Motion of submicron particles in a supersonic laminar boundary layer under hypergravity

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Abstract

The process of submicron particle movement in laminar boundary layers has many applicable backgrounds including high-speed centrifugal devices. Although a big progress has been made on this problem during the last decades, many mechanisms in this process still remain unclear. Here, we developed a theoretical model to understand how submicron particles will behave when they are in a supersonic laminar boundary layer above an adiabatic plate along with the main stream under both zero gravity and hypergravity. In this model, we applied the Lagrangian method to track the particles and calculate their trajectories, and the Eulerian method was used to calculate the flow field. Because of the large velocity and temperature gradient near the wall and the small size of the particle in this question, and the high gravity field, five forces (e.g., drag force, Saffman lift force, thermophoretic force, Brownian force and gravitational force) acting on the particle are considered, and the role of each force is studied. The effects of entering position, intensity of gravity field, the size and density of particles are investigated. As a result, we discovered that there are three particle movement patterns when they enter the supersonic boundary layer under regular gravity and other patterns under high gravity. This research gives a better understanding of the particle movement process in the supersonic laminar boundary layer, which can be a useful instruction for the industrial processes relating to this phenomenon.

Keywords: supersonic laminar boundary layer; submicron particles; hypergravity

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1. Introduction

The process of submicron particles movement in a high speed laminar boundary layer has a wide range of application, such as the particle deposition on turbine blades and tracer particle movement in high speed boundary layer experiments, yet the direct motivation of this research is its application in the supersonic separator. The supersonic separator is a new technology in the area of natural gas processing. The physical progress in this device is very complex. In the separator, the feed gas is accelerated to supersonic and then the condensable components form into droplets due to the low temperature in high speed (i.e. the condensation process). At the same time, the condensed droplets are driven to move towards the wall under huge centrifugal force generated by the swirling generator, which is the separation process. The droplet in the supersonic is usually submicron. Actually before the droplets reach the wall, they have to cross the boundary layer. Sometimes the boundary layer in the separator is laminar. The motion of submicron droplets in the supersonic laminar boundary layer in the separator is of great importance as it affects the deposition process which determines the separation efficiency of the device.

Several mechanisms are involved in the particle deposition process [1]. For submicron particles, Brownian diffusion can be important. At the wall, the particle concentration is zero due to the continuous deposition of the particles. For this reason, in the thin area near the wall, there exists a particle concentration gradient which results in Brownian diffusion of particles from the main stream to the wall. If a temperature difference exists between the wall and the main stream, the thermophoresis must be considered. In the case the wall temperature is higher, the particles are expelled from the wall, forming a “dust free layer” where there is almost no particles near the wall. While the wall temperature is lower, the particle deposition rate increases. This effect is considered in the following research because the temperature gradient in a supersonic boundary layer is large. In addition, sometimes external force fields like electrostatic field exist in the flow region, in which case the electrostatic force must be considered if the particles are charged. In the laminar flow, turbulent diffusion, which is very important in turbulent flows, is not considered here.

Many scholars have investigated the particle deposition process in laminar boundary layers, especially when Brownian diffusion and thermophoresis coexist. Goren came up with the concept of “dust free layer” [2]. Mills et al. researched the deposition rate under the condition of wall suction as well as thermophoresis in a laminar boundary layer [3]. Epstein et al. studied the case of thermophoretic transport in a natural convection flow [4]. Gokoglu and Rosner presented a series of papers on the thermophoretic deposition in both laminar and turbulent boundary layers [5-7]. Garg and Jayaraj studied the thermophoresis of small particles over a cylinder and inclined plates [8, 9]. Most of the above researches are using the Eulerian method to obtain the macroscopic variables like deposition rate and mean deposition velocity. However, the behavior of a single particle and the particle group stay unknown. In some cases, the behaviors of different particles are not the same, resulting in a complex behavior of particle group. In general, some underlying mechanisms are not revealed yet.

