Novel optical super-resolution pattern with upright edges diffracted by a tiny thin aperture

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Abstract: In the past decade numerous efforts have been concentrated to achieve optical imaging resolution beyond the diffraction limit. In this letter a thin microcavity theory of near-field optics is proposed by using the power flow theorem firstly. According to this theory, the near-field optical diffraction from a tiny aperture whose diameter is less than one-tenth incident wavelength embedded in a thin conducting film is investigated by considering this tiny aperture as a thin nanocavity. It is very surprising that there exists a kind of novel super-resolution diffraction patterns showing resolution better than $\lambda/80$ ($\lambda$ is the incident wavelength), which is revealed for the first time to our knowledge in this letter. The mechanism that has allowed the imaging with this kind of super-resolution patterns is due to the interaction between the incident wave and the thin nanocavity with a complex wavenumber. More precisely, these super-resolution patterns with discontinuous upright peaks are formed by one or three items of the integration series about the cylindrical waves according to our simulation results. This novel optical super-resolution with upright edges by using the thin microcavity theory presented in the study could have potential applications in the future semiconductor lithography process, nano-size laser-drilling technology, microscopy, optical storage, optical switch, and optical information processing.

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References and links


1. Introduction

The past decade has seen numerous efforts to achieve optical imaging resolution beyond the diffraction limit. For near-field optics, evanescent waves have to be taken into account because the energy exchange between the vibration surface and the electromagnetic wave at the near field prevents all the near-field energy to be transferred totally to the far field. Recently, the evanescent waves containing fine detail of the electromagnetic field distribution were exploited to form subwavelength hotspots, such as near-field scanning optical microscope [1–3], various field concentrators [4–7], and negative index superlens [8–13]. More recently, instead of using evanescent waves in these works, the super-oscillatory phenomenon was found [14] and a super-oscillatory lens with a high-throughput binary masks was exploited for subwavelength imaging showing resolution better than $\lambda/6$ [15].

As the key concept of near-field optics, evanescent electromagnetic waves makes it difficult to use any simple approximation in Maxwell’s equations. In our previous paper [3], a theoretical model of near-field optics diffracted from a subwavelength aperture in a thin conducting film was proposed. As its continuous research, by considering the subwavelength aperture within a thin film as a thin microcavity, this paper first presents a thin microcavity theory to describe this kind of near-field optical diffraction and then a novel optical super-resolution pattern with upright edges is revealed.

2. The thin microcavity theory for near-field optical diffraction

For the near-field optical diffraction from a subwavelength aperture in a thin conducting film, the subwavelength aperture within the thin film could be considered as a thin microcavity. Therefore in this section, the governing equation of the electromagnetic field distribution in a thin microcavity under an external light incidence is derived in details. Because of the small thickness of the thin microcavity, we make the assumptions that the magnetic field component $H_z$ in the $z$ direction, i.e., perpendicular to the symmetric plane of the microcavity with the origin at the center, is equal along the thickness, and the other components in $x$ and $y$ directions are $H_x = z \partial H_x / \partial y$ and $H_y = -z \partial H_y / \partial x$, respectively. Thus the electric field components in the microcavity can be expressed as

$$
-\imath \omega \varepsilon E_i = \frac{\partial H_i}{\partial y} + \frac{\partial H_i}{\partial x}, \quad -\imath \omega \varepsilon E_2 = \frac{\partial H_2}{\partial y} - \frac{\partial H_2}{\partial x}, \quad -\imath \omega \varepsilon E_3 = -z \frac{\partial^2 H_3}{\partial y^2} - z \frac{\partial^2 H_3}{\partial x^2} \quad (1)
$$

where the parameter $\omega$ is the circular frequency, $\varepsilon$ is the permittivity of the microcavity material and $i$ is the imaginary unit. It can be easily verified that $\nabla \cdot H = 0$ and $\nabla \cdot E = 0$, thus all the Maxwell’s equations are satisfied. Comparing Maxwell’s equations $-\imath \omega E_i = \partial H_i / \partial y - \partial H_i / \partial z$ and $-\imath \omega E_2 = \partial H_2 / \partial y - \partial H_2 / \partial z$ with Eq. (1), combining $H_x = z \partial H_x / \partial y$ and $H_y = -z \partial H_y / \partial x$, we can confirm the above assumptions, because $\partial H_x / \partial z = H_x / z$ and $\partial H_y / \partial z = H_y / z$ are both correct for small thickness. Thus all the electromagnetic field components in the thin microcavity can be expressed with respect to $H_3(x, y)$.

