The Source of de Broglie Matter Wave

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Abstract: Comparing Schrödinger equation and Maxwell equation, Born probability density is regarded as the source of de Broglie matter wave. The source-denpendent Schrödinger equation is established for the matter wave radiation process for the first time. A two-level system is reinvestigated considering the effect of matter wave radiation. A modified Stern-Gerlach setup is proposed to verify the existence of matter wave radiation.

Keywords: source of de Broglie matter wave; matter wave radiation; source-dependent Schrödinger equation; two-level system; modified Stern-Gerlach setup

0 Introduction

In 1923 de Broglie brought forward that microcosmic particles have the duality of wave and particle enlightened by Einstein's light quantum theory[1]. Microcosmic particles' relation between wave and particle is the same to the one of light[2,3]. de Broglie matter wave is a key concept which is the cornerstone of wave mechanics founded by Schrödinger 1926[4,5,6,7]. As all know, Schrödinger equation is derived from common wave equation with de Broglie relation,

$$i\hbar\frac{\partial\psi}{\partial t}=(-\frac{\hbar^2}{2m}\nabla^2+V)\psi \quad (1).$$

Mechanical vibration generates mechanical wave, electromagnetic oscillation generates electromagnetic wave, Mass of accelerated motion generates gravitational wave, what generates de Broglie matter wave? A kinetic particle always takes its matter wave, how? Matter wave should also be generated by some source, the process that some source generates matter wave is called matter wave radiation(MWR) throughout this paper.

The paper is organized as follows. In section II the MWR equation named the source-dependent Schrödinger equation is established. The equation is applied to a two-level system. In section III a modified Stern-Gerlach setup to verify MWR existence is proposed. A brief summary is given in section IV.

1 The Source-dependent Schrödinger Equation and Applications to a Two-level System

In order to give equation of MWR, Maxwell equation is reviewed here. In Lorentz gauge $\partial_\mu A_\mu=0$, Maxwell equations can be written as follows:

$$\Box A_\mu=-\mu_0 J_\mu \quad (2),$$

where $A_\mu=(\vec{A},i\frac{\varphi}{c}),J_\mu=(\vec{j},ic\rho)$ are respectively four-dimension vector potential and four-dimension electron current density. When the right hand of Equ (2) equals zero, that's the situation in vacuum. It is the electron current density that radiates electromagnetic wave. Schrödinger equation (1) can be written as follows:

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$$i\hbar\frac{\partial\psi}{\partial t}-(-\frac{\hbar^2}{2m}\nabla^2+V)\psi=0\,.$$

Schrödinger equation corresponds to Maxwell equation without source in vacuum. Electron current density on the right hand of Maxwell equation (2) is electromagnetic field source. We suppose that the source of matter wave should be some density. Maybe Born matter wave density is a good choice. So MWR equation has the form below:

$$i\hbar\frac{\partial\psi}{\partial t}-(-\frac{\hbar^2}{2m}\nabla^2+V)\psi=k\psi^*\psi \quad (3),$$

where real number k measured experimentally is called by MWR coefficient. Equ (3) is so called the source-dependent Schrödinger equation and is a main equation in this paper.

The source-dependent Schrödinger equation can be resolved with Green function method. Firstly we find the solution of the following equation that Green function satisfies:

$$[i\hbar\frac{\partial}{\partial t'}-(-\frac{\hbar^2}{2m}\nabla'^2+V(x'))]G^+(x',t',x,t)=\hbar\delta(t'-t)\delta^3(x'-x)$$

where $G^+(x',t',x,t)$ is retarded Green function which can be evaluated by the the method of Feynman path integral. $G^+(x',t',x,t)$ is such that

$$G^+(x',t',x,t)=-i\theta(t'-t)\sum_E U^*{}_E(x)U_E(x')\exp[-iE(t'-t)/\hbar] \quad (4),$$

in which $\theta(t'-t)=\begin{cases}1 & t'-t>0\\ 0 & t'-t<0\end{cases}$ is step function. In Equ (4), E and $U_E(x)$ are respectively eigen value and eigen function of the steady state Schrödinger equation. So the solution of equation (3) is written below

$$\psi(x,t)=\psi_0(x,t)+\frac{k}{\hbar}\int d^3x'dt'G^+(x',t',x,t)\psi^*(x',t')\psi(x',t') \quad (5)$$

where $\psi_0(x,t)$ is just the solution of the time-dependent Schrödinger equation (1). Equ (5) can be evaluated iteratively. Time-independent Hamiltonian $H(x)$ acts on the wave function (5), we get

$$H(x)\psi(x,t)=H(x)\psi_0(x,t)+\frac{k}{\hbar}\int d^3x'dt'[H(x)G^+(x',t',x,t)]\psi^*(x',t')\psi(x',t')$$

$$=E[\psi_0(x,t)+\frac{k}{\hbar}\int d^3x'dt'G^+(x',t',x,t)\psi^*(x',t')\psi(x',t')]=E\psi(x,t)\,.$$

The above equation demonstrates that our MWR theory does not conflict with the accepted quantum mechanics.