The objective of this paper is to investigate the motion of submicron particles in a supersonic laminar boundary layer above an adiabatic flat plate under hypergravity, the results of which can reveal the mechanism and provide useful instructions for the particle separation process in the supersonic separator with high centrifugal force. In this research, we focus on the laminar case and do not consider the case when it turns into turbulent. We built a two-dimensional model to study this issue. The flow field was solved in an Eulerian grid and the particles were tracked in a Lagrangian way. The force models for the particle motion are elaborated and the accuracy of the theoretical model is then verified. In the results section, we analyze the role of each force and the effects of initial position, Mach number and particle diameter.

2. Problem description

The problem is simplified as gas-particle two phase flow above a flat plate, as shown in Fig. 1. Submicron particles enter the zone above the plate in a velocity the same as that of the flow, i.e. there is no velocity difference between the disperse phase and the continuous phase at the initial position. Then some particles will enter the boundary layer and some will not. Here we only investigate the former case. Because the gas phase is supersonic, it is treated as compressible. For the disperse phase, the particles are thought of as spherical and monodisperse. The effect of the particle on the fluid and the particle-particle interaction are neglected (one-way coupled), given the small size and dilute dispersion of the particles.

For the forces acting on the particles, the drag force must be considered. In addition, as the flow speed is very
high, the velocity and temperature gradient in the boundary layer are very large. So we also considered the Saffman lift force and thermophoretic force. Besides, the Brownian force is also considered for the submicron size of the particles. Other forces including added mass force, Basset force, Magnus force and pressure gradient force are neglected in this research. Because we focus on the particle movement process, the heat and mass transfer between the disperse phase and continuous phase are not considered here. In addition, we assume that the particles do not rebound as they touch the wall.

Fig. 1. Problem description.

3. Computational model

3.1. Fluid force models

The motion of the particles is described by the Newton’s second law of motion:

$$m_p \frac{d^2 x}{dt^2} = F_D + F_T + F_S + F_B + F_G$$

(1)

where $m_p$ is the particle mass, $F_D$ is the drag force, $F_T$ is the thermophoretic force, $F_S$ is the Saffman lift force, $F_B$ is the Brownian force and $F_G$ is the gravitational force. In order to accurately predict the movement of the particles, each force model is carefully chosen.

3.1.1. Drag force

The drag force on a spherical particle in a flow has been studied for a very long time and dates back to Stokes who obtained an expression valid for $Re_p<<1$ in continuum flow by neglecting the effect of inertia [10]. But for higher particle Reynolds number, the inertia cannot be neglected. In addition, if the continuum assumption fails in high Knudsen number, rarefaction of the gas must be considered. Schiller and Nauman used a semi-empirical coefficient and proposed an expression valid for $Re_p<200$. Cunningham obtained a form by considering the effect of rarefaction that valid for $Re_p<<1$, $Map<<1$ and $Kn<0.1$ [11]. However, in the problem we study, both the particle Reynolds number and the Knudsen number are relatively high, and the relative Mach number, which should be considered in compressible flows, is also high. Under this condition, Henderson proposed some expressions for various flow conditions [12, 13]. But the expression proposed by Tedeschi which is valid for $Re_p<200$, $Ma_p<1$ agrees better with the experiment [14]. For this reason, we applied his expression to calculate the drag force on particles. The expression is

$$F_D = -6\pi \mu a k \Delta U [1 + 0.15(k Re_p)^{0.687}] \tilde{\xi}(Kn)C$$

(2)

where $a$ is the radius of the particle, $\mu$ is the dynamic viscosity of the fluid, $\Delta U$ is the relative velocity between the particle and the fluid, $\tilde{\xi}(Kn)$ is the rarefied correction coefficient and $C$ is the correction coefficient for high relative Mach number. The expressions of $\tilde{\xi}(Kn)$ and $C$ are
1.16
and
\[ C = 1 + \frac{\text{Re}_p^2}{\text{Re}_p^2 + 100} e^{-0.225/\text{Ma}_p^{2.5}} \]  

[4]

3.1.2. Thermophoretic force
In 1884, Aitken found that the particles are experiencing a net force when they suspend in the fluid with a temperature gradient [15]. The reason is that the molecules of the fluid with higher temperature have larger kinetic energy, and they exert more impact on the particle. Due to the net force, the particles tend to move from the hot zone to the cold one, which is called thermophoresis. Since then, many scholars have tried to find a way to calculate the thermophoretic force.