Therefore, the electric field energy $U$ and the magnetic field energy $T$ can be expressed as
\[ U = \int \frac{1}{2} \epsilon |E|^2 dV = \frac{1}{2} \int \int \int_{\Omega/2} \epsilon |E|^2 dxdy = \frac{1}{2a \epsilon} \int \int_{\Omega/2} \left[ 2h \left( \frac{\partial H_z}{\partial y} \right)^2 + \left( \frac{\partial H_z}{\partial x} \right)^2 \right] + h^2 \left( \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} \right) dxdy \]  

(2a)

And

\[ T = \int \frac{1}{2} \mu |H|^2 dV = \frac{1}{2} \int \int \int_{\Omega/2} \mu |H|^2 dxdy = \frac{\mu}{2} \int \int_{\Omega/2} \left[ h^2 \left( \frac{\partial H_x}{\partial y} \right)^2 + \left( \frac{\partial H_x}{\partial x} \right)^2 \right] + hH_z^2 dxdy \]  

(2b)

where \( \mu \) is the permeability and \( h \) is the thickness of the microcavity.

With an incident light, time-harmonic electromagnetic field of the thin microcavity satisfies the power flow theorem, the complex form of the power flow theorem can be expressed as

\[ -\oint_S (E \times H^*) \cdot dS = i\omega \oint_S (B \cdot H^* - E \cdot D^*) dV + \int_v E \cdot J^* dV \]  

(3)

where \( H^*, D^*, J^* \) are the conjugate complex vectors of \( H, D, J \), respectively. Considering Eqs. (2a) and (2b), Eq. (3) can be written as

\[ -\frac{1}{2i\omega} \oint_S (E \times H^*) \cdot dS = T - U \]  

(4)

Due to the small thickness of the microcavity, we can ignore the surface integral term parallel to \( z \) direction. Thus, we can obtain that

\[ \oint_{S_1} (E \times H^*) \cdot dS = \oint_{S_2} (E \times H^*) \cdot dS + \oint_{S_3} (E \times H^*) \cdot dS \]  

(5)

where \( S_1 \) and \( S_2 \) indicate the upper and lower interface of the thin microcavity. It can be noticed that

\[ \oint_{S_1} (E \times H^*) \cdot dS = \oint_{S_1} \left( E_1 H_{2}^* - E_2 H_{1}^* \right) dS = -\int_{S_1} \left( \frac{h}{2} \frac{\partial H_z}{\partial x} E_1 \bigg|_{l = -\frac{h}{2}} - \frac{h}{2} \frac{\partial H_z}{\partial y} E_2 \bigg|_{l = -\frac{h}{2}} \right) dS \]  

(6)

and

\[ \oint_{S_2} (E \times H^*) \cdot dS = \oint_{S_2} \left( E_1 H_{2}^* - E_2 H_{1}^* \right) \cdot dS = -\int_{S_2} \left( \frac{h}{2} \frac{\partial H_z}{\partial x} E_1 \bigg|_{l = -\frac{h}{2}} + \frac{h}{2} \frac{\partial H_z}{\partial y} E_2 \bigg|_{l = -\frac{h}{2}} \right) dS \]  

(7)

Then Eq. (7) can be expressed as

\[ -\oint_{S_3} (E \times H^*) \cdot dS = -\int_{S_3} \left( \frac{h}{2} \frac{\partial H_z}{\partial x} E_1 \bigg|_{l = -\frac{h}{2}} + \frac{h}{2} \frac{\partial H_z}{\partial y} E_2 \bigg|_{l = -\frac{h}{2}} \right) dS + \int_{S_3} \left( \frac{h}{2} \frac{\partial H_z}{\partial x} E_1 \bigg|_{l = -\frac{h}{2}} + \frac{h}{2} \frac{\partial H_z}{\partial y} E_2 \bigg|_{l = -\frac{h}{2}} \right) dS \]  

