Low-frequency locally resonant band-gaps in phononic crystal plates with periodic spiral resonators

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Low-frequency locally resonant band-gaps in phononic crystal plates with periodic spiral resonators

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In this paper, low-frequency band-gaps (BGs) in a phononic crystal (PC) thin plate with periodic spiral resonators are investigated numerically and experimentally. The formation mechanisms of the BGs in the proposed structure are explained based on the modal analysis. We find that the interaction between the local resonances and the traveling wave modes in the plate is responsible for the formation of the BG in low-frequency range. This interaction strength greatly affects the bandwidth of the BG, of which the lower edge depends on the corresponding local resonance frequency. It is shown that the out-of-plane BG can be modulated by changing the geometrical parameters. The proposed PC plate is demonstrated to possess a broad out-of-plane BG in low-frequency range from 42 Hz to 150 Hz, by combining the numerical calculations with experimental measurements. The structure design and its results provide an effective way for phononic crystals to obtain broad BGs in low-frequency range, which has potential applications in the low-frequency vibration and noise reduction. © 2013 AIP Publishing LLC.

I. INTRODUCTION

Over the past two decades, phononic crystals (PCs) have attracted much attention due to their excellent performance in controlling elastic waves.1–7 The existence of band-gaps (BGs), in which non-propagating modes can be activated in the system, leads to a variety of potential applications, such as vibration and noise reduction, sound filters, waveguides, and high-Q resonators.4–7 The formation of BGs is mainly attributed to Bragg scattering and local resonance in the structure.4,8–10 For the mechanism of Bragg scattering, the BG usually falls into the wavelength region of the order of the structural period. But for the mechanism of local resonance, resonances of scattering units play the dominant role, which depend less on the periodicity and symmetry of the structure. Generally, the frequency range of locally resonant (LR) BG is almost two orders of magnitude lower than that of the usual Bragg gap. A 2-centimeter slab of the LR composite material is proved to possess BGs around 400 Hz.4

The existence of low-frequency LRBGs in such geometries gives rise to the applications of LRPCs in the reduction of low-frequency vibration and noise.11–13 which has been a challenging task because of the long wavelength and weak attenuation of the low-frequency waves.14,15 In the last decade, various LR structures were proposed to obtain BGs in low frequency range. Ho11 demonstrated a class of broadband (200–500 Hz) sonic shield materials based on the principle of LR microstructures. Lai12 and Mei13 presented different designs of elastic metamaterials, each of which exhibits a large BG above 100 Hz. In recent years, a large number of works have been devoted to the study of BGs in PC plates due to the widespread applications of plate structures in engineering, where the propagated elastic wave modes are also called Lamb wave modes. Wu16 and Pennec17 investigated the elastic behavior of PC plates with stubbed surface and reported LRBGs in these structures. Oudich18,19 provided experimental evidence of the existence of a LRBG in a two-dimensional stubbed plate and developed a kind of PC plate with two-layer stubs. Hsu20 investigated the Lamb wave propagation in a two-dimensional PC structure composed of an array of stepped resonators on a thin slab. But the BGs of these PC plate are usually located in the frequency range above 1 kHz even higher. Unfortunately, the frequencies of most of the ambient vibrations in practical cases are distributed over a wide frequency range from 20 Hz to 250 Hz.21 However, few studies have been done on low-frequency LRBGs in the range below 100 Hz up to date.

In this paper, a spiral resonator is introduced into the PC plate structures to obtain BGs in lower frequency range. The low-frequency BG characteristics of the PC plate with periodic spiral resonators are investigated numerically and experimentally. The band structure and eigenmode shapes are calculated by using finite element (FE) methods, based on which the formation mechanisms and the geometrical parameter dependence of the BGs are analyzed and explained. What’s more, evidences of the low-frequency LRBG in finite structures are shown and discussed.