Epstein theoretically derived the expression with the continuum fluid assumption when \( Kn \to 0 \) [16]. For the free-molecular case, Waldmann proposed the expression of thermophoretic force in the limit of \( Kn \to \infty \) [17]. After that many scholars proposed some modified expressions. However, the case we study here is in the transition regime. Talbot proposed an interpolation formula which agrees within 20% with the experimental data through the entire range of Knudsen number [18]. In addition, Sone and Aoki [19], Yamamoto and Ishihara [20], Loyalka [21], Takata [22] obtained more accurate way to calculate the thermophoretic force, of which the one proposed by Yamamoto and Ishihara is chosen as our solution because it agrees better and easy to use. The dimensionless form is

\[ \xi(Kn) = 1.177 + 0.177 \frac{0.851Kn^{1.16} - 1}{0.851Kn^{1.16} + 1} \]  

[3]

3.1.3. Saffman lift force
In 1962, Segré and Silberberg discovered the phenomenon of lateral migration of neutrally buoyant particles in Poiseuille flow, since when many scholars tried to reveal its mechanism. Among them Saffman proposed the concept of lift force in a shear flow [23]. When there is a slip velocity between the particle and the surrounding fluid in the shear flow, the particle experiences a lateral force. For the cases of particle moving faster and slower than the fluid, the directions of the lift force are opposite. The expression of lift force proposed by Saffman is:

\[ f_L = \frac{16\pi}{5} \left[ A_w H_o - A_o (H_w + \frac{5\sqrt{\pi}}{4} Kn\hat{k}) \right] (H_w + \frac{5\sqrt{\pi}}{4} Kn\hat{k})^{-1} \]  

[5]

where \( A_w, A_o, H_w, \) and \( H_o \) are functions of Knudsen number, \( \hat{k} \) is the ratio of thermal conductivity of the particle to that of the gas.

3.1.4. Brownian force
Albert Einstein [24] and Marian Smoluchowski [25] independently revealed the mechanism of Brownian motion. The random movement of Brownian particles is due to the random thermal motion of surrounding fluid molecules. The Brownian force can be modeled as a Gaussian white noise random process:

\[ F_S = 6.46\mu a^2 \Delta U \left( \frac{1}{\nu} \frac{dU}{dy} \right)^{1/2} \]  

[6]

where \( \Delta U \) is the particle slip velocity, \( \nu \) is the fluid kinematic viscosity and \( dU/dy \) is the velocity gradient of the shear flow. This expression can satisfy the accuracy requirement and is widely used. We used this expression to calculate the lift force.
\[ S^n_i = S_o \delta_{ij} \] (7)

where \( \delta_{ij} \) is the Kronecker delta function, \( S^n_i \) is the spectral intensity and

\[ S_o = \frac{216v k_B T}{\pi^2 \rho d_p^6 \left( \frac{P_p}{P} \right)^2 C_c} \] (8)

in which \( k_B \) is the Boltzmann constant and \( C_c \) is the Stokes-Cunningham correction. The form of Brownian force is

\[ F_{Bi} = \zeta_i \sqrt{\frac{\pi S_o}{\Delta t}} \] (9)

where \( \zeta_i \) is the zero-mean, unit-variance-independent Gaussian random numbers [26, 27].