(8)
According to the variation of Eq. (4), and considering the arbitrariness of \( \delta H_3 \), the following governing equation for the time-harmonic magnetic field of the thin microcavity can be obtained that

\[
\nabla^4 H_z - k^2 H_z = \frac{-3i\omega e}{\hbar^2} \left[ \frac{\partial E_{3u}}{\partial z} \Bigg|_{z=h/2} - \frac{\partial E_{3d}}{\partial z} \Bigg|_{z=-h/2} \right] \tag{9}
\]

where \( \nabla^4 = \nabla^2 \nabla^2 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \), \( k^2 = (\mu\varepsilon \omega^2 - k_0^2) \mu_0 \varepsilon_0 \omega^2 / \hbar^2 \), \( k \) is the propagation constant in the microcavity. \( E_{3u}(x, y, z) \) and \( E_{3d}(x, y, z) \) are the electric field components in the \( z \) direction on both sides of the microcavity, the subscripts \( U \) and \( D \) denote the upper and lower spaces, respectively. This derived equation has an obvious physical meaning that the magnetic field distribution inside the microcavity is determined by the difference between the electric field gradients at the interfaces.

### 3. The near-field optical diffraction from a tiny aperture

Here a near-field optical diffraction system that considers a perfect electrical conducting (PEC) thin film with a subwavelength aperture embedded will be investigated based on the thin microcavity theory. The sketch of the system is shown in Fig. 1.

![Fig. 1. Sketch of a thin film with a subwavelength size aperture.](image)

A plane wave vector \( k_0 (k_0 = k_{0s} = 2\pi/\lambda) \) illuminates the system along the \( z \) axis. For simplicity the theory will be described for an \( E \)-polarized field incident upon the perfectly conducting metallic film. The incident electric field is given by \( E_0 = E_{0s}y = \exp(ik_0z)y \), with the use of a time dependence in \( \exp(-i\omega t) \). In Fig. 1, the reflected and transmitted field components in the upper (region I) and lower (region III) space can be expressed, respectively, as

\[
\Phi'(r, \theta, z) = \sum_{n=-\infty}^{\infty} \int_0^\infty \Phi' (\rho) J_n (\rho r) e^{i\sqrt{k_0^2 - \rho^2} (z-h/2)} \rho d\rho e^{i\omega t} \tag{10a}
\]

\[
\Phi'(r, \theta, z) = \sum_{n=-\infty}^{\infty} \int_0^\infty \Phi' (\rho) J_n (\rho r) e^{-i\sqrt{k_0^2 - \rho^2} (z+h/2)} \rho d\rho e^{i\omega t} \tag{10b}
\]
where $J_n$ is the Bessel function of the first kind, $\Phi'(\rho)$ is the Hankel transform of $\Phi'(r,0,h/2)$ defined as $\Phi'(\rho) = \int_0^\infty \Phi'(r,0,h/2) J_n(\rho r) r dr$, and $\Phi'(\rho)$ is the Hankel transform of $\Phi'(r,0,-h/2)$ defined as $\Phi'(\rho) = \int_0^\infty \Phi'(r,0,-h/2) J_n(\rho r) r dr$. Here $\Phi'$ represents for $E_{1U}, E_{2U}, H_{1U}$ and $H_{2U}$, and $\Phi'$ does for $E_{1D}, E_{2D}, H_{1D}$ and $H_{2D}$. Eqs. (10a) and (10b) both satisfy the Helmholtz equation and include all the evanescent waves reflected and transmitted, respectively.

Thus the magnetic field component $H_3$ in region II (vacuum) can be solved from Eq. (9) in a simple way as

$$H_3(r, \theta) = \sum_{n=0}^{\infty} \left[ A_n J_n(kr) + C_n I_n(kr) + F_n \cdot r^n \right] \exp(in\theta)$$

(11a)

where

$$F_n = \frac{3k^2}{\pi} \exp(ik_0 \rho/2) \frac{(-1)^{n+1} - 1}{(n+1)^2 \pi (2n+1)}$$

(11b)

and $I_n$ is the modified Bessel function of the first kind, the coefficient $A_n$ and $C_n$ can be determined by the PEC boundary conditions $H_3(r, \theta) \mid_{r=a} = 0$ and $\partial H_3 / \partial r \mid_{r=a} = 0$.

Here notice that the relation $1/2 = \sum_{n=0}^{\infty} [1 - (-1)^n] \sin(n\pi) / n\pi$ is used when Eq. (11a) is derived. Therefore, all the electromagnetic components in Regions I, II, and III can be obtained by using the Hankel transform and the continuous boundary conditions at the interfaces.

