Two-phase mass flow coefficient of V-Cone throttle device

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A B S T R A C T

The V-Cone throttle device has been paid more and more attentions on the measurement of the wet gas flow in recent years. However, it tends to over-read the gas flow rate when directly used to measure the wet gas. How to correct this over-reading and obtain an accurate yet simple wet gas correlation is still one of the challenges in the investigation on the V-Cone throttle device. In this paper, the two-phase mass flow coefficient of the V-Cone throttle device was proposed to correct the deviation of the readings produced by the device. The parameters affecting the coefficient were determined and discussed experimentally. Two V-Cone throttle devices with the respective equivalent diameter ratios of 0.65 and 0.75 were investigated under the condition of the wet gas flow. The test fluid was air and tap water. The gas-to-liquid density ratio ranged from 0.002445 to 0.006083, and the gas densiometric Froude number ranged from 0.3 to 2.0. The influencing parameters including the Lockhart–Martinelli parameter, the gas-to-liquid density ratio, the gas densiometric Froude number and the equivalent diameter ratio were studied. Then the correlations of the two-phase mass flow coefficient and the influencing parameters were proposed for the two V-Cone throttle devices and evaluated experimentally. The results show that the relative deviations of the correlations are less than ±5.0% and the average deviations of the correlations are 2.06% and 2.22% for the V-Cone throttle devices with the equivalent diameter ratio of 0.65 and 0.75, respectively. It can be concluded that the two-phase mass flow coefficient of the V-Cone throttle device is crucial for the development of the wet gas correlation.

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1. Introduction

As a particular gas–liquid two phase flow, the wet gas has drawn increasing attention in industrial processes in the last few decades, especially in the oil–gas industry [1]. The measurement of the wet gas flow rate is one of the challenges in the field of multiphase flow owing to its high gas volume fraction (GVF, more than 90%) and low liquid volume fraction [2,3]. Although the liquid volume fraction is considerably low in the wet gas, significant deviation will occur if the measurement correlation for the dry gas is used to meter the wet gas flow rate without the necessary modification. The commonly used meter for the wet gas is the differential pressure (DP) meter which is believed to tend to over-read the gas flow rate [1]. Many researchers investigated the over-reading and proposed the correction correlations [4–10]. These correlations are based on the homogeneous flow model and the separated flow model. The homogeneous flow model is to measure the total wet gas flow rate by assuming that the wet gas is a homogeneous mixture and has the “single density”. But in fact, the mixing density of the wet gas is difficult to measure, for it is dependent on the void fraction, the flow pattern and many other factors. The majority of the wet gas correlations are based on the separated flow model which is primarily to measure the gas flow rate of the wet gas flow and in which the DP of the wet gas is taken as the DP of the dry gas, and the deviation of the readings is corrected by some correlations. This method is generally employed to investigate the classical orifice plate and the Venturi tube throttle devices. However, the correlations are not applicable to the V-Cone throttle device [11].

As a new type of DP throttle device, the V-Cone throttle device has the advantages of great accuracy, excellent repeatability, and wide turndowns, and requires shorter straight length and outputs stable signals, which has been proved in the dry gas and liquid flow measurement [10,12,13]. In real oil–gas metering, the wet gas flow in the horizontal natural gas pipeline is practically the stratified flow and the annular (or annular-mist) flow. The V-Cone throttle device has little effects on the wet gas flow pattern because of its unique cone structure installed in the center of the pipe, which makes the measurement of the DP more stable. So the V-Cone throttle device is a promising differential pressure device for the measurement of the wet gas. The V-Cone throttle device is a non-standard differential pressure device, and there is no generally accepted standard concerning its structure, including the optimum...
Nomenclature

**English symbols**

- \( A \): area of the throttle device (\( \text{m}^2 \))
- \( a \): coefficient in Eq. (27) (-)
- \( b \): intercept in Eq. (25) (-)
- \( C_d \): discharge coefficient of a DP device (-)
- \( D \): Diameter of the DP device inlet (\( \text{m} \))
- \( d \): maximum diameter of the V-Cone (\( \text{m} \))
- \( \text{DR} \): Gas-to-liquid density ratio (-)
- \( f \): some function (-)
- \( Fr \): denisiometric Froude number (-)
- \( g \): gravitational constant (\( \text{m/s}^2 \))
- \( K \): mass flow coefficient (-)
- \( k \): slope in Eq. (25) (-)
- \( m \): mass flow rate (\( \text{kg/s} \))
- \( \text{OR} \): over-reading (-)
- \( P \): pressure (\( \text{Pa} \))
- \( \text{Re} \): Reynolds number (-)
- \( U \): velocity (\( \text{m/s} \))
- \( \text{X}_{LM} \): Lockhart–Martinelli parameter (-)
- \( \text{We} \): modified Weber number (-)