3.2. Flow field calculation

The gas phase flow field in the problem we study is solved in a Eulerian grid. Because the flow is compressible, energy equation is included align with the mass and momentum equation. The real-gas R-K equation of state is solved under this condition. Because the influence of the particles on the fluid is not considered here, there are no source terms in the gas phase equations.

The two dimensional structured mesh is used for domain discretization. And the coupled solver based on FLUENT codes is used to calculate the flow field. The gas phase field is calculated first and when it is converged, the particles are injected and their trajectories are calculated along with the gas phase flow. The force models are inserted through the user defined function.

4. Computational parameters and validation

4.1. Domain and calculation parameters

The range of the domain is \( 0 \leq X \leq 50mm, 0 \leq Y \leq 50mm \). The y direction length is made the same as that in the x direction in order to eliminate the effect of the upper bound. Particles are injected along the inlet. As mentioned above, the influencing factor like Mach number, particle size and the gravity are investigated. The Mach number varies from 1.68 to 3.01. And the diameter of the particles ranges from \( 0.05 \mu m \) to \( 1.0 \mu m \). The gravitational acceleration is from \( 0 \) to \( 500000g \). The fluid material is air, and the material of the particles is water. The total pressure and total temperature of the inlet flow is 101 kPa and 421 K, respectively.

4.2. Validation

We operated a grid dependence check in order to ensure the accuracy of the simulation. The y position with time of the particle injected from a certain position is considered as the measuring parameter. Figure 2 shows that the grid with element number exceeding 110000 can meet the accuracy. So the grid with 119201 elements is selected for the calculation.
Fig. 2. Grid-dependence check.

As no relevant experimental data on the movement of submicron particles in a supersonic boundary layer is available so far, the simulation result of the velocity distribution in the supersonic laminar boundary layer above an adiabatic flat plate with the main flow of Mach 2.0 is compared with the theoretical result proposed by Crocco [28] in order to validate the accuracy of the simulation, as shown in Fig. 3. The result shows a good prediction of the simulation.

Fig. 3. Velocity distribution in the supersonic laminar boundary layer above an adiabatic flat plate. The Mach number of the main flow is 2.0.

5. Results and discussion

5.1. Role of each force

As we mentioned above, five forces on particles are considered. To examine in what condition these forces are important, the trajectories of particles with different force types are compared. Figure 4 shows the particle trajectory with the thermophoretic force is almost the same with the one without, meaning that the thermophoretic force is not important during the process. With the same method, we notice that Saffman lift force plays an important role. Another phenomenon we notice is that Brownian motion of the smaller particles is severe.
Fig. 4. Y position of particles with time on the condition of all forces, without Saffman lift force, without thermophoresis, without Saffman lift force and thermophoresis at: a) $Ma_0 = 2.05$, $d_p = 1.0\,\mu m$; b) $Ma_0 = 3.01$, $d_p = 0.05\,\mu m$. The oscillation in b) is due to Brownian motion. The gravitational acceleration is 0.

There is a phenomenon worth noticing in Fig. 4 that the particles tend to move upward at the beginning after they enter the domain. The reason is due to the oblique shock wave induced by the boundary layer, or the “induced shock wave” as shown in Fig. 5. The flow direction is not the same on the two sides of this shock wave. The original flow parallel with the plate obtains a positive y-direction velocity after the shock. And it’s this velocity that brings the particle upward.

Fig. 5. Induced shock wave and the change of the particle trajectory.

Another interesting phenomenon in Fig. 4 is that the particle without the Saffman lift force on it tends to move away from the wall, implying that the direction of the Saffman force is towards the wall. It is because the particle is leading the fluid when it runs into the boundary layer with main flow velocity at the beginning, i.e. the particle velocity is larger than the fluid velocity.