For the incident $E$-polarized plane wave $E_0 = E_{0y} y = \exp(ik_0 y - i\omega t) y$, by using the thin microcavity theory presented above, we calculate the transmitted electric fields at different observation points $(r_0, \theta_0, z_0)$ with the following parameters: film thickness $h = 0.2 nm$, and wavelength of the polarized incident plane wave $\lambda = 20 nm$ in Figs. 2 and 3.

Figure 2 shows the optical super-resolution pattern of total transmitted electric field magnitude by a less-than-one-tenth-wavelength aperture in a thin conducting film under different radii of the aperture along the radial distance at $z_0 = -14 nm$ and $\theta_0 = \pi / 2$. From Fig. 2(a) it can be seen that there is only one abrupt peak with upright edge when the aperture radius $a = 0.660 nm$, and the width of the peak is just 0.25 nm, i.e. $\lambda/80$. When the radius is increased to $a = 0.666 nm$, there are two abrupt peaks with upright edge, the width of the first peak is enlarged to 0.70 nm and another is only 0.20 nm, i.e. $\lambda/100$. With the radius increasing to $a = 0.668 nm$, three upright peaks appear with different widths which are 0.80 nm, 0.40 nm, and 0.125 nm one by one, and the minimum resolution is up to $\lambda/160$. Additionally, in our calculation these discontinuous upright peaks in Fig. 2(a) are formed extremely mainly only by $n = 1$ in Eq. (12), or only by one item of the integration series about the cylindrical waves. Figs. 2(b) and 2(c) show the transition process of the super-resolution pattern from discontinuous peaks to continuous peaks when the radius of the aperture is increased from $a = 0.680 nm$ to $0.800 nm$. At $a = 0.680 nm$, there is a series of upright pulses with different widths from $r = 0$ to $r = 17 nm$, which are also formed only by $n = 1$ in Eq. (12). Meanwhile, when $a = 0.700 nm$, there appears a series of upright pulses all over the radial distance from $r = 0$ to $r = 50 nm$, which are formed mainly by $n = 2, 3, 4$ in Eq. (12). However, at $a = 0.750 nm$, the transmitted electric field magnitude is becoming continuous only with
several pin peaks varying along the radial distance, and here \( n = 2, 3, \) and 4 still play extreme role in Eq. (12). When \( a = 0.800\,nm \), it’s totally becoming continuous with some near-field diffractional peaks, which is formed by the interaction among most items of the integration series about the cylindrical waves in Eq. (12).

Corresponding to the sectional results of Fig. 2, Fig. 3 shows the three-dimensional (3D) super-resolution diffraction patterns of the total transmitted electric field magnitude by the less-than-one-tenth-wavelength apertures with different radii along the radial distance at \( z_0 = -14\,nm \). At \( a = 0.668\,nm \), a 3D discontinuous polarized diffraction pattern is displayed in Fig. 3(a), which is corresponding to the lowest plot of Fig. 2(a). When \( a = 0.680\,nm \), Fig. 3(b) also shows a 3D discontinuous polarized diffraction pattern corresponding to the first plot of Fig. 2(b). When the radius of the aperture is increased to \( a = 1.000\,nm \), the 3D diffraction pattern in Fig. 3(c) is becoming totally continuous, but the polarization phenomenon is weakened. From Fig. 3(c), it can be found that there are two intensity lobes along the aperture edge for the total electric field due to the polarized incident wave. This result is very similar with the experimental result for much larger parameters in the previous literature [3, 16]. When \( a = 1.500\,nm \) in Fig. 3(d), the 3D diffraction pattern is continuous without the polarization phenomenon.

These novel optical refined patterns diffracted by a less-than-one-tenth-wavelength aperture within a thin conducting film can be explained by the interaction between the polarized incident plane wave and the tiny aperture, which is a resonant transmission phenomenon.

Figure 4 shows the maximum transmitted electric field magnitude varying with different aperture radii under the different film thicknesses at the incident wavelength \( \lambda = 20\,nm \), and the observation plane \( z_0 = -14\,nm \) with \( \theta_0 = \pi/2 \). It can be seen that the maximum transmitted electric field magnitudes have a steep decreasing from 1.93329E-4, 1.9996E-4, and 2.47609E-
Fig. 2. Optical refined pattern diffracted by a less-than-one-tenth-wavelength aperture in a thin conducting film with different radii.