**Greek symbols**

- \( \Delta P \): differential pressure (\( \text{Pa} \))
- \( \beta \): equivalent diameter ratio (-)
- \( \delta \): deviation index (%) 
- \( \epsilon \): gas expansibility coefficient (-)
- \( \kappa \): gas isentropic exponent (-)
- \( \mu \): viscosity (\( \text{Pa} \cdot \text{s} \))
- \( \rho \): density (\( \text{kg/m}^3 \))
- \( \sigma \): interfacial tension (\( \text{N/m} \))
- \( \Pi \): dimensionless group (-)

**Subscripts**

- \( 0, 1, 2 \): different coefficient
- \( 0.65, 0.75 \): V-Cone throttle devices with \( \beta = 0.65 \) and \( \beta = 0.75 \)
- \( 1', 2' \): different cross sections
- \( a \): average
- \( d \): discharge
- \( g \): gas
- \( l \): liquid
- \( r \): relative
- \( t \): tendency or throat
- \( \text{sg} \): superficial gas
- \( \text{sl} \): superficial liquid
- \( \text{tp} \): two-phase

front and back cone angles, the size of the high and low pressure port. In addition, studies on the discharge coefficient \((C_d)\) and the gas expansibility coefficient \((\epsilon)\) of the V-Cone throttle device in the dry gas flow [14–19] are comparatively few and \(C_d\) and \(\epsilon\) are not tabulated yet. In the light of these research disadvantages, a simple yet accurate wet gas correlation for the V-Cone throttle device is urgently needed.

In this study, a new attempt was made to study the V-Cone throttle device in the measurement of the wet gas. The two-phase mass flow coefficient of the V-Cone throttle device was proposed to correct the measurement deviation rendered by the theoretical measurement correlation. The parameters affecting the two-phase mass flow coefficient were determined and experimentally investigated. The relationship of the coefficient and these parameters was then concluded and the performance of the proposed two-phase mass flow coefficient was analyzed.

## 2. Two-phase mass flow coefficient fundamental

### 2.1. Dry gas mass flow coefficient \((K_g)\)

The horizontal gas flow through the V-Cone throttle device shown in Fig. 1 is assumed to be incompressible, inviscid and reversible. The density of the gas is supposed to remain unchanged on the cross sections of \(1' \rightarrow 1\) and \(2' \rightarrow 2\). According to the mass continuity equation (Eq. (1)) and the Bernoulli equation (Eq. (2)), the following gas flow rate equation can be concluded (Eq. (3)):

\[
m_g = \frac{\rho g A_{1' \rightarrow 1} U_{1' \rightarrow 1} = \rho g A_{2' \rightarrow 2} U_{2' \rightarrow 2}}{2}
\]

\[
P_{1' \rightarrow 1'} + \frac{U_{1' \rightarrow 1}^2}{2} = \frac{P_{2' \rightarrow 2} + U_{2' \rightarrow 2}^2}{\rho g}
\]

\[
m_g = \frac{\rho g A_{1' \rightarrow 1} \sqrt{2 \rho g (P_{1' \rightarrow 1} - P_{2' \rightarrow 2})}}{\sqrt{1 - \beta^4}}
\]

where \(m_g\text{theoretical}\) is the theoretical gas mass flow rate; \(\rho g\), the gas density; \(A_{1' \rightarrow 1}\) and \(A_{2' \rightarrow 2}\), the maximum and minimum flow cross section area of the V-Cone throttle device, respectively; \(P_{1' \rightarrow 1}'\) and \(P_{2' \rightarrow 2}'\), the velocity of the maximum and minimum flow cross section area of the V-Cone throttle device, respectively; \(P_{1' \rightarrow 1}\), the high pressure; \(P_{2' \rightarrow 2}\), the low pressure and \(\beta = \sqrt{A_{2' \rightarrow 2}/A_{1' \rightarrow 1}}\) is the equivalent diameter ratio of the V-Cone throttle device.