The paper is organized as follow. In Sec. II, the model of the structure is presented and explained. And the calculation methods of band structure and transmission coefficient based on FE methods are described. In Sec. III, the formation mechanisms of the BGs are explained based on the modal analysis of plate modes and local resonances. And the effects of geometrical parameters on the out-of-plane BG are analyzed. Section IV shows the evidences of low-frequency BG characteristics in finite PC plates through numerical
be expressed by the matrix equation

\[
\begin{bmatrix}
  m & 0 \\
  0 & M
\end{bmatrix}
\begin{bmatrix}
  \ddot{x} \\
  \ddot{X}
\end{bmatrix}
+ \begin{bmatrix}
  k & -k \\
  -k & k + K
\end{bmatrix}
\begin{bmatrix}
  \dot{x} \\
  \dot{X}
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0
\end{bmatrix}
\rho_0 \sin(\omega t).
\]

Substituting general solutions \( x = x_0 \sin(\omega t) \) and \( X = X_0 \sin(\omega t) \) into Eq. (2), the amplitudes of the harmonic response of the vibrating system are obtained as

\[
\begin{bmatrix}
  x_0 \\
  X_0
\end{bmatrix}
= \frac{\rho_0}{Z(\omega)}
\begin{bmatrix}
  -k \\
  k - m \omega^2
\end{bmatrix},
\]

where |\( Z(\omega) \)\| = \( mM(\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2) \), and \( \omega_1 \) and \( \omega_2 \) are the nature mode frequencies. It can be seen from Eq. (3) that, when the frequency \( \omega \) of elastic waves is near \( \omega_0 = \sqrt{k/m} \), the motion of the plate \( M \) reduces to zero while the cylinder \( m \) resonates with the spiral beam \( k \). This is because the excitation applied on the plate is counteracted by the force from the vibrating \( m-k \) system, which is the principle of dynamic vibration absorbers. Using this spiral resonator as a structural unit of PC plates, elastic waves in some frequency range may not be supported to propagate in the PC plate, which implies the formation of complete BGs. The main purpose of our work is to provide theoretical and experimental evidences for the low-frequency BGs in this proposed PC structure.

In order to investigate the BG characteristics of the proposed PC plate, the band structures are calculated by using the FE method, which has been proved in previous works\(^{12,13,16}\) to be an efficient method. In the calculations, an infinite periodic structure is considered. According to Bloch-Floquet theorem, periodic boundary conditions are applied on the interfaces among all adjacent unit cells, and the Bloch wave vector \( k \) is introduced and only needs to have a value along the border line of irreducible Brillouin Zone (BZ), because of the periodicity of the PC plate in both \( x \) and \( y \) directions and the symmetry of the unit cell. By varying the value of \( k \) in the irreducible BZ and solving the eigenvalue problem, the band structures (i.e., dispersion relations) are obtained. Moreover, the FE method allows the calculation of modal displacements, by which we can identify the eigenmode shapes of each branch in the band structure.

The band structure of the PC plate is calculated based on an infinite structure, but all the structures are finite for

\[
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\end{bmatrix}
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\begin{bmatrix}
  -k \\
  k - m \omega^2
\end{bmatrix},
\]

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The band structure of the PC plate is calculated based on an infinite structure, but all the structures are finite for
TABLE I. Physical parameters of the used materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\rho$/kg-m$^{-3}$</th>
<th>Young modulus $E$/Gpa</th>
<th>Poisson ratio $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>7780</td>
<td>210.6</td>
<td>0.3</td>
</tr>
<tr>
<td>PMMA</td>
<td>1142</td>
<td>2</td>
<td>0.389</td>
</tr>
</tbody>
</table>

practical applications. To illustrate the transmission properties of BGs in finite PC structures, the frequency response function (FRF) is also calculated using the FE method. In the numerical model, eight layers of unit cells are arranged in a row along $x$ direction. Periodic or free boundary conditions are considered on the edges in $y$ direction when the PC structure is infinite or finite in $y$ direction. The excitation is applied perpendicular to the plate surface on one side, while the acceleration on the other side is recorded to deduce the transmission coefficient.

III. LOW-FREQUENCY BGs OF THE PC PLATE

A. Modal analysis of local resonances

In the PC structure considered here, the plate and the cylinders are made of polymethylmethacrylate (PMMA) and steel, respectively, whose physical parameters are listed in Table I. The band structure and the corresponding eigenmodes are calculated by using the FE method. Figure 2 shows the calculated band structure of the proposed PC plate. In the calculation, the endpoint of the reduced Bloch wave vector $k$ sweeps along the border line of the Brillouin zone shown on the right of the band structure.

