent nonlinear wave equation:

$$u'' = \pm \left\{ \frac{\Gamma}{b} + \frac{J_0 - u^2}{1 + u^2} \left[1 - \exp\left(-\frac{1 + u^2}{T_d}t\right) \right] \right\} u, \tag{18}$$

where $u'' = d^2 u/d\xi^2$, etc. The integral of Eq. (18) leads to

$$p^{2} - p_{0}^{2} = \pm \left\{ \left(\frac{\Gamma}{b} - 1 \right) \left(u^{2} - u_{0}^{2} \right) + \left(J_{0} + 1 \right) \ln \left(\frac{u^{2} + 1}{u_{0}^{2} + 1} \right) - \left(J_{0} + 1 \right) \left[Ei \left(-\frac{u^{2} + 1}{T_{d}} t \right) - Ei \left(-\frac{u_{0}^{2} + 1}{T_{d}} t \right) \right] - \frac{T_{d}}{t} \left[\exp \left(-\frac{u^{2} + 1}{T_{d}} t \right) - \exp \left(-\frac{u_{0}^{2} + 1}{T_{d}} t \right) \right] \right\},$$
(19)

where $Ei(\theta) = -\int_{-\theta}^{\infty} [\exp(-\vartheta)/\vartheta] d\vartheta$, p = u', and $p(\xi = 0) = p_0$.

For dark solitons one requires that the boundary conditions are $u(\infty) = u_{\infty} \neq 0$, $u'(\infty) = u''(\infty) = 0$, and $u_0 = 0$. Using boundary conditions $u_{\infty} \neq 0$ and $u''(\infty) = 0$ and substituting $\xi \to \infty$ into Eq. (18) lead to

$$\frac{\Gamma}{b} = -\frac{J_0 - u_{\infty}^2}{1 + u_{\infty}^2} \left[1 - \exp\left(-\frac{1 + u_{\infty}^2}{T_d}t\right) \right].$$
(20)

Substituting Eq. (20), $\xi \to \infty$, and conditions $u_{\infty} \neq 0$, $u'(\infty) = 0$, and $u_0 = 0$ into Eq. (19) leads to

$$p_{0}^{2} = \pm \left\{ (J_{0}+1) \left[\frac{u_{\infty}^{2}}{1+u_{\infty}^{2}} - \ln\left(1+u_{\infty}^{2}\right) \right] + \frac{(J_{0}-u_{\infty}^{2})u_{\infty}^{2}}{1+u_{\infty}^{2}} \\ \times \exp\left(-\frac{1+u_{\infty}^{2}}{T_{d}}t\right) + (J_{0}+1) \left[Ei\left(-\frac{1+u_{\infty}^{2}}{T_{d}}t\right) \\ -Ei\left(-\frac{t}{T_{d}}\right) \right] + \frac{T_{d}}{t} \left[\exp\left(-\frac{1+u_{\infty}^{2}}{T_{d}}t\right) - \exp\left(-\frac{t}{T_{d}}\right) \right] \right\},$$
(21)

which reduces to the expression for steady-state dark photovoltaic solitons when $t \to \infty$. The reality of p_0 can be obtained only for the negative (lower) sign [9]. In the limit of $p_0^2 > 0$ and $u_0 = 0$, Eqs. (19)–(21) yield

$$p^{2} = (J_{0} + 1) \left[\frac{u^{2} - u_{\infty}^{2}}{1 + u_{\infty}^{2}} + \ln\left(\frac{1 + u_{\infty}^{2}}{1 + u^{2}}\right) + Ei\left(-\frac{1 + u^{2}}{T_{d}}t\right) - Ei\left(-\frac{1 + u_{\infty}^{2}}{T_{d}}t\right) \right] - \frac{(J_{0} - u_{\infty}^{2})(u^{2} + u_{\infty}^{2})}{1 + u_{\infty}^{2}} \times \exp\left(-\frac{1 + u_{\infty}^{2}}{T_{d}}t\right) + \frac{T_{d}}{t} \left[\exp\left(-\frac{1 + u^{2}}{T_{d}}t\right) - \exp\left(-\frac{1 + u_{\infty}^{2}}{T_{d}}t\right)\right],$$
(22)