**Fig. 1.** Sketch of generic V-Cone throttle device.
where \( m_g \) is the actual dry gas flow rate. When \( A = A_{11} \), and \( P_1 \) and \( P_2 \) replace \( P_{11} \) and \( P_{22} \), respectively, \( K_g \) is expressed by Eq. (5):

\[
K_g = \frac{m_g \sqrt{1 - \beta^4}}{A_0 \sqrt{2 \beta_1 (P_1 - P_2)}} = \frac{m_g \sqrt{1 - \beta^4}}{A_0 \sqrt{2 \beta_2 \Delta P}}
\]

(5)

where \( \Delta P = P_1 - P_2 \) is the actual differential pressure of the V-Cone throttle device in the dry gas flow. In Eq. (5), \( K_g = C_{dg} \). \( C_{dg} \) is the gas discharge coefficient. For a set geometry and material V-Cone throttle device, studies on the \( C_{dg} \) and \( \epsilon \) under the condition of the dry gas have proved that \( C_{dg} \) is independent of the Reynolds number (Re), and \( \epsilon \) is dependent on \( \Delta P, P_1 \) and the gas isentropic exponent (\( \kappa \)) and [14–19]. Thus, \( K_g \) is determined by the \( \Delta P, P_1 \) and \( \kappa \).

### 2.2. Two-phase mass flow coefficient (\( K \))

When the horizontal fluid flowing through the V-Cone throttle device is the wet gas flow, the wet gas flow can be supposed to be the quasi-single phase flow. According to the mass continuity equation and the Bernoulli equation, the wet gas flow rate equation is as follows in Eq. (6):

\[
m_{tg, \text{theoretical}} = \frac{A_1 \sqrt{1 - \beta^4}}{2 \beta_1 (P_1 - P_2 - \Delta P)}
\]

(6)

where \( m_{tg, \text{theoretical}} \) is the theoretical wet gas mass flow rate, \( \rho_{tg} \) is the density of the wet gas flow, and \( P_1 \) and \( P_2 \) are the high pressure and the low pressure of the V-Cone throttle device in the wet gas, respectively. Similarly, Eq. (6) is theoretical. When used in the practical measurement of the wet gas flow, it produces significant errors. The influencing factors are much more complex than those in the dry gas flow. In addition to the above factors, the density of the wet gas flow, \( \rho_{tg} \), is a significant one. It relates to the flow pattern, the void faction and other parameters. So it is hard to determine \( \rho_{tg} \) in practical wet gas measurement. Hence, the gas density, \( \rho_g \) is generally used in practice instead of \( \rho_{tg} \). This simplification is a main source of the measurement errors.

In the light of the discharge coefficient of the V-Cone throttle device in the dry gas, a dimensionless parameter, \( K \), is proposed here to revise Eq. (6). \( K \) is the two-phase mass flow coefficient and is defined by Eq. (7):

\[
K = \frac{m_{tg}}{m_{tg, \text{theoretical}}} = \frac{m_g \sqrt{1 - \beta^4}}{A_1 \sqrt{2 \beta_1 (P_1 - P_2 - \Delta P)}}
\]

(7)

where \( m_{tg} \) is the actual wet gas flow rate. When \( A = A_{11} \), \( P_2 \) replaces \( P_2 \), and \( P_1 \) and \( P_2 \) replace \( P_{11} \) and \( P_{22} \), respectively, it follows that

\[
K = \frac{m_g \sqrt{1 - \beta^4}}{A_0 \sqrt{2 \beta_2 (P_1 - P_2)}} = \frac{m_g \sqrt{1 - \beta^4}}{A_0 \sqrt{2 \beta_2 \Delta P_g}}
\]

(8)

where \( \Delta P_g = P_1 - P_2 \) is the actual differential pressure of the V-Cone throttle device in the wet gas. Owing to the great number of the factors affecting the measurement error, the dimensional analysis is needed to determine these factors.

### 2.3. Influencing parameters analysis

Suppose a single liquid phase component in the wet gas is to be measured. When the effects of the geometric structure and the surface roughness of the device are ignored, the possible influencing parameters are the actual wet gas differential pressure of the V-Cone throttle device (\( \Delta P_g \)), the gas and total mass flow rate (\( m_g \) and \( m_t \)), the gas density (\( \rho_g \)), the difference between the gas and liquid (\( \Delta \rho = \rho_g - \rho_l \)), the gas and liquid viscosity (\( \mu_g \) and \( \mu_l \)), the liquid interfacial tension (\( \sigma_l \)), the maximum diameter of the V-Cone element (\( d \)), the pipe diameter (\( D \)) and the gravitational constant (\( g \)). \( \Delta P \) contains the information of the liquid density (\( \rho_l \)), so it is used here instead of \( \rho_l \) and \( m_{tg} \) is used instead of the liquid mass flow rate (\( m_l \)) [20].