It is known that a finite thickness plate admits the propagation of a number of anti-symmetric (A mode) and symmetric (S mode) Lamb waves and shear-horizontal (SH mode) waves with cutoff frequencies, including three fundamental plate-modes (i.e., $A_0$, $S_0$, and $SH_0$ modes) and infinite overtones ($A_n$, $S_n$, and $SH_n$ modes, $n = 1, 2, 3, \ldots$).\(^{20}\) In the band structure, the dispersion relation branches originating from $\Gamma$ point can be identified as the $A_0$, $S_0$, and $SH_0$ (i.e., the lowest anti-symmetric, symmetric, and shear-horizontal) modes in the plate. In the low frequency range, i.e., in the long-wave limit, the symmetric (S) Lamb waves and SH waves propagate through the PC plate like in a uniform thin plate. So the two branches are nearly linear, and their slopes reveal the phase velocities of waves. These traveling wave modes can propagate through the structure very easily.

In the band structure shown in Fig. 2, six flat bands labeled in A–F are found appearing in the frequency range below 250 Hz, which almost spread all over the directions and indicate resonances of the spiral resonators. To illustrate this, the corresponding eigenmodes are calculated and shown in Figs. 3(a)–3(f). From the eigenmode shapes and displacement vector fields of the unit cell, we can find that the flat bands labeled in A, D, and E are attributed to the out-of-plane, longitudinal, and transverse resonances of the steel cylinders, respectively, while the flat bands labeled in B, C, and F result from the torsional resonances of the steel cylinders. Most obviously, all these modes have a static plate and a resonance inclusion, making the vibration energy localized in each unit cell. This is the reason why we call them local resonances.

In any of the local resonances, the cylinder and the spiral beam vibrate in a rigid plate as an internal spring-mass system, which can also be simplified as the system shown in Fig. 1(d). As explained in Sec. II, the frequencies of the local resonances only depend on the equivalent mass and stiffness, which almost remain unchanged in any direction. It follows that the bands corresponding to local resonances keep flat throughout all the directions, as shown in the band structure in Fig. 2.

To understand the band structure and the corresponding modes better, the flat bands induced by local resonance modes and other branches with high slopes corresponding to the traveling wave modes in the plate are painted in different colors according to the polarization directions of their modes. The local resonance mode A and the anti-symmetric Lamb wave mode $A_0$ are out-of-plane modes polarized in $z$ direction; the local resonance mode D and the symmetric Lamb wave mode $S_0$ are longitudinal in-plane modes polarized in $x$ direction; the local resonance mode E and the shear-horizontal wave mode $SH_0$ are transverse in-plane modes polarized in $y$ direction; the local resonance mode F is in-plane torsional mode; and the local resonance modes B and C are hybrid modes, having both in-plane and out-of-plane displacement components.

B. Formation mechanisms of the low-frequency BGs

When the frequency of incident elastic waves is close to one of the nature frequencies of the internal spring-mass system, the corresponding local resonance will be activated, with a reacting force applied on the plate against the harmonic elastic wave excitation. For the same reason explained in Sec. II, the vibration of the plate is reduced or even counteracted due to the interaction between the local resonances and the traveling wave modes. As a result, elastic waves with their frequency close to any nature frequency of the local resonances are not supported by the plate to propagate through the PC, which means the formation of BGs. Taking the local resonance mode A as an example, the internal spring-mass system resonates in $z$ direction and its nature...
frequency indicated by the corresponding flat band is around 42 Hz. When the frequency of the incident out-of-plane waves is close to 42 Hz, the Lamb wave mode $A_0$ of the plate and the mode $A$ of the spiral resonator are activated simultaneously and display a strong interaction because of their same polarization direction. In the band structure, the branches of the mode $A_0$ and $A$ cut of each other due to the strong interaction, with an out-of-plane BG open in the frequency range above the flat band. The BG spreads all over the frequency range (i.e., the bandwidth of BG), in which the reacting forces from the spiral resonators are still out-of-phase with the elastic wave excitation, and stops at the frequency where the interaction disappears and the antisymmetric Lamb mode $A_0$ is activated again. The displacement vector fields of mode $A_0$ are shown in Fig. 3(g).