from which the dark soliton's temporal behavior can be obtained by numerical integration. Recall now that for opencircuit condition $J_0 = 0$ and for short-circuit condition $J_0 \to W u_\infty^2 / [\Delta x (1 + u_\infty^2) + W]$, where W is the width of the photovoltaic crystal between the electrodes [9]. In the $W \gg \Delta x$ case and for $u_{\infty}^2 \ll 1000$, one quickly obtain $J_0 \approx u_{\infty}^2$ from the expression we have just found for J_0 . Obviously, J_0 can vary continuously from zero to the maximum value of J_0 . Fig. 1 shows the intensity FWHM of the dark solitons as a function of t/T_d for open-circuit conditions when $u_{\infty}^2 = 0.1$, 1, 10, and 100. Note that the time needed to reach the minimum transient intensity FWHM of the quasi-steady-state soliton is defined as its formation time [16], whereas the formation time of the steady-state soliton is given by t/T_d corresponding to the plateau starting point of the intensity FWHM curves in Fig. 1. A careful examination of Fig. 1 reveals that the formation time of open-circuit quasi-steady-state and open-circuit steadystate dark solitons decreases with an increase in u_{∞}^2 and open-circuit quasi-steady-state dark solitons exist only for big u_{∞}^2 . Moreover, two types of spatial soliton can be determined from Fig. 1. The first kind involves the so-called quasi-steady-state soliton, which is the transient character. The second category of soliton is the steady-state soliton, occurring in the steady state. The effect of J_0 on the formation time of the dark soliton has several interesting phenomena. First, consider the time-dependent nonlinear wave equation that exhibits only an open-circuit steadystate dark soliton. Fig. 2 depicts the intensity FWHM of dark solitons as a function of t/T_d for $u_{\infty}^2 = 1$ when



Fig. 1. Intensity FWHM of the dark soliton as a function of t/T_d for short-circuit conditions when $u_0^2 = 0.1$, 1, 10, and 100.

 $J_0 = 0, 0.5, \text{ and } 1$. This figure also shows that changing J_0 does not generate quasi-steady-state dark solitons and affects the formation time of steady-state dark solitons. Second, consider the time-dependent nonlinear wave equation that exhibits an open-circuit quasi-steady-state dark soliton. Fig. 3 depicts the intensity FWHM of dark solitons as a function of t/T_d for various values of J_0 between $0\leqslant J_0\leqslant u_\infty^2$ when $u_\infty^2=100.$ It reveals that changing J_0 gives rise to three different intensity FWHM time evolution regimes. Fig. 3a shows that the formation time of steadystate dark solitons increases with J_0 whereas the formation time of quasi-steady-state dark solitons is independent of J_0 . Fig. 3b shows that the formation time of steady-state dark solitons decreases with an increases in J_0 and the formation time of quasi-steady-state dark solitons increases with J_0 . Fig. 3c shows that changing J_0 enables only steady-state dark solitons in the time-dependent nonlinear wave equation, of which the formation time increases in J_0 .

Finally, it is instructive to explain that our results are correct. The expression for the time-dependent nonlinear wave equation of previously studied dark solitons in open-circuit photovoltaic media can be obtained from Eq. (22). Let us assume that the soliton's optical field amplitude is defined as $A(x,z) = \sqrt{rI_d}w(x) \exp(i\Gamma z)$, where r is the ratio of soliton peak intensity to dark irradiance. In this case, substituting $J_0 = 0$ into Eq. (22) leads to the same expression obtained from Eqs. (11) and (12) in Ref. [19]. Moreover, when $t \to \infty$, the physical system of quasisteady-state photovoltaic solitons. By substituting $t \to \infty$ into Eqs. (15), (18), and (22), we obtain Eqs. (13), (14), and (26) of Ref. [9], in which closed-circuit steady-state photovoltaic solitons were studied.

In conclusion, we have shown that the time-dependent nonlinear wave equation in closed-circuit photovoltaic media can exhibit quasi-steady-state and steady-state solitons. We have demonstrated that the formation time of open-circuit quasi-steady-state and open-circuit steadystate dark solitons decreases with an increase in u_{∞}^2 . We have found that for the time-dependent nonlinear wave



Fig. 2. Intensity FWHM of dark solitons as a function of t/T_d for $u_{\infty}^2 = 1$ when $J_0 = 0, 0.5$, and 1.



Fig. 3. Intensity FWHM of the dark soliton as a function of t/T_d for (a) $J_0 = 0$, 0.1, and 1, (b) $J_0 = 20$, 25, and 30, and (c) $J_0 = 60$, 80, and 100 when $u_{\infty}^2 = 100$.

equation exhibiting only an open-circuit steady-state dark soliton, changing J_0 does not generate quasi-steady-state dark solitons and affects the formation time of steady-state dark solitons and that for the time-dependent nonlinear wave equation exhibiting an open-circuit quasi-steady-state dark soliton, changing J_0 gives rise to three different intensity FWHM time evolution regimes, of which the properties were also shown.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (No. 10674176).

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