Eleven parameters are investigated in the terms of the primary parameters, i.e., mass, length and time. Eight independent dimensionless groups are obtained.

\[
f(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6, \Pi_7, \Pi_8) = 0
\]

(9)

where \( f \) is some function. According to the Buckingham Pi theorem, the two-phase mass flow coefficient, \( K \), can be expressed as follows:

\[
K = f(X_{LM}, DR, Fr_g, \beta, Re_{tg}, Re_{lg}, We_{tg})
\]

(10)

where \( X_{LM} \) is the Lockhart–Martinelli parameter; DR, the gas-to-liquid density ratio; \( Fr_g \), the gas densiometric Froude number; \( Re_{tg} \) and \( Re_{lg} \) the gas and liquid Reynolds number, respectively, and \( We_{tg} \) is the modified Weber number for the two-phase flow. These parameters are defined in Eqs. (11)–(17):

\[
X_{LM} = \sqrt{\frac{\text{superficial liquid inertia}}{\text{superficial gas inertia}}} = \frac{m_l}{m_g} \sqrt{\frac{\rho_g}{\rho_l}}
\]

(11)

\[
DR = \frac{\rho_g}{\rho_l}
\]

(12)

\[
Fr_g = \sqrt{\frac{\text{superficial gas inertia}}{\text{liquid gravity force}}} = \frac{U_{lg}}{\sqrt{gD}} \sqrt{\frac{\rho_g}{\rho_l}}
\]

(13)

\[
U_{lg} = 4m_l \pi D \rho_g
\]

(14)

\[
Re_{tg} = \frac{\text{superficial gas inertia}}{\text{gas viscous force}} = \frac{\rho_g U_{lg}}{\mu_g}
\]

(15)

\[
Re_{lg} = \frac{\text{superficial liquid inertia}}{\text{liquid viscous force}} = \frac{\rho_l U_{lg}}{\mu_l}
\]

(16)

\[
We_{tg} = \frac{\text{superficial gas inertia}}{\text{liquid interfacial tension force}} = \frac{\rho_g U_{lg}^2 D}{\sigma_l}
\]

(17)

where \( U_{lg} \) is the superficial gas velocity (Eq. (14)).

The above-listed dimensionless parameters are the possible parameters affecting the wet gas measurement response of the V-Cone throttle device. Among them the most influential ones and the negligible ones can be determined in practice. The gas and the liquid viscosity have been proved to impose little influence on the measurement [21]. Hence, \( Re_{tg} \) and \( Re_{lg} \) are negligible. The liquid interfacial tension does affect the V-Cone throttle device wet gas response, but it is ignored in most correlations. The reason is that it is almost impossible to corporate all of the influencing parameters into the correlation and more parameters in the correlation certainly will increase the overall uncertainty of the measurement. However, in practice, an accurate coefficient is required in the studies on different types of liquid [22].

In the present work, the influences of \( K, X_{LM}, DR, Fr_g \) and \( \beta \) on the measurement of the wet gas with the V-Cone throttle device are taken into account. Thus \( K \) is reduced to the Eq. (18).

\[
K = f(X_{LM}, DR, Fr_g, \beta)
\]

(18)

With the function \( f \) available, the wet gas flow rate is obtainable by the following equation:
When the data about the liquid mass flow rate or the gas (liquid) mass fraction are obtained with some techniques (e.g., the phase fraction measurement), and then the gas and liquid flow rate can be measured simultaneously. Our work is focused on the study of $K$ and intended to obtain the function $f$ experimentally.

3. Experimental facilities and scheme

3.1. V-Cone throttle device

The structure of the V-Cone throttle device in our work is shown in Fig. 2. A pair of “V” shaped Cones (the front-cone and the back-cone) are connected and held by a supporting bar. The low pressure is obtained on the apex of the back-cone. The distance between the low pressure port and the high pressure port is 56 mm. Two equivalent diameter ratios, i.e., $\beta = 0.65$ and $\beta = 0.75$, are tested. The two V-Cone throttle devices (see Fig. 3) have the same geometry except that their equivalent diameter ratio is different. The front-cone and back-cone angle is 45° and 135°, respectively. The devices are positioned horizontally. The inside diameter of the V-Cone device inlet, $D$, is 50 mm.