Contrary to the local resonances, the cylinder keeps still while the plate vibrates with large amplitude, as if the whole plate is elastically supported on a static peg in this mode. As the shadow area shown in the band structure in Fig. 2, an out-of-plane BG from 42 Hz to 157 Hz is obtained.

However, the branches of wave modes $S_0$ and $SH_0$ cannot be cut off by the local resonance mode $A$, and no BG appears around them, due to the decoupling of in-plane and out-of-plane modes. It is the same for the cases between the in-plane torsional mode $F$ and the out-of-plane shear-horizontal wave mode $A_0$. For the two torsional modes, $B$ and $C$, the steel cylinder twists around an in-plane axis. Compared with the excitation from $S_0$ or $SH_0$ wave mode, the resonances-induced in-plane force components applied on the plate are very small. So there is no in-plane BG appearing around their flat bands. But a narrow pass band around 60 Hz is induced in the out-of-plane BG and breaks the gap into two parts, due to the out-of-plane displacement component of these two modes.

To illustrate the interaction of the in-plane modes, the band structure in $\Gamma X$ direction is enlarged to show details in Fig. 4(a). The frequency bands whose modes are polarized in the same direction are also painted in the same color. The two dashed lines originating from $\Gamma$ point represent the frequency bands of the symmetric Lamb wave mode $S_0$ and shear-horizontal wave mode $SH_0$ in a uniform thin plate. The other two horizontal dashed lines indicate the flat bands of local resonances $D$ and $E$. It can be observed clearly from the figure that the frequency bands of $S_0$ and $D$ cross each other at point $O_1$, and those of $SH_0$ and $E$ cross each other at point $O_2$. The corresponding modes display a strong interaction around the crossing point due to the same polarization direction, giving rise to repulsion between their frequency bands. So the actual frequency bands of $S_0$ and $SH_0$ in the band structure deviate from the dashed lines and connect with the flat bands, as the thick dotted blue and red lines shown in Fig. 4(a). It follows that a symmetric Lamb wave ($S_0$) and shear-horizontal wave ($SH_0$) BG are created in the frequency range above the flat bands $D$ and $E$, respectively.

It is interesting for us to consider the interaction of the shear-horizontal wave mode $SH_0$ and the longitudinal local resonances $D$, whose frequency bands cross each other at point $O_3$. Although they are polarized in different directions, a weak interaction exists around the crossing point, resulting in a small local gap between the two frequency bands. The weak interaction can be proved by the eigenmode shapes of the unit cell in the coupling area, which are the hybrid of the mode $D$ and $SH_0$, as shown in Figs. 4(b) and 4(c). The interaction between the modes polarized in different directions may be attributed to the anisotropy of the spiral beam. When the beam moves along $x$ (or $y$) direction following the steel cylinder, longitudinal and transverse deformations are produced simultaneously.

As mentioned above, the formation of BGs depends on the interaction between the local resonances and the traveling wave modes. The lower edge of the BG is determined by the corresponding local resonance frequency, and the bandwidth of the BG is relevant to the interaction strength, which depends on the polarization directions. The modes polarized in the same direction display a stronger interaction than those polarized in different directions.

C. Effects of the geometrical parameters on the BG

In engineering, the flexural vibration of plates is the widely considered motion in plate structures. So out-of-plane BGs of the proposed PC structures are considered in the

![Fig. 3](image-url)  
FIG. 3. Eigenmode shapes and displacement vector fields of the branches labeled in Fig. 2. (a) Mode $A$; (b) Mode $B$; (c) Mode $C$; (d) Mode $D$; (e) Mode $E$; (f) Mode $F$; and (g) Mode $A_0$.  