For a two-level system, the system wavefunction is written as $|\psi>=\begin{pmatrix}a\\ b\end{pmatrix}$, and the Hamiltonian is $H=\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}$ where λ_1,λ_2 are eigenvalues of exciting state $\begin{pmatrix}1\\ 0\end{pmatrix}$ and ground state $\begin{pmatrix}0\\ 1\end{pmatrix}$. The source-dependent Schrödinger equation (3) is written as follows

$$\begin{cases} i\hbar\dot{a} = \lambda_1 a + k(a^* a + b^* b) \\ i\hbar\dot{b} = \lambda_2 b + k(a^* a + b^* b) \end{cases}$$

where k is the matter wave radiation coefficient, and probability amplitudes a and b are complex numbers. Without generality atomic unit(a.u.) is adopted, i.e. $\hbar = 1$, $\lambda_1 = -\lambda_2 = 1$. We suppose that system is in ground state at first, so the initial conditions are $b^* b = 1$ and $a^* a = 0$. The probability evolutions with time in ground state and exciting state are show in Fig.1, where coefficient k=0.1 in atomic unit.

From Fig.1 we observe that the system will not always be in the ground state and about 180 a.u. later the system is in the exciting state. The probability density is not a smooth curve but a variance. The oscillating period is about 360 a.u., several tens femtoseconds in System International. Both of the phenomena are due to the effect of matter wave radiation. If there is not

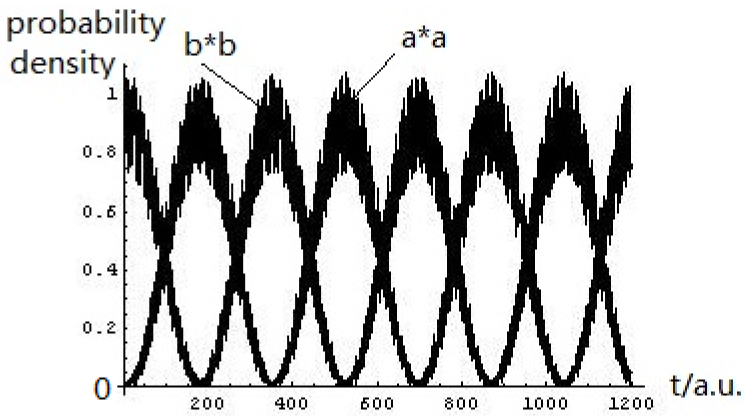

Fig.1 probability evolution with time k=0.1, initially a*a=0,b*b=1

matter wave radiation, the system will always in the ground state. The matter wave generated by Born probability makes system in ground state be distributed in the exciting state. The total probability of ground state and exciting state equals to 1, i.e. $|a|^2 + |b|^2 = 1$. It is apparent that the system is always in transition between exciting state and ground state.

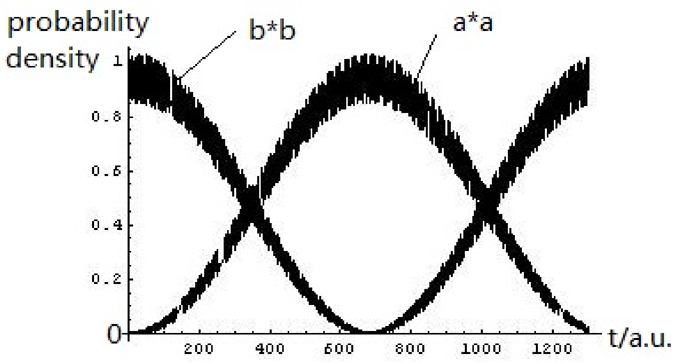

Fig.2 probability evolution with time k=0.05, initially a*a=0,b*b=1.

When radiation coefficient k=0.05 in atomic unit, with the same initial conditions as k=0.1 the system probability evolutions with time in ground state and exciting state are show in Fig.2. Comparing it with Fig.1 we find that the oscillating period is about 1400 a.u. The MWR coefficient k affects the oscillating period of system probability. The smaller coefficient k is, the longer the oscillating period is.

If the system is iniatially in exciting state, i.e. $b^* b = 0$ and $a^* a = 1$, the system probability evolutions with time of ground state and exciting state are show in Fig.3 where the coefficient k=0.1 too. The situation in Fig.3 is similar with the one iniatially in ground state in Fig. 2. The probabilities of exciting state or ground state are both oscillating curves, however, their periods is not different. The period in Fig.3 is about 260 a.u. and the period in Fig.2 is about 360 a.u. The system in exciting state initially transits to ground state faster than the one in ground state initially to exciting state.