3.2. Experimental setup

The experiments were conducted in the gas–liquid two phase flow loop as is shown in Fig. 4. The test fluid was air and tap water. The compressed air was supplied by an air compressor and flowed into the air storage tank, and then was filtered and cooled by the air filters and the air freezing dryer. Two Siemens Coriolis mass flow meters (with the uncertainty of 0.1%) and two Siemens Coriolis mass flow meters with an uncertainty of 0.5% were employed to meter the air mass flow rate. The water was pumped by three centrifugal pumps and measured by one Yokogawa electromagnetic flow meter (its uncertainty is 0.2%) and two Siemens Coriolis mass flow meters (with the uncertainty of 0.1%). The water meters were calibrated with the weighting method. A gas–liquid mixer was used to mix the compressed air and the tap water. The V-Cone test section was located approximately 100 pipe diameters away from the outlet of the gas–liquid mixer. After testing, the gas–liquid mixtures were separated by the gas–liquid separator, the water returned to the water tank for recycling and the gas was discharged into the atmosphere. The high resolution camera recorded the flow pattern in the V-Cone throttle device.

The upstream pressure of the V-Cone throttle device was measured by the Rosemount 3051CG pressure transmitter with the uncertainty of less than 0.075%; the differential pressure was measured by the Rosemount 3051CD differential pressure transmitter with the uncertainty of less than 0.075%. The temperature of the wet gas was measured by a Pt100 temperature transmitter with the uncertainty of ±0.15 °C. The data were collected by the NI USB-6229 data acquisition module and the LabVIEW based software.

3.3. Experimental scheme

The experimental conditions are tabulated in Table 1. The flow distribution in the Mandhane flow pattern map is consistent with the one shown in the photographs taken by the camera.

4. Results and analysis

4.1. Validation of previous correction coefficient, OR

Like the other generic DP devices, when the V-Cone throttle device is used in wet gas flow, it will produce a higher differential pressure than when used in the dry gas flow, i.e., an over-reading (OR). This uncorrected gas mass flow rate prediction is generally referred to as the “apparent” gas mass flow, $m_{g,apparent}$, which is shown in Eq. (20).

$$m_{g,apparent} = \frac{Ap^2C_{gV}\sqrt{2\rho_g\Delta P_p}}{\sqrt{1 - \beta^4}}$$

A correction coefficient, OR, is proposed to correct the over prediction of the gas mass flow rate. OR is the ratio of the apparent gas mass flow rate to the actual gas mass flow rate (Eq. (21)).

$$OR = \frac{m_{g,apparent}}{m_g}$$

Two popular OR correlations for the V-Cone throttle device were shown in Appendix A [24,10]. The Steven OR correlation is developed by data fitting using the experimental data from the 0.75 diameter ratio V-Cone devices with gas/light hydrocarbon liquids. The modified de Leeuw OR correlation is obtained by changing the value of the exponent “n” in the de Leeuw correlation [8]. The data used to develop the modified de Leeuw OR correlation is from
the 0.75 diameter ratio V-Cone devices with gas/light hydrocarbon and/or water liquids (the water cut is from 0% to 100%). The comparisons of the two OR correlations with the present experimental data is shown in Fig. 6. It is can be seen that the two correlations predict the OR of the 0.65 diameter ratio V-Cone device better than that of the 0.75 one.

Three indexes are employed to quantitatively evaluate the Steven and the modified de Leeuw OR correlations. The first is the relative deviation index, \( \delta_r \), which indicates the deviation of the predicted value from the experimental value. It is determined by the following formula:

\[
\delta_r(i) = \left( \frac{\text{Estimated } X(i) - \text{Experimental } X(i)}{\text{Experimental } X(i)} \right) \times 100\% 
\]

where \( X(i) \) is the parameter to be evaluated. The second index is the tendency index, \( \delta_t \), which indicates that a correlation overestimates or underestimates the experimental data. When the correlation overestimates the data, the index is positive; otherwise, the index is negative.

\[
\delta_t(i) = \left( \frac{1}{N} \sum_{i=1}^{N} \delta_r(i) \right) \times 100\% 
\]

where \( N \) is the total number of the test data. The third index is the average deviation index, \( \delta_a \), which indicates the accuracy of the correlation prediction compared with the experimental value.

\[
\delta_a(i) = \left( \frac{1}{N} \sum_{i=1}^{N} |\text{abs}(\delta_r(i))| \right) \times 100\% 
\]

The relative deviation index, the tendency index and the average deviation index of the two OR correlations are listed in Table 2. It

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Table 1
Experimental conditions in present study.