Downloaded 29 Apr 2013 to 117.32.153.162. This article is copyrighted as indicated in the abstract. Reuse of AIP content is subject to the terms at: http://jap.aip.org/about/rights_and_permissions
following parts. As explained above, the lower and upper edges of the out-of-plane BG are determined by the frequency of the local resonance A and the lowest frequency of the overtones mode $A_0$, respectively, which are relevant to the geometrical parameters of the structure. To illustrate the effects of geometrical parameters on the BG, we perform the calculation of band structures with one of the parameters changing. In the calculation, the lattice constant is fixed to $a = 30$ mm. Figures 5(a)–5(e) show the changes of the LRGB with the thickness of the plate, the width and angle of the spiral beam, the radius and height of the cylinder, respectively. The pass band induced by the flat bands B and C in the BG is ignored due to the narrow bandwidth except for the case in Fig. 5(c), in which the bands B and C stretch over a larger frequency range.

As shown in Figs. 5(a)–5(c), the lower and upper edges as well as the bandwidth of the out-of-plane BG increase nearly linearly with the increase of the thickness of the plate.
and the width of the spiral beam due to the increase of the equivalent stiffness, but decrease rapidly with the increase of the spiral angle of the beam, due to the decrease of the equivalent stiffness. The sensitivity of the upper edge to the equivalent stiffness of the plate and the spiral beam reveals that the vibrations of the upper edge modes are mainly in the plate, as anti-symmetric Lamb wave mode $A_0$ shown in Fig. 3(g).

From Figs. 5(d) and 5(e), one can observe that the effects of the radius and height of the cylinder are nearly the same. An increase in the radius or height of the steel cylinder significantly reduces the lower edge of the BG while the upper edge is slightly decreased slowly, giving rise to the increase of the bandwidth. Contrary to the case of changing the geometrical parameters of the plate, the lower edge of the BG is more sensitive than the upper edge to the variations in the size of the cylinder, which confirms that the lower edge of the BG depends on the local resonance of the cylinder. The equivalent mass of the local resonance system increases with the size of the cylinder, leading to the decrease of the local resonance frequency and the increase of the interaction strength between the local resonance and the traveling wave modes. It follows that the lower edge of the BG decreases while the bandwidth of the gap increases.

According to the analysis of the effects of geometrical parameters on the out-of-plane BG, one can choose optimal parameters of the structure to obtain the BG meeting the requirement of practical applications. However, changing the equivalent mass of the cylinder is a better choice to obtain a broad BG in low-frequency range.

IV. EVIDENCES OF THE LOW-FREQUENCY LRBG

A. Frequency response (FR) of the PC plate

To verify the low-frequency BG in the finite PC structures, the transmission properties of out-of-plane elastic waves in a PC plate with eight periodic spiral resonators are simulated by using the FE method. In the numerical model, there are eight layers of unit cells along $x$ direction and only one layer along $y$ direction. For one type of the structure model, free boundary conditions are used on the edges in $y$ direction. For another type, periodic boundary conditions are employed to simulate the structure arranged infinitely in $y$ direction. The harmonic excitation with its frequency sweeping from 0 to 300 Hz is applied perpendicularly on the plate surface near one side. The FRF of a test point on the plate surface near the other side and the FR of the steel cylinder in the first cell are shown in Fig. 6.

From the figure, it can be observed that both of the two FRF curves have a sharp drop in the frequency range from 42 to 150 Hz and a resonant peak around 60 Hz, which are in good agreement with the band structure shown in Fig. 2. The two resonant peaks appearing at points P1 and P2, whose frequencies correspond to the A, B, and C flat bands in the band structure, result from the out-of-plane resonances of the cylinder. It can be confirmed by the $z$-displacement FR of the first cylinder as the dotted curve shown in Fig. 6. We can find the vibration of the first cylinder has large amplitudes at the frequencies corresponding to the two resonant peaks P1 and P2. Determining the upper edge of the BG, the resonant peak at point P3 is caused by the resonant mode of the whole plate. For each of the eight unit cells, the PMMA plate resonates in $z$ direction while the cylinders almost keep still, as the mode $A_0$ shown in Fig. 3(g). But for the whole PC structure, eight piece plates are connected together in a row along $x$ direction, making the structure seems like a bridge supported on eight piers. Around these resonant peaks, there are three FRF troughs, T1, T2, and T3, which result from the anti-resonant modes of the plate and the attached eight spiral resonators. At the frequencies of these points, the vibration of the whole plate is counteracted by the vibration of spiral resonators most effectively, giving rise to the minima in the FRF curves.