In order to examine the importance of the gravity, particle trajectories under different gravitational accelerations are investigated, as shown in Fig. 6. We can see that as the gravitational acceleration increases, the particle tends to deposit on the wall, which is easy to predict. In addition, the divergence between the particle trajectory with the Saffman force and that without is decreasing, showing that the gravitational force is overwhelming other forces and becoming the dominate one.
5.2. Effect of initial position

It is easy to imagine that particles with different initial injected position will behave in different ways. In order to find out how they will behave, an investigation is made. Figure 7 shows the results under both zero and 100000 times of the regular gravitational acceleration. In Fig. 7 a), the particle injected closer to the wall tends to deposit on it, while the one injected farther tends to move away. It is because when a particle enters the boundary layer at a closer position to the wall, the acting period of the positive drag force on it is shorter, and at the meanwhile, the Saffman force is larger due to the bigger slip velocity. So if the particle is injected higher, the acting period of positive drag force increases and the Saffman lift force decrease, resulting in the particle’s tendency to move away from the wall. Actually there exists an equilibrium position, from which the particle being injected by the square does not touch the wall. So there are three particle movement patterns: I) the particle moves upward and won’t touch the wall; II) the particle moves upwards first, then downwards and finally moves along the stream, and the distance between the particle and the wall is zero (i.e. the equilibrium position); III) particle moves upward first, and then downward until it reaches the wall.

However, in Fig. 7 b), we cannot find such a position, as the gravity is large enough to make all the particles, regardless of the initial injected position, move downwards to the wall. Unlike the case in zero gravity, there are no three particle movement patterns under hypergravity.

5.3. Effect of initial Mach number

Sometimes the Mach number of the flow changes and affects the movement of the particles in the boundary layer. As the speed of the main flow increases, the velocity gradient in the boundary layer increases, thus the Saffman force exerted on the particle is larger. At the same time, the induced shock wave becomes more oblique and
the positive y velocity of the fluid after the shock wave becomes smaller, resulting in a smaller drag force in the positive y direction. Based on these inferences, it can be speculated that the particle tends to move towards the wall with a larger main flow Mach number. As we can see, results in Fig. 8 confirm our speculation. Under different gravitational conditions, the effects of the Mach number the main flow are similar. But as gravitational acceleration increases, the divergence of the trajectories of particles injected from different initial positions becomes smaller, once again proving the more dominating role of the gravity.

![Graphs](image_url)

**Fig. 8.** The trajectories of particles with different Mach numbers of the main flow at:

a) 0g; b) 100000g; c) 300000g; d) 500000g. The particle diameter is 0.5μm.

5.4. Effect of particle size

The effect of particle size can be very large because almost all the important forces exerted on the particle considered are related with its size, including the drag force, the Saffman force and the gravitational force. In Fig. 9 we can see that as the particle size increases, the particle tends to move towards the wall. The reason lies in that the Saffman force increases faster than the drag force as the particle size increases. Besides, the Saffman force acts a longer period than the drag force, so the increasing Saffman force means a lot for the particle to move downward. It is also shown in Fig. 9 that as the gravity increases, all the particles tend to deposit on the wall, but the divergence of the trajectories of the particles injected from different initial positions is still large. It is because the change in the particle size also brings the change in the gravitational force besides other forces. The bigger particle has a more tendency to deposit.
6. Conclusion

In this paper, we investigated the submicron particle movement in a supersonic boundary layer above an adiabatic flat plate under hypergravity. The importance of each force is studied. The drag, Saffman force and gravitational force play dominating roles in this process, and the gravity becomes more and more important when it gets larger. The effects of initial position, Mach number and particle size are investigated. Without gravitational force, there exists an equilibrium position for the particle, but not under hypergravity. The particle tends to move towards the wall as the initial position gets lower and the Mach number is larger, but the effects of these two parameters becomes weaker when the gravity increases, under which condition all the particles tend to deposit on the wall. The particle size is related to the gravitational force experienced by the particles, so the difference among the particles of different sizes is large.

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References


[28] Crocco, L., Sullo strato limite laminare nei gas lungo una lamina plana, Rend Math Appl Ser. 5 (1941) 138-152.