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>( \beta )</th>
<th>Pressure (gauge, MPa)</th>
<th>DR</th>
<th>( Fr_g )</th>
<th>( X_{LM} )</th>
<th>GVF (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.65, 0.75</td>
<td>0.10</td>
<td>0.002445</td>
<td>0.5–2.0</td>
<td>0.01–0.31</td>
<td>98.5–99.9</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.003656</td>
<td>0.5–1.5</td>
<td>0.01–0.31</td>
<td>98.1–99.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>0.004870</td>
<td>0.3–1.2</td>
<td>0.01–0.32</td>
<td>97.8–99.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>0.006083</td>
<td>0.3–1.2</td>
<td>0.01–0.32</td>
<td>97.6–99.9</td>
<td></td>
</tr>
</tbody>
</table>

---


Table 2

The flow pattern of experimental conditions. (a) Experimental conditions distribution in Mandhane flow pattern map and (b) typical flow pattern taken by the camera.
is demonstrated that both the OR correlations fail to accurately predict the OR and they underestimate the experimental values. Three primary reasons account for this: (a) the operating pressure in the present study is 0.1–0.4 MPa, which is beyond the pressure range (more than 1.3 MPa) that used to develop the correlations [10]. The correlations cannot be extrapolated to the low pressure conditions as described in this study; (b) the liquid phase properties: the tap water is employed in the present experiment, while the light hydrocarbon liquids are used to develop the correlations. Studies showed that the liquid properties (e.g., the surface tension force) has effects on the wet gas measurement [10,21,22]; (c) the V-Cone throttle device is a nonstandard differential pressure device, whose structure parameters (e.g., the front cone angle and the back cone angle) may be different in different studies. These differences also cause the deviation in predicting the OR.

4.2. Experimental results on K

According to Section 2, the two-phase mass flow coefficient, K, is primarily affected by the Lockhart–Martinelli parameter (XLM), the gas-to-liquid density ratio (DR), the gas densiometric Froude number (FrG) and the equivalent diameter ratio (β) for a set geometry and material V-Cone throttle device. Hence, the following discussions focus on the influences of these four parameters on K.

The influence of XLM on K shown in Fig. 7 suggests that K consistently increases with XLM in the cases of both β = 0.65 and β = 0.75. In fact, they exhibit a good linear relationship which can be expressed by Eq. (25).

\[ K = kX_{LM} + b \] (25)

where k is the slope of the line and b is the intercept. When there is no liquid present in the wet gas, i.e., XLM = 0, b equates the dry gas flow coefficient, Kg, as shown in Eq. (26).

\[ b = K_g = C_{dg} \] (26)

<table>
<thead>
<tr>
<th>β</th>
<th>Correlation</th>
<th>Index (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Relative deviation, δt</td>
</tr>
<tr>
<td>0.65</td>
<td>Steven</td>
<td>−7.20 to 2.21</td>
</tr>
<tr>
<td></td>
<td>Modified de Leeuw</td>
<td>−5.48 to 6.73</td>
</tr>
<tr>
<td>0.75</td>
<td>Steven</td>
<td>−12.37 to 3.16</td>
</tr>
<tr>
<td></td>
<td>Modified de Leeuw</td>
<td>−12.76 to 9.68</td>
</tr>
</tbody>
</table>

As shown in Fig. 8, in the present study Kg can be treated as independent of the gas Reynolds number, ReG, and can be averaged at 0.8214 and 0.7642, respectively. The relative deviation of Kg ranges from −1.90% to 3.61% and from −3.30% to 4.20% for the 0.65 and 0.75 diameter ratio V-Cone devices, respectively. The average value of b is equal to 0.8214 and 0.7642 in the cases of β = 0.65 and β = 0.75, respectively.