By comparing the curves of the $y$-finite and $y$-infinite PC plates, we find boundary conditions in $y$ direction do not make a significant impact on the transmission properties of the finite PC plate. This is because the BG characteristics of the PC plate depend on the local resonances of single unit cells, which have nothing to do with the periodicity. However, some differences appear in the gap bottom between the peaks P1 and P2. It can be observed that the pit on the FRF curve of the $y$-infinite PC plate is deeper, because more counterforces from the adjacent resonant cylinders in $y$ direction act on the whole plate.

B. Experimental measurement of the transmission properties

To further verify the low-frequency BG of the proposed structure, we perform the transmission-measuring experiments of a PC plate with eight periodic spiral resonators. As explained above, boundary conditions in $y$ direction do not make a significant impact on the transmission properties. So we make the PC plate sample with only one layer in $y$ direction. The spiral beams are cut out from the PMMA plate by laser cutters. And steel cylinders are fixed in the center of the spiral beams, as the test sample shown in Fig. 7(a). In the transmission-measuring experiment, we place the samples in
vertical direction to avoid the influence of the weight of steel cylinders on the out-of-plane vibration of the PC plate. LMS test system with a vibration exciter (MB MODAL 2), a power amplifier, and two acceleration transducers (Dytran 3145AG) is employed in this experiment. As the setup in the numerical simulation, a harmonic force is applied perpendicularly on the plate surface near one side. Two acceleration transducers are placed on the spare surface, one on the excitation point and the other on a test point near the other end, as shown in Fig. 7(b). A white-noise signal with bandwidth from 0 to 300 Hz is amplified to drive the vibration exciter, which transmits vibrations to the left side of the PC plate. Then, flexural waves propagate through the plate to the right side, where the acceleration of the plate is measured by the acceleration transducer.

The FRF is worked out based on LMS Test.Lab software and shown as the blue solid curve in Fig. 8. As the same reason explained above, the attenuation of the BG in the frequency range from 42 Hz to 58 Hz is hardly noticeable due to the boundary effects. But a strong attenuation is found obviously in the frequency range above 60 Hz. Comparing with the FE calculation result (black solid curve), one can see that the peak frequencies of the experimental result in low-frequency range are in good agreement with those of the FE calculation result except for some differences in high frequency. The attenuation of the experimental result in high frequency range is larger than that of the FE calculation result, broadening the BG to high frequency range. These differences result from the effect of the material damping in the PMMA plate. To illustrate this, the transmission FRF of the PC plate is calculated by using the FE method, with different levels of damping added on the PMMA plate. The colored dashed lines in Fig. 8 represent the FRF of the PC plate with material damping coefficients of the PMMA plate $\eta = 0.05, 0.1,$ and $0.3$, respectively. We can find the FE calculation FRF curve in high frequency range moves close to the experimental result as the value of damping coefficient increases.

Based on the numerical and experimental results presented above, a broad out-of-plane BG in the low-frequency range, as the shadow area shown in Fig. 8, is obtained in the proposed PC plate with periodic spiral resonators.

V. CONCLUSIONS

In this paper, low-frequency BGs in a phononic crystal thin plate with periodic spiral resonators are investigated numerically and experimentally. The formation mechanisms of the BGs are explained based on the modal analysis of plate modes and local resonances. We find that the interaction between the local resonances and the traveling wave modes in the plate is responsible for the formation of the BG in low-frequency range. This interaction strength is relevant to the polarization directions of the two interactional modes and greatly affects the bandwidth of the BG, of which the lower edge depends on the corresponding local resonance frequency. It is shown that the lower and upper edge of the out-of-plane BG can be modulated by changing the geometrical parameters of the structure. By combining the numerical calculations and experimental measurements, the proposed PC plate is demonstrated to possess a broad out-of-plane BG in low-frequency range from 42 Hz to 150 Hz. The numerical calculation results are in good agreement with those of experimental measurement except for some differences due to the material damping of the PMMA plate. The
structure design and its results provide an effective way for phononic crystals to obtain broad BGs in low-frequency range, which has potential application prospects in the low-frequency noise and vibration reduction.

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