Azimuthons in weakly nonlinear waveguides

Yiqi Zhang,1,2 Stefan Skupin,1,3 Keqing Lu,2 Wieslaw Królikowski4
1Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany
2State Key Laboratory of Transient Optics and Photonics, Xi’an Institute of Optics and Precision Mechanics of Chinese Academy of Sciences, 710119 Xi’an, China
3Friedrich Schiller University, Institute of Condensed Matter Theory and Optics, 07742 Jena, Germany
4Laser Physics Centre, Research School of Physics and Engineering, Australian National University, Canberra, ACT 0200, Australia
E-mail: zhangyq@pks.mpg.de

Abstract: We show that a weakly guiding nonlinear waveguide supports propagation of stable rotating solitons, azimuthons. We calculate analytically the rotation frequency of these solitons and find it to be in agreement with numerical simulations.

©2010 Optical Society of America

OCIS codes: (190.0190) Nonlinear Optics; (190.4420) Nonlinear Optics, transverse effects in

1. Introduction

Recently, there has been a lot of interest in a generalized type of spatial solitons, so-called azimuthons [1-3]. Those multiple peak ring-shaped solitons, which exhibit angular rotation during propagation, have been studied almost exclusively in nonlocal nonlinear media, because higher order solitonic structures are generally unstable in material with local (Kerr) response. In spite of the fact that there are various physical settings exhibiting nonlocality [4-6], their experimental realization is always quite involved. Moreover, from the theoretical point of view, nonlocal media are quite challenging for numerical modeling and analytical treatment. Here, we propose a much simpler optical system to study the propagation of azimuthons: a weakly nonlinear optical waveguide. We show that in such system azimuthons occur as the natural nonlinear counterparts of linear waveguide modes. Following [7], we can expect that weakly nonlinear azimuthons are stable in multi-mode waveguides.

2. Mathematical modeling

The propagation of the slowly varying optical field envelope $\Psi$ in a weakly-guiding optical waveguide with Kerr nonlinearity is described by the following dimensionless equation,

$$i\frac{\partial}{\partial Z}\Psi + \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)\Psi + |\Psi|^2\Psi + V\Psi = 0,$$

(1)

with $V$ representing the waveguide index profile, and $\lim_{X,Y \to \infty} V = 0$. In this work, we will restrict ourselves to a circular step-index waveguide, with $V(\chi^2 + \chi^2 \leq 1) = V_o$, and $V = 0$ elsewhere. Then, $V_o = 20$ guarantees a multi-mode waveguide with stable vortex soliton in the weakly nonlinear regime [7].

Let us now introduce the azimuthon ansatz in cylindrical coordinates, which reads

$$\Psi(r,\phi,Z) = U(r,\phi-o\lambda Z)e^{i\phi},$$

(2)

where $U$ is the complex amplitude function, $\omega$ the angular frequency, and $\lambda$ the propagation constant [1]. If we insert this ansatz into Eq. (1), multiply by $\Psi^*$ and $\partial^2\Psi^*/\partial\phi$ respectively, and integrate over the transverse coordinates, we obtain two coupled equations which allow us to compute the angular frequency $\omega$ for given amplitude function $U$ [8]. The simplest azimuthon is the so-called rotating dipole, which will serve as an illustrative example in this work. In the weakly nonlinear regime, a reasonable ansatz is

$$U(r,\phi-o\lambda Z) = AF\left(r\right)\left[\cos(\phi-o\lambda Z) + iB\sin(\phi-o\lambda Z)\right]$$

(3)

where $F$ is the radial shape of the linear vortex mode and $\pi\int r|F(r)|^2 \, dr = 1$. By tuning the amplitude factor $A$ we can adjust the strength of the nonlinearity, and $0 \leq B \leq 1$ determines the strength of the amplitude modulation along the ring of the vortex. For the chosen step-index waveguide, it is possible to express $F$ analytically by using Bessel functions inside the waveguide and modified Bessel functions for the outside region. Hence, we can give an analytical estimate for the angular frequency.
Fig. 1. Propagation of the azimuthon with $A=2$ and $B=0.2$. The beam rotates about $\pi/4$ over a propagating distance of 3.3, so the angular frequency is about $\omega \approx 0.24$.

Fig. 2. Analytical estimates for the angular frequency $\omega$ in the weakly nonlinear limit versus azimuthon parameters $A$ and $B$ [Eq. (4)]. In the range of stability of the azimuthons we find excellent agreement with simulations.

$$\omega = \frac{\int r |F(r)|^2 \, dr}{2 \int r |F(r)|^2 \, dr} A^2 B. \quad (4)$$

As expected, for $A$ or $B$ equal zero the azimuthon does not rotate: $A=0$ refers to the linear regime, and $B=0$ to the stationary dipole soliton.

3. Numerical simulations

Direct numerical simulations of Eq. (1) reveal that the dipole azimuthons are stable in the chosen parameter range and moderate amplitudes. Figure 1 shows the exemplary propagation of a rotation dipole azimuthon in the nonlinear waveguide. If we choose $A$ too large ( $A>2$ in our example of a circular step-index waveguide with $V_0=20$), azimuthons become unstable upon propagation and decay into a perturbed ground state mode (not shown). In the stability domain of the azimuthons the angular frequencies predicted by Eq. (4) coincide with those obtained from the simulations (cf. Fig. 2).

4. Conclusion

In conclusion, we have demonstrated by means of numerical simulations that stable propagation of azimuthons can be observed in weakly nonlinear waveguides. We believe that these findings may open a relatively easy route to a first experimental observation of stable rotating solitons as well as a more profound theoretical understanding of rotating nonlinear localized structures.

5. References