Figs. 9 and 10 show that k is dependent upon DR and FrG. The influence of the gas-to-liquid density ratio (DR) on K is investigated with other parameters kept constant. Two gas densiometric Froude number (FrG) sets, i.e., FrG = 0.5 and FrG = 1.2, are used here in each case of the V-Cone throttle devices (see Fig. 9). It is found that K increases with the decrease of DR, and the slope, k, thus decreases as DR increases. The effect of the gas densiometric Froude number (FrG) on K is not so obvious as that of DR on K (see Fig. 10). And this finding under some flow conditions (e.g., Fig. 10(c)) is different from that under some other flow conditions. This is probably caused by the influence of the V-Cone element on the flow pattern. During the experiment we observed that the flow pattern of the wet gas was different before and after the V-Cone element, and the flow pattern after the V-Cone element was closely related to the structure of the V-Cone. The flow pattern after the V-Cone element caused by the influence of the V-Cone element on the flow pattern. Nevertheless, it can still...
4.3. Two-phase mass flow coefficient correlation

According to Figs. 9 and 10, the slope $k$ of the graph of $K$ vs. $X_{LM}$ decreases with $DR$ or $F_{rg}$ increasing, and with $\beta$ increasing. It has been found that for the V-Cone throttle devices with $\beta = 0.65$ and $\beta = 0.75$, the relationships of the slope $k$ and $DR$ and $F_{rg}$ are the same as is shown in Eq. (27), and the coefficients, $a_0$, $a_1$ and $a_2$, are different for different V-Cone throttle devices. Three coefficients are listed in Table 3 for the V-Cone throttle devices with $\beta = 0.65$ and $\beta = 0.75$.

$$K = a_0 + \frac{a_1}{\rho_g/\rho_l} - a_2F_{rg}$$

Therefore, the two-phase mass flow coefficient correlation in terms of $X_{LM}$, $DR$ and $F_{rg}$, is concluded as follows: For the V-Cone throttle device with $\beta = 0.65$, the two-phase mass flow coefficient $K_{0.65}$:

$$K_{0.65} = \frac{4.1031 + 0.01568}{\rho_g/\rho_l} - 0.1891F_{rg}X_{LM} + 0.8214$$

For the V-Cone throttle device with $\beta = 0.75$, the two-phase mass flow coefficient $K_{0.75}$:

$$K_{0.75} = \frac{3.9104 + 0.01339}{\rho_g/\rho_l} - 0.05151F_{rg}X_{LM} + 0.7642$$

be concluded that as $F_{rg}$ increases, $K$ tends to decrease with other parameters held constant under present testing conditions.

Fig. 11 shows the effect of the equivalent diameter ratio, $\beta$, on $K$. According to the figure, $K$ increases with $\beta$ increasing. In practice, when the geometry of a V-Cone throttle device is given, it follows that $\beta$ is constant. The influence of $\beta$ can be considered negligible for a given V-Cone throttle device. Thus the influences of $X_{LM}$, $DR$ and $F_{rg}$ on $K$ are exclusively taken into account for a given V-Cone throttle device. This simplified consideration reduces the number of the influencing parameter in the correlation and the overall uncertainty. Actually, similar suggestion was made by Steven in his correlation for the wet gas flow measurement [20].

4.3. Two-phase mass flow coefficient correlation

According to Figs. 9 and 10, the slope $k$ of the graph of $K$ vs. $X_{LM}$ decreases with $DR$ or $F_{rg}$ increasing, and with $\beta$ increasing. It has been found that for the V-Cone throttle devices with $\beta = 0.65$ and $\beta = 0.75$, the relationships of the slope $k$ and $DR$ and $F_{rg}$ are the same as is shown in Eq. (27), and the coefficients, $a_0$, $a_1$ and $a_2$, are different for different V-Cone throttle devices. Three coefficients are listed in Table 3 for the V-Cone throttle devices with $\beta = 0.65$ and $\beta = 0.75$.

$$K = a_0 + \frac{a_1}{\rho_g/\rho_l} - a_2F_{rg}$$

Therefore, the two-phase mass flow coefficient correlation in terms of $X_{LM}$, $DR$ and $F_{rg}$, is concluded as follows: For the V-Cone throttle device with $\beta = 0.65$, the two-phase mass flow coefficient $K_{0.65}$:

$$K_{0.65} = \frac{4.1031 + 0.01568}{\rho_g/\rho_l} - 0.1891F_{rg}X_{LM} + 0.8214$$

For the V-Cone throttle device with $\beta = 0.75$, the two-phase mass flow coefficient $K_{0.75}$:

$$K_{0.75} = \frac{3.9104 + 0.01339}{\rho_g/\rho_l} - 0.05151F_{rg}X_{LM} + 0.7642$$

Fig. 9. Influence of $DR$ on $K$ (a) $\beta = 0.65$, $F_{rg} = 0.5$; (b) $\beta = 0.65$, $F_{rg} = 1.2$; (c) $\beta = 0.75$, $F_{rg} = 0.5$ and (d) $\beta = 0.75$, $F_{rg} = 1.2$.