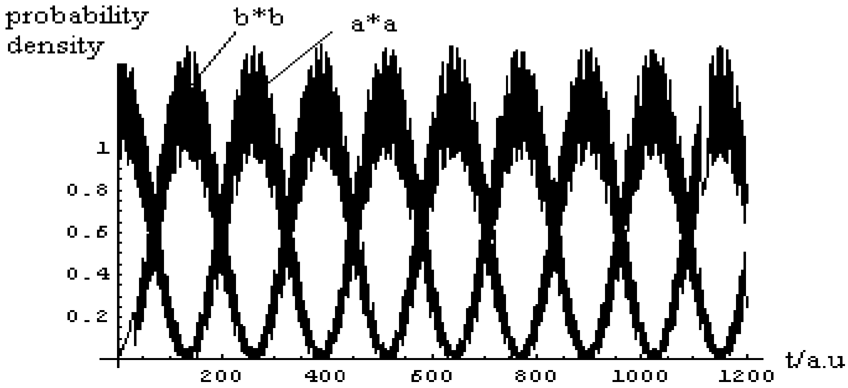

Fig. 3 probability evolution with time k=0.1, initially a*a=1,b*b=0

From the equ. (3) and the normalized condition $\int \psi^* \psi d\tau = 1$, we conclude that the dimension of the coefficient *k* is $J \cdot m^{3/2}$.

2 A modified Stern-Gerlach setup to Verify MWR Existence

The modified Stern-Gerlach setup is shown in Fig.4. The two-level system is an atomic nucleus of half spin, and the total electron angular momentum of the corresponding atom is zero without generality, for example ^{3}He nucleus. The heated stove produces atomic beams, ABC are all Stern-Gerlach magnets. Their magnetic induction directions are all up, however, the A gradient $\left(\frac{\partial B}{\partial z}\right)_A$ direction is down, B's is up and C's is down. If necessary magnets D,E,F,G… are placed in modified Stern-Gerlach setup. All the magnetic induction directions are up, but their gradient directions are all down, up, down, up….The force from magnetic field to atomic beams is $F = \mu_z \frac{\partial B}{\partial z} = m_I g_I \mu_N \frac{\partial B}{\partial z}$, so the above beams' nuclear spin must be up i.e. $m_I = 1/2$ then can get to the detector, the below beams' nuclear spin should be down i.e. $m_I = -1/2$.

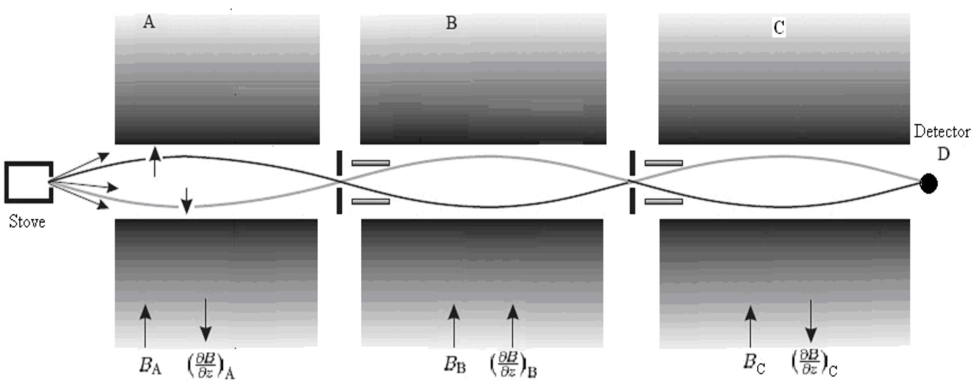

Fig. 4 a modified Stern-Gerlach setup

According to our theoretic analysis in section II, the above and below beams' spin will change i.e. from up to down and from down to up because of MWR. We **expect** that at some position the detector will not receive any atomic beams through the Stern-Gerlach magnets series, because the atoms changing spins will collide with magnets and can not get to the detector. Comparing Fig.2 and Fig.4 we know that the above up-spin beams will collide with magnets earlier that the below down-spin beams. After measuring beams flying displacement at the end of which the detector receives none, we can obtain the MWR coefficient k.

3 Summery

The concept of matter wave radiation is put forward, and the source-dependent Schrödinger equation is established for the first time. A two-level system are investigated considering MWR effect. A modified Stern-Gerlach setup verifying MWR are presented. We call on the experimental physicists all over the world to perform such experiments in contrast with the above theoretical expectatation because MWR may change the modern physics paradigm.

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德布罗意物质波的源

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摘要：比较薛定谔方程和麦克斯韦方程后，提出玻恩概率密度为德布罗意波的发射源。第一次建立了含源的薛定谔方程。考虑物质波辐射效应后重新研究了两能级系统，提出了修正的斯特恩-盖拉赫实验装置证实物质波辐射的存在。

关键词：德布罗意波的源；物质波辐射；含源薛定谔方程；两能级系统；修正的斯特恩-盖拉赫实验装置
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