(a)

(b)

(c)

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Fig. 3. Three-dimensional super-resolution diffraction patterns of the total transmitted electric field magnitude with different aperture radii along the radial distance at $z_0 = -14 \text{nm}$, (a) $a = 0.668 \text{nm}$, (b) $a = 0.680 \text{nm}$, (c) $a = 1.000 \text{nm}$, and (d) $a = 1.500 \text{nm}$.
4 at the aperture radius \( a = 0.661 \text{ nm}, 0.6585 \text{ nm}, \) and \( 0.652 \text{ nm} \) to \( 1.13233 \times 10^{-24}, 1.0698 \times 10^{-24}, \) and \( 1.02369 \times 10^{-24} \) at the aperture radius \( a = 0.6605 \text{ nm}, 0.658 \text{ nm}, \) and \( 0.651 \text{ nm} \) when the film thickness \( h = 0.1 \text{ nm}, 0.2 \text{ nm}, \) and \( 0.5 \text{ nm} \), respectively. With the film thickness increasing, the critical aperture radius with this kind of steep decreasing is becoming lower. Thus the aperture volume could keep the resonant condition with the incident wavelength, from which the resonant transmission phenomenon can be also verified.

![Fig. 4. Maximum transmitted electric field magnitude varying with different aperture radii under the different film thicknesses at the incident wavelength \( \lambda = 20 \text{ nm} \), \( z_0 = -14 \text{ nm} \), and \( \theta_0 = \pi/2 \).](image)

![Fig. 5. The total transmitted electric field magnitude by a tiny aperture within a thin conducting film with the thickness \( h = 0.2 \text{ nm} \) under different incident wavelength at the observation plane \( z_0 = -14 \text{ nm} \) with \( \theta_0 = \pi/2 \).](image)

Figure 5 shows the transmitted optical pattern by a tiny aperture within a thin conducting film with the thickness \( h = 0.2 \text{ nm} \) under the different incident wavelength \( \lambda = 15 \text{ nm}, 20 \text{ nm}, \) and \( 30 \text{ nm} \) at the observation plane \( z_0 = -14 \text{ nm} \) with \( \theta_0 = \pi/2 \). When the incident wavelength is decreased from \( 30 \text{ nm} \) to \( 15 \text{ nm} \), the abrupt peaks with upright edges are
becoming narrower and higher. In this case, although the resonant condition is still kept to some extent, the transmitted peaks are changed. For example, at $\lambda = 15$ nm there are only two narrow peaks, and the narrow peaks would become narrower and narrower so that there would be no peak left when the incident wavelength is decreased so that the resonant condition is not satisfied. On the other hand, at $\lambda = 30$ nm there are three low peaks, and there is also no peak left when the incident wavelength is increased so that the low peaks are becoming lower and lower without satisfying the resonant condition.

This kind of novel optical super-resolution pattern diffracted by a tiny thin aperture presented in Figs. 2 and 3 is with upright edges and with much higher transmitted magnitude, which could be applied in the future semiconductor lithography process, nano-size laser-drilling technology, and so on.

In addition, Fresnel diffraction will happen for this system in Fig. 1 when the radius of the aperture is enlarged and the distance far from the thin film is increased. Here by using the thin microcavity theory presented above we also plot the total transmitted electric field magnitude along the radial distance with the parameters $a = 30$ nm, $h = 0.2$ nm, and $z_0 = -100 \text{nm}$, as shown in Fig. 6, which is very similar to the general Fresnel diffraction pattern.

![Fig. 6. The far-field Fresnel diffraction pattern for the total transmitted electric field along the radial distance at $z_0 = -100 \text{nm}$, $a = 30$ nm, $h = 0.2$ nm.](image)

### 4. Conclusion

In conclusion, the thin microcavity theory of near-field optics is presented by using the power flow theorem firstly, then the near-field optical diffraction from a tiny aperture whose diameter is less than one-tenth incident wavelength embedded in a thin conducting film is investigated by considering this tiny aperture as a thin nanocavity, and a kind of novel super-resolution diffraction patterns with upright edges is revealed showing resolution better than $\lambda/80$ ($\lambda$ is the incident wavelength). This kind of super-resolution diffraction patterns with upright edges could have potential applications in the future semiconductor lithography process, nano-size laser-drilling technology, microscopy, optical storage, optical switch, and optical information processing.