Fig. 10. Influence of $F_{rg}$ on $K$ (a) $\beta = 0.65$, $DR = 0.002445$; (b) $\beta = 0.65$, $DR = 0.006083$; (c) $\beta = 0.75$, $DR = 0.002445$ and (d) $\beta = 0.75$, $DR = 0.006083$.

Fig. 11. Influence of $\beta$ on $K$ for different $DR$ and $F_{rg}$ (a) $F_{rg} = 0.5$ and (b) $F_{rg} = 1.2$. 
and decreases with $DF$ or $0 < 0.027$, regions $q_1 + q_2$ is 2.06% and 2.22%, respectively. According to $q_1 > q_2$, $q_1$ increases in the whole region. This may be one of the primary reasons (i.e., the previous studies on the OR of the V-Cone device, the V-Cone simplification of the dry gas mass flow coefficient, produced the errors. Another primary reason leads the errors is the tendency and deviation indexes of correlations for V-Cone throttle devices with $0.75$. The average deviation of the correlations is less than ±5.0% and the average deviation of the correlations is 2.06% and 2.22% for the V-Cone throttle devices with $q_1 = 0.65$ and $q_1 = 0.75$, respectively. The results demonstrate that the introduction of the two-phase mass flow coefficient of the V-Cone throttle device makes a simple and accurate wet gas measurement correlation feasible.

As a result of some experimental limitations, in the present study the effect of the liquid interfacial tension on $K$ is not investigated, which will be the focus in our further investigations with the CFD method. Our ultimate objective is to develop a wet gas correlation based on the two-phase mass flow coefficient. Moreover, the two-phase mass flow coefficient and its development method can also be used to other standard differential pressure devices, such as the orifice plate and the Venturi throttle device.

**Acknowledgment**

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**Appendix A. V-Cone throttle device OR correlation**

A.1. Steven OR correlation [24]

$$\text{OR} = \frac{1 + A X_{LM} + B F_{rg}}{1 + C X_{LM} + B F_{rg}}$$

(A1)

where for density ratio, $\rho_g/\rho_l \geq 0.027$,

$$A = -0.0013 + \frac{0.3997}{\sqrt{\rho_g/\rho_l}}$$

(A1a)

$$B = 0.0420 - \frac{0.0317}{\sqrt{\rho_g/\rho_l}}$$

(A1b)

$$C = -0.7157 + \frac{0.2819}{\sqrt{\rho_g/\rho_l}}$$

(A1c)

and for $\rho_g/\rho_l < 0.027, A = +2.431, B = -0.151, C = +1.0$.

A.2. Modified de Leeuw OR correlation [10]

$$\text{OR} = \sqrt{1 + C X_{LM} + X_{LM}^2}$$

(A2)

where

$$C = \left(\frac{\rho_2}{\rho_l}\right)^n + \left(\frac{\rho_l}{\rho_g}\right)^n$$

(A2a)

for $F_{rg} \leq 0.5, n = 0.143$.

for $F_{rg} > 0.5, n = \frac{1}{2} \left(1 - \frac{0.83}{\exp(0.3 F_{rg})}\right)$

(A2b)

**Table 3**

Coefficients $a_0$, $a_1$, and $a_2$ for V-Cone throttle devices with $q_1 = 0.65$ and $q_1 = 0.75$.

<table>
<thead>
<tr>
<th>$q_1$</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.65</td>
<td>4.103</td>
<td>0.0156</td>
<td>0.1891</td>
</tr>
<tr>
<td>0.75</td>
<td>3.9104</td>
<td>0.01339</td>
<td>0.05151</td>
</tr>
</tbody>
</table>

**Table 4**

Tendency and deviation indexes of correlations for V-Cone throttle devices with $q_1 = 0.65$ and $q_1 = 0.75$.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Index</th>
<th>Value (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{0.65}$</td>
<td>Relative deviation, $\delta_r$</td>
<td>±5.0</td>
</tr>
<tr>
<td></td>
<td>Tendency, $\delta_t$</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>Average deviation, $\delta_a$</td>
<td>2.06</td>
</tr>
<tr>
<td>$K_{0.75}$</td>
<td>Relative deviation, $\delta_r$</td>
<td>±5.0</td>
</tr>
<tr>
<td></td>
<td>Tendency, $\delta_t$</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>Average deviation, $\delta_a$</td>
<td>2.22</td>
</tr>
</tbody>
</table>

**Fig. 12.** Relative deviation of predicted $K$ and experimental $